AN EXPLICIT INVERSION TECHNIQUES FOR USE IN SHALLOW WATER

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ABSTRACT

The extraction of bottom properties by inversion of sound field data usually requires the use of either iterative or matrix inversion techniques. Straightforward application of Weston's effective depth approximation is used here to show how a comparison between measured and predicted pressure fields can be used to infer the sediment properties using a simple explicit technique. The water depth can also be found separately if not known. It is shown how the method can be applied to problems with arbitrary (but gradual) variation of the parameters in range, by means of the adiabatic approximation.

1. INTRODUCTION

The problem of measuring the geo-acoustic properties of the ocean bottom is a long-standing one, and the difficulty of performing experiments on the sea-bed means that the measurements are often carried out remotely by inverse methods [1].

This paper describes a method of inverting the sound field by a simple and explicit method to obtain the sediment density ρ and its sound speed (or more precisely the ratio $\rho/\sin\theta_c$ where θ_c is the critical angle). In its simplest form, the method relies on accurate knowledge of the water depth H, but a variant using broadband measurements is able to extract the water depth separately.

The approach is basically to compare the measured (cw) sound field with a numerical prediction (using nominal sediment properties). The difference between the observed and predicted beat patterns is then used to calculate the true geo-acoustic parameters. Given the sediment type, trial values suitable for use as nominal parameters can be obtained from the literature [2-4].

The method is illustrated here by measuring the properties of fine sand in a laboratory tank, using a frequency of 500kHz in very shallow water (10mm depth). The results are found to be in good agreement with the actual sand parameters which had been measured previously [5].

2. RANGE-INDEPENDENT CASE (FLUID BOTTOM)

2.1 Theory

We consider first the simple case when the sound field consists entirely of two beating modes in water of constant depth. The beat spacing Δr is determined by the difference between the two eigenvalues

$$\Delta r = 2\pi/(\kappa_1 - \kappa_2)\,. \tag{2.1}$$

For a sediment with critical angle θ_c and specific gravity ρ, the eigenvalues can be found using Weston's effective depth approximation [6]

$$k\,(H + \rho/k\sin\theta_c)\,\sin\theta_n = n\pi \tag{2.2}$$

where H is the water depth, k is the acoustic wavenumber in the water and θ_n is the eigen-angle, equal to $\cos^{-1}\kappa_n/k$. Approximating κ_n by $k(1 - \frac{1}{2}\sin^2\theta_n)$, eq (2.1) then becomes

$$\Delta r \approx \frac{4k}{3\pi}\,(H + \varepsilon/k)^2 \tag{2.3}$$

where

$$\varepsilon = \rho/\sin\theta_c\,. \tag{2.4}$$

Assuming that the source and receiver are both nearer the surface than the bottom, the condition for destructive interference between the first two modes to occur at range r_j is easily seen to be

$$r_j = \left(j - \frac{1}{2}\right)\Delta r = \frac{2(2j-1)}{3\pi}\,k\,(H + \varepsilon/k)^2 \tag{2.5}$$

Now consider two sets of r_j values: The first $\{(r_j)_0\}$ based on a numerical prediction of the field using trial sediment parameters; the second $\{(r_j)_{meas}\}$ taken from an experimental measurement. Defining δr as the difference between the two

$$\delta r = (r_j)_{meas} - (r_j)_0 \tag{2.6}$$

it follows that

$$\begin{aligned}(H + \varepsilon/k)_{meas} &= (H + \varepsilon/k)_0\left[1 + \delta r/(r_j)_0\right]^{1/2}\\ &\approx (H + \varepsilon/k)_0\left[1 + \delta r/2(r_j)_0\right]\end{aligned} \tag{2.7}$$

where the zero subscript refers to the trial parameter values and $(H + \varepsilon/k)_{meas}$ represents the new measurement. Although we have derived eq (2.7) for the rather special two mode case here, it is shown in the Appendix to apply more generally.

We now proceed to make use of eq (2.7), initially for the case when the water depth H is known.

2.2 Known Water Depth

If the water depth is given, then the only unknown is ε_{meas} and eq (2.7) can be rearranged as

$$\varepsilon_{meas} = \varepsilon_0 + (kH + \varepsilon_0)\, \delta r/2(r_j)_0. \tag{2.8}$$

An example of the application of this formula is shown in Fig 1 for a water depth of 9.87mm (3.34 wavelengths at 500kHz). The trial values used for the sediment parameters were ρ=1.90 and θ_c = 24.6° (corresponding to a sound speed ratio of 1.1), giving ε_0 = 4.56, and the transmission loss predicted using these values is plotted vs range in Fig 1a alongside the laboratory measurement.

The fractional range shift $\delta r/r_j$ was estimated from Fig 1a to be -0.0248 and our prediction for ε (from eq 2.8) is therefore ε_{meas} = 4.24

Two more numerical predictions were made, both with this predicted value of ε, but using two different densities. In one the density was kept fixed (ρ=1.90), corresponding to a critical angle of 26.6° (see Fig 1b) and in the other the trial critical angle of 24.6° was used, giving ρ=1.77 (Fig 1c). For reasons which are not yet clear, the agreement in Fig 1b is significantly better than that in Fig 1c. The actual values of the two floating parameters as measured in Ref 5 are ρ=1.90 and θ_c=27.5°, so that ε is equal to 4.12, close to our prediction here of 4.24.

2.3 Unknown Water Depth

If the water depth is not known, then in the unlikely event that $\rho/\sin\theta_c$ is known accurately, eq (2.7) can be rearranged as

$$H_{meas} = H_0 + \frac{\delta r}{2r_j}(H_0 + \varepsilon/k)\ . \tag{2.9}$$

A more likely scenario though is that the bottom properties are also unknown and for this we revert to eq (2.7). The quantity $\delta r/r_j$ can still be measured as before, but in order to separate the H measurement from that of ε, the experiment must be repeated at several frequencies. If the quantity $(H + \varepsilon/k)_{meas}$ is then plotted vs 1/k the resulting graph should look like Fig 2 with intercept H and gradient $\rho/\sin\theta_c$.

Note that the source (strength) does not necessarily need to be calibrated because the alignment of interference nulls can still be carried out with an unknown source level.

3. RANGE-DEPENDENT CASE (FLUID BOTTOM)

If either the water depth or the sediment properties vary with range, it can be shown by using the adiabatic approximation that, provided the variations are gradual, and using the shorthand $D(r) = H(r) + \varepsilon(r)/k$ eq (2.7) becomes [7]

$$D_{meas}(r) = D_0(r)\left[1 + \frac{D_0^2(r)}{2}\frac{d\left(\delta r / D_0^2(r)\right)}{dr}\right]. \tag{3.1}$$

An example of the application of this formula is shown in Fig 3. It corresponds to a case for which the sediment properties were known, so that eq (3.1) for $D_{meas}(r)$ amounts to a measurement of the bathymetry profile $H(r)$. Full details of the calculation can be found in Reference 7 (Figs 10, 12 and surrounding discussion).

In the general case, $\varepsilon(r) = \rho/\sin\theta_c$ is also an unknown function of range and the procedure for measuring $H(r)$ and $\varepsilon(r)$ separately is then as follows (following Section 2.2)

i) Calculate $D_{meas}(r)$ from eq (3.1) as in Fig 3.
ii) Repeat at several frequencies.
iii) At each range, plot a graph of $D_{meas}(r)$ vs $1/k$ as in Fig 2. The intercept is $H(r)$ and the gradient $\varepsilon(r)$.

4. SOLID BOTTOM

The discussion so far has assumed a fluid sediment throughout. However, all the derivations can be applied equally well to a (low shear speed) solid bottom by redefining the parameter ε as [8, 9]

$$\varepsilon = \frac{\rho\left(1 - 2c_s^2/c_0^2\right)^2}{\sin\theta_c} \tag{4.1}$$

where c_s is the sediment shear speed and c_0 the sound speed in the water ($c_0 = 2\pi f/k$). The critical angle θ_c is that for the sediment compressional wave speed c_p so that $\cos\theta_c = c_0/c_p$.

5. ATTENUATION COEFFICIENT

Measurement of the sediment attenuation coefficient is best carried out at a relatively low frequency when a single mode only can propagate so that

$$p(r) = \text{constant} \times e^{-\beta r}/r^{1/2} \,. \tag{5.1}$$

It is a straightforward matter to measure the decay rate β from a graph of pressure vs range (or ideally log $(pr^{1/2})$ vs r) and the sediment attenuation coefficient α (in Np/λ) can then be found from the relationship (putting m=1 into eq (A7) of Reference 7)

$$\beta = \frac{\pi\,\alpha\,\rho\,\cot^3\theta_c\,\sec\theta_c\left[1-(\pi/kD)^2\right]^{-1/2}}{2k^2D^2H\left[\sigma^3+\sigma\,(\varepsilon\pi/kD)^2+\varepsilon/kH\right]} \tag{5.2a}$$

where

$$\sigma = \left[1-(\pi/kD\sin\theta_c)^2\right]^{1/2} . \tag{5.2b}$$

For a fluid bottom, α in eq (5.2) is simply α_p, the compressional wave attenuation coefficient (in units of nepers per wavelength). For a solid bottom, assuming the shear speed c_s is small compared with the compressional wave speed, it is given by (eq 15 of Reference 9)

$$\alpha = \alpha_p + 4(c_s/c_0)^3\sin\theta_c\tan\theta_c\,(2\alpha_s + k\sin\theta_c) \tag{5.3}$$

where c_0 $(=2\pi f/k)$ is the sound speed in water and α_s is the shear wave attenuation coefficient.

6. DISCUSSION (PROS AND CONS)

The main advantage of the proposed inversion technique is that it gives the answer in an explicit form. There is also no need to calibrate the strength of the source because the method relies on alignment of interference nulls and not of absolute levels. The method copes with a solid bottom (provided that the shear speed is small) and with an environment whose parameters vary (gradually) in range.

The main disadvantage is that on its own, the technique cannot distinguish between density ρ, compressional speed c_p and shear speed c_s. Instead it collects them all into a single parameter ε given by eq (4.1). Furthermore the approach relies on the existence of a sufficiently thick homogeneous sediment.

It should be pointed out that it is essential that the theoretical predictions used be free of phase errors. If the predicted beat positions are not in the right place for the trial parameters, then the measured values will be in error. The numerical models used here were SNAP [10] and IFD [11], and care was taken to ensure that phase errors were negligible in both for the cases considered [7].

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APPENDIX - BOTTOM INVERSION FOR MULTIPLE MODES

Eq (2.7) of the main text, relating the interference range shift r'-r to a change in effective water depth D'-D, was derived for a simple situation with just two propagating modes. In fact it applies more generally than this (for an arbitrary number of modes) as shown below.

The phase difference Δ_{nm} between any pair of modes n and m at range r is

$$\Delta_{nm} \equiv (\kappa_n - \kappa_m)r = kr\,(\cos\theta_n - \cos\theta_m)\,. \tag{A1.1}$$

Using the effective depth formula for the eigenvalues (eq 2.2) and assuming small angles we then have

$$\Delta_{nm} = \frac{\pi^2(m^2-n^2)r}{2kD^2} \tag{A1.2}$$

$$D = H + \rho/k\sin\theta_c\,. \tag{A1.3}$$

For a slightly different depth D', the same phase difference occurs at a different range r' given by

$$\Delta_{nm} = \frac{\pi^2(m^2-n^2)r'}{2kD'^2}\,. \tag{A1.4}$$

Equating right hand sides of eqs (A1.2), (A1.4) and rearranging for r' gives

$$r' = rD'^2/D^2 \tag{A1.5}$$

independent of both n and m. In other words if a given interference feature (say a null) occurs at range r with effective depth D, then the same feature will appear slightly displaced (at range r') for a depth D'. Rearranging eq (A1.5) for D' we then have

$$D' = D[1 + (r'-r)/r]^{1/2} \tag{A1.6}$$

which is identical in form to eq (2.7) and has been derived more generally for an arbitrary number of modes.

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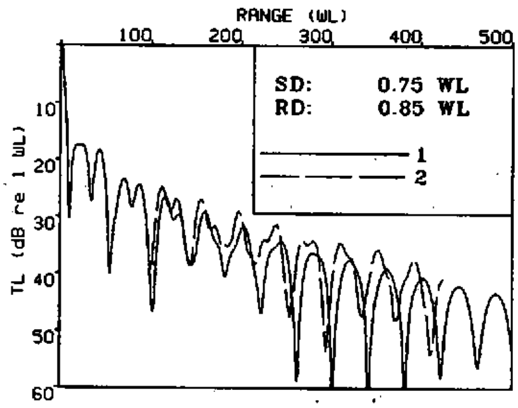
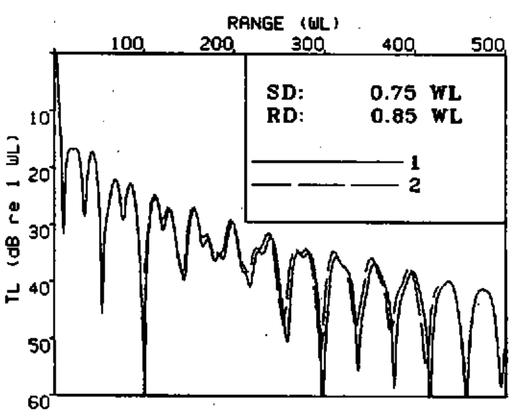
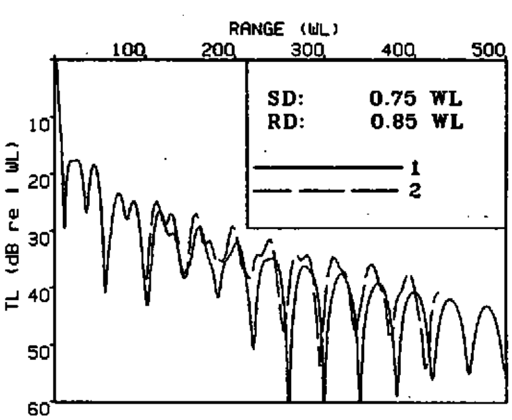


a) **Trial values (ρ = 1.90, θ_c = 24.6°)**



b) **Modified sound speed (θ_c = 26.6°)**



c) **Modified density (ρ = 1.77)**

***Figure 1** -*
Theory (___1) vs experiment (_ _ _2) for three sets of range-independent parameters (cw)

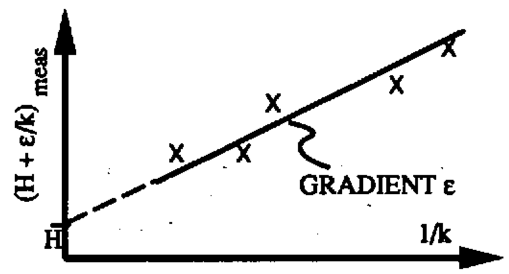
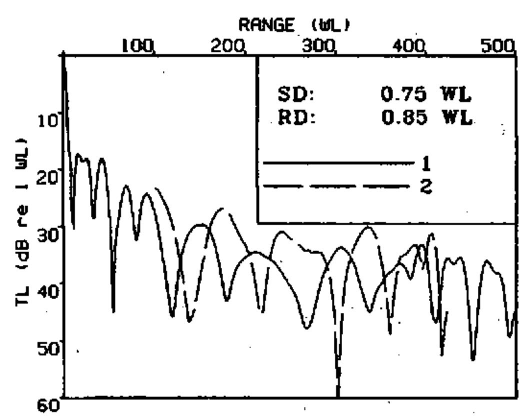
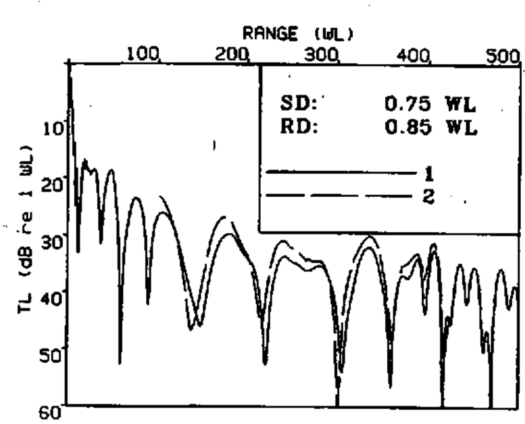


Figure 2 - Graph of $(H+\varepsilon/k)_{meas}$ vs $1/k$. The intercept is the water depth H and the slope is $\varepsilon = \rho/\sin\theta_c$



a) **Trial bathymetry**



b) **Modified bathymetry**

***Figure 3** -*
Theory (___1) vs experiment (_ _ _2) for two sets of range-dependent parameters (cw)