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REFLECTION OF PLANE WAVES FROM INHOMOGENEOUS SOLID SEDIMENTS

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1. INTRODUCTION

In order to be able to use acoustic or seismic methods to probe inhomogeneous media such as the sea-bed it is first necessary to understand and predict how the reflected pressure field varies with the sediment properties. A common way of quantifying this is by means of the plane wave reflection coefficient (denoted E: the ratio of reflected to incident power) and a number of computer programs exist for evaluating this quantity (or bottom loss $BL = -10 \log_{10}E$) numerically for arbitrary stratification of density, compressional and/or shear sound speed and attenuation [1, 2].

In Reference 3, a very simple analytical method, involving power addition over ray multipaths, was introduced for calculating BL in the case of inhomogeneous fluid sediments. The results and conclusions of Reference 3 are summarised in Section 2. The purpose of this present paper is to generalise that result to the case of a solid sediment and this is done in Sections 3 and 4. Advantages and disadvantages of the technique are discussed in Section 5, with particular reference to the assumption of incoherence implicit in the power addition.

2. FLUID-FLUID-SOLID

As mentioned above , Reference 3 assumes that the inhomogeneous sediment behaves as a fluid. Nevertheless, the method does allow for the possibility of a homogeneous solid substrate and for this reason, the result is referred to here as FFS for Fluid (water) - Fluid (sediment) - Solid (substrate). For the geometry shown in Figure 1 the reflection coefficient is

$$E_{FFS} = R_{12} + \frac{R_{23}(1-R_{12})^2}{1-R_{23}R_{12}} \tag{1}$$

with R_{12}, R_{23} (denoted R, $10^{-A/10}$ in equation (2.1) of Reference 3) defined as in the caption of Figure 1. The definition of R_{23} incorporates the effect of volume attenuation in the sediment. An example of the application of this formula can be seen in Figure 2.

The simplicity of equation (1) and its success in describing the broad bottom loss features predicted by the exact numerical model SAFARI [2] encourages a similar approach for more complicated problems. In the next section we examine the effect of allowing the sediment to support shear waves.

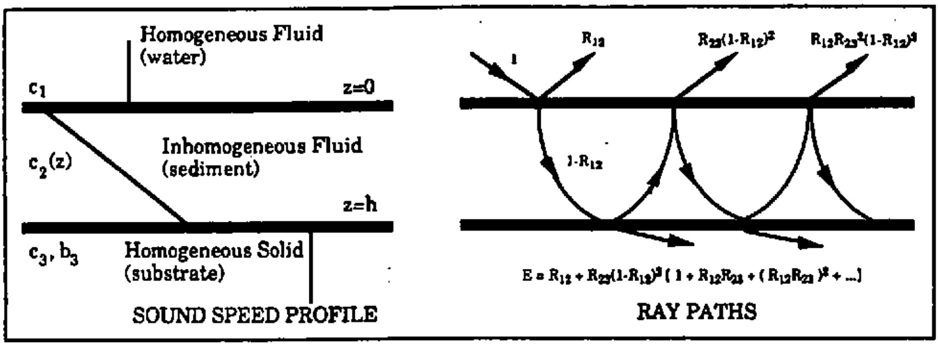

Figure 1 - Sound speed profile and ray diagram for calculating the FLUID-FLUID-SOLID reflection coefficient. The basement reflection coefficient R_{23} incorporates volume attenuation in the sediment

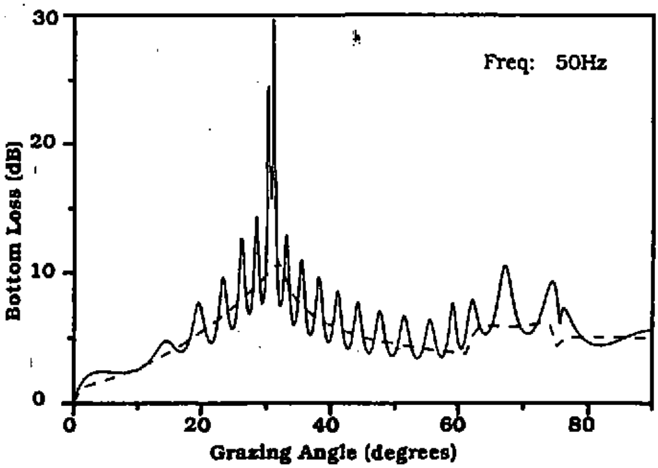

Figure 2 - Bottom loss calculated by SAFARI (solid line) and by power addition (dashed line) for a clay sediment overlying a basalt substrate. Details of parameters used are given in Reference 3

3. FLUID-SOLID-SOLID

Letting the sediment layer become a solid results in a proliferation of new multipaths (Figure 3) which complicate the problem considerably.

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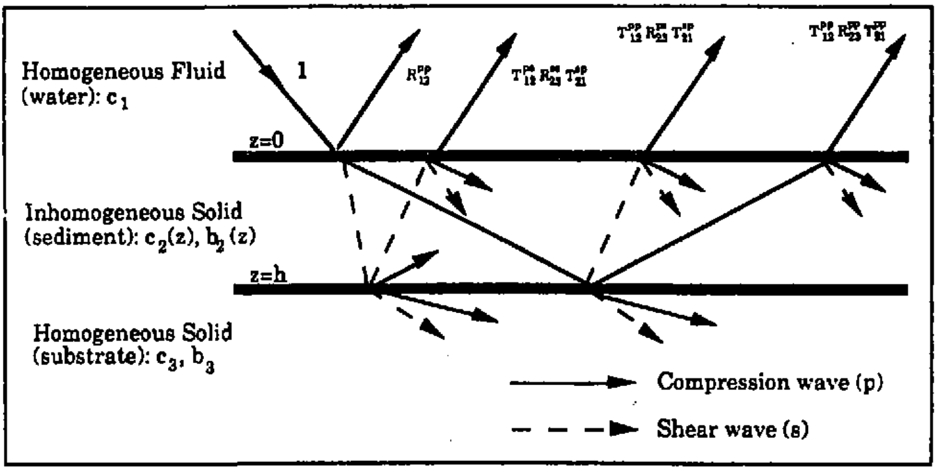

Figure 3 - Ray diagram for the FLUID-SOLID-SOLID case. In this and all subsequent Figures the ray paths are drawn straight for simplicity but in general they are curved according to the sediment profiles $c_2(z)$, $b_2(z)$

To cope with the extra degrees of freedom we introduce the new notation as illustrated by Figure 4.

R_{ij}^{pp} (R_{ij}^{ss}) - Compressional-p (shear-s) reflection coefficient

R_{ij}^{ps} (R_{ij}^{sp}) - p to s (s to p) conversion coefficient: reflection.

T_{ij}^{pp} (T_{ij}^{ss}) - p (s) transmission coefficient.

T_{ij}^{ps} (T_{ij}^{sp}) - p to s (s to p) conversion coefficient: transmission.

A simple non-trivial generalisation of equation (1) above can be obtained by assuming shear waves are generated at each boundary interaction but that they do not reach the next boundary (because of high shear attenuation) and hence do not contribute to the energy returned to the water. The result is

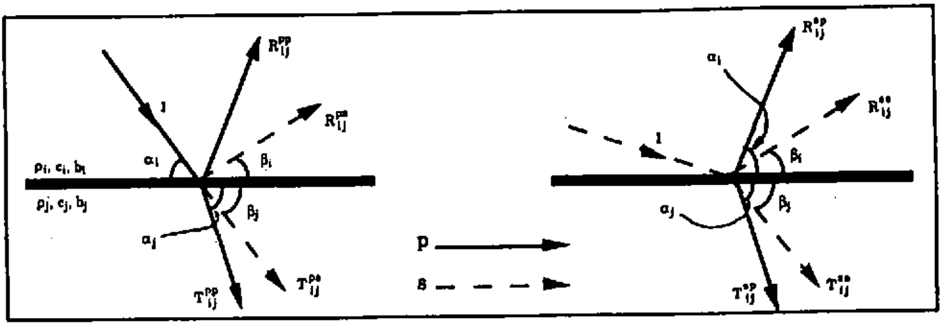

Figure 4 - Ray diagram illustrating the definition of the R and T coefficients

$$E = R_{12}^{pp} + T_{12}^{pp} R_{23}^{pp} T_{21}^{pp} \sum_{n=0}^{\infty} [R_{21}^{pp} R_{23}^{pp}]^n$$

$$= \quad R_{12}^{pp} + \frac{T_{12}^{pp} R_{23}^{pp} T_{21}^{pp}}{1 - R_{21}^{pp} R_{23}^{pp}} \quad . \qquad (2)$$

Note that the FFS result equation (1) can be recovered from equation (2) by using the fluid-fluid relationships

$$T_{12}^{pp} = T_{21}^{pp} = 1 - R_{12}^{pp} \quad \text{and} \quad R_{12}^{pp} = R_{21}^{pp}.$$

While equation (2) doubtless has its applications for realistic sediments, in order to make further progress, a slightly different approach is necessary. To reduce the number of paths which must be considered explicitly, consider the first pair of transmitted rays as secondary sources of plane waves (Figure 5) such that the compressional (shear) source σ_p (σ_s) contributes a proportion P(S) of its energy to the reflected field. In other words

$$E_{FSS} = R_{12}^{pp} + PT_{12}^{pp} + ST_{12}^{ps}. \qquad (3)$$

The fractions P and S can be found by analysis of the ray paths of Figure 6

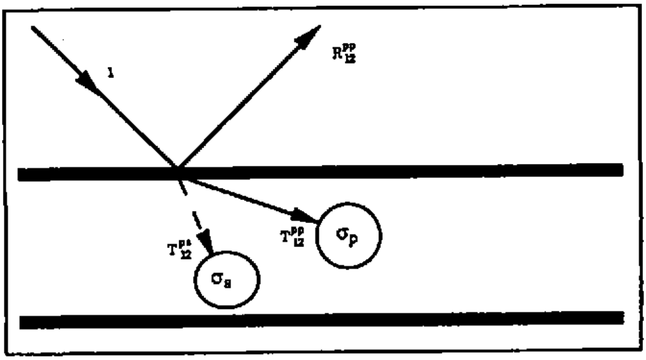

Figure 5 - Ray diagram for equation (3). The incident ray generates a reflected ray of strength R_{12}^{pp} and two secondary source σ_p and σ_s

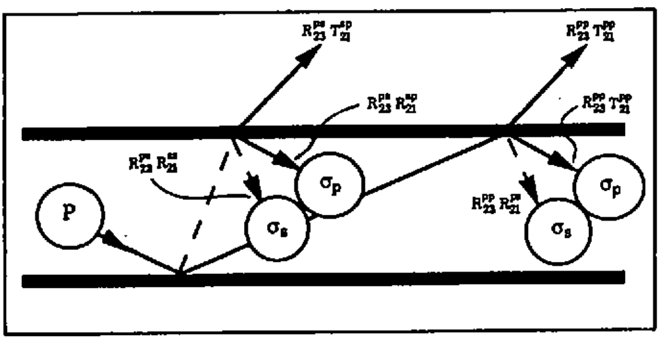

Figure 6 - Ray diagram for equation (4). Each secondary source generates two reflected rays and four more secondary sources

from which we can see that

$$P = R_{23}^{pp}\left[\, T_{21}^{pp} + PR_{21}^{pp} + SR_{21}^{ps} \,\right] + R_{23}^{ps}\left[\, T_{21}^{sp} + PR_{21}^{sp} + SR_{21}^{ss} \,\right] \quad (4)$$

and similarly

$$S = R_{23}^{sp}\left[\, T_{21}^{pp} + PR_{21}^{pp} + SR_{21}^{ps} \,\right] + R_{23}^{ss}\left[\, T_{21}^{sp} + PR_{21}^{sp} + SR_{21}^{ss} \,\right]. \quad (5)$$

Equations (4) and (5) can be rewritten in matrix form

$$\begin{pmatrix} \rho^{pp}-1 & \rho^{ps} \\ \rho^{sp} & \rho^{ss}-1 \end{pmatrix} \begin{pmatrix} P \\ S \end{pmatrix} = -\begin{pmatrix} \tau^{pp} \\ \tau^{sp} \end{pmatrix} \tag{6}$$

where

$$\rho^{\alpha\beta} = R_{23}^{\alpha p} R_{21}^{p\beta} + R_{23}^{\alpha s} R_{21}^{s\beta}$$

$$\tau^{\alpha\beta} = R_{23}^{\alpha p} T_{21}^{p\beta} + R_{23}^{\alpha s} T_{21}^{s\beta} \tag{7}$$

and the solution is therefore

$$P = \frac{\tau^{sp}\rho^{ps} - \tau^{pp}(\rho^{ss}-1)}{(\rho^{pp}-1)(\rho^{ss}-1) - \rho^{sp}\rho^{ps}}$$

$$S = \frac{\tau^{pp}\rho^{sp} - \tau^{sp}(\rho^{pp}-1)}{(\rho^{pp}-1)(\rho^{ss}-1) - \rho^{sp}\rho^{ps}} \tag{8}$$

with the reflection coefficient E given by substituting equations (7) in (8) and then (8) in (3).

No numerical results are presented here but a simple test of this formula can be carried out by considering the special case for which equation (2) was derived. Bearing in mind that the reflection coefficients $R_{23}^{\alpha\beta}$ include any losses due to volume attenuation in medium 2 (this point is explained in more detail in Section 4 - see in particular equations (12)), a high shear attenuation will mean that R_{23}^{ps}, R_{23}^{sp}, $R_{23}^{ss} \rightarrow 0$, from which it follows that

$$\rho^{pp} \rightarrow R_{23}^{pp} R_{21}^{pp}$$

$$\tau^{pp} \rightarrow R_{23}^{pp} T_{21}^{pp} \tag{9}$$

$$\rho^{sp}, \tau^{sp} \rightarrow 0$$

and hence

$$S = 0$$

$$P = \frac{\tau^{pp}}{1-\rho^{pp}} = R_{23}^{pp} T_{21}^{pp} \,/\, (1-R_{23}^{pp} R_{21}^{pp}) \tag{10}$$

and equation (2) follows by substituting equations (10) in (3).

4. EVALUATION OF R AND T COEFFICIENTS

In Section 3 we derived an expression for the reflection coefficient E in terms of R and T, the local reflection and transmission coefficients at the water-sediment and sediment-substrate boundaries. Their values can be related to the density ρ and sound speeds c, b in the various layers by means of the amplitude reflection (transmission) coefficients V(W) given by Brekhovskikh [4] (equations (8.14) to (8.21)) as follows (using his notation):

$$R^{pp} = |V_{ll}|^2 \qquad T^{pp}_{ij} = \left(\frac{\rho_j}{\rho_i}\right)^2 \frac{Z_i}{Z_j} |W_{ll}|^2$$

$$R^{ps} = |V_{lt}|^2 \qquad T^{ps}_{ij} = \left(\frac{\rho_j}{\rho_i}\right)^2 \frac{Z_i}{Y_j} |W_{lt}|^2$$

$$R^{sp} = |V_{tl}|^2 \qquad T^{sp}_{ij} = \left(\frac{\rho_j}{\rho_i}\right)^2 \frac{Y_i}{Z_j} |W_{tl}|^2 \tag{11}$$

$$R^{ss} = |V_{tt}|^2 \qquad T^{ss}_{ij} = \left(\frac{\rho_j}{\rho_i}\right)^2 \frac{Y_i}{Y_j} |W_{tt}|^2$$

in which ρ_i is the density, $Z_i = \rho_i c_i \operatorname{cosec}\alpha_i$ the compressional impedance and $Y_i = \rho_i b_i \operatorname{cosec}\beta_i$ the shear impedance of layer i (cf equation (7.12) of Reference 4). The grazing angles α, β are defined as in Figure 4.

Strictly speaking, equations (11) above apply to the fluid-sediment but not the sediment-substrate boundary. For the latter case, the T_{23} coefficients are as per equations (11) (although not required for evaluation of E), but the R_{23} values must incorporate the effects of volume attenuation in the sediment, ie

$$R^{pp}_{23} = a_p^2 |V_{ll}|^2 \qquad R^{ss}_{23} = a_s^2 |V_{tt}|^2 \tag{12}$$

$$R^{ps}_{23} = a_p a_s |V_{lt}|^2 \qquad R^{sp}_{23} = a_s a_p |V_{tl}|^2$$

where, if h is the sediment depth (Figure 3) and $\varepsilon_p(\varepsilon_s)$ the imaginary part of the sediment compressional (shear) wave number, then the attenuation factors are

$$a_p = \exp\left[-\int_0^h \varepsilon_p(z) \cot\alpha(z)\, dz\right]$$

$$a_s = \exp\left[-\int_0^h \varepsilon_s(z) \cot\beta(z)\, dz\right]. \tag{13}$$

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Equations (12) and (13) assume that the ray paths are steep enough to reach the substrate before they reach a turning point, otherwise the $|V|^2$ factors are omitted and h becomes the ray turning depth. It goes without saying that sediment sound speeds c_2, b_2 for substitution in V and W are to be evaluated at the relevant boundary.

5. COHERENCE

Although Sections 2 and 3 are ostensibly about the incoherent reflection coefficient, Ainslie and Harrison [3] have shown that the analysis works equally well for coherent addition, at least for the FFS case. The coefficients R and T must simply be equated to the amplitudes V and W instead of (eg) $|V|^2$ as in Section 4, taking due account of phase. In other words it is no more difficult to calculate the coherent reflection coefficient than it is the incoherent one, the only extra work involved being calculation of the phase differences between successive returns - in principle no more difficult than the attenuation integrals. Nevertheless, unless coherence effects are known to be important (e.g. for very thin sediments or low angle anomaly [3,5]), it is frequently preferable to use power addition in order to reveal trends which may otherwise be obscured by inteference patterns. Other circumstances which might suggest the use of the incoherent formula are the use of a wide band source or the presence of rough surfaces.

6. SUMMARY

The method described in Reference 3 for calculating incoherent plane wave reflection coefficients has been generalised to incorporate an inhomogeneous solid sediment. No numerical results are presented for comparison but the previous success with a fluid sediment gives us some confidence in the outcome. Similarly, the technique is believed to be applicable to the coherent reflection coefficient if required. Current research for future publication includes evaluation of the proposed formulae for comparison with exact solutions.

ACKNOWLEDGEMENT

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