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ACOUSTIC OPTIMISATION OF A MULTIPLE WALL

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ABSTRACT

A multiple wall of four panels separated by air gaps is considered. The thicknesses of the panels and the air gaps are variable. They are determined in such a way that the transmission coefficient (A-weighted and averaged over the angle of incidence) of the multiple wall is minimized under the constraint that the total thickness of the multiple wall is constant. This optimisation problem is treated numerically. The influence of frequency range, loss factor of the panels and flow resistance in the air gaps is discussed.

1. INTRODUCTION

If a sound wave with pressure amplitude p_i is incident on a wall, it is partly transmitted with a transmitted pressure amplitude p_t. The transmission coefficient

(1.1)
$$\tau = \left|\frac{p_t}{p_i}\right|^2$$

describes the transmitting properties of a wall. For a thin single wall, which is hit perpendicularly by a plane wave of a single frequency, τ is given by the well-known mass law, which states that τ decreases with the frequency of the incident wave and also with the mass per unit area of the wall. If the incident wave is oblique, bending waves on the wall are excited which can have a strong influence on the transmitting properties. The sound transmission is particularly high at coincidence, i.e. if the trace wavelength (the distance between pressure maxima of the air-borne wave along a line parallel to the wall) coincides with the bending wavelength of the wall (Cremer and Heckl 1988, p. 444). A double wall (two thin panels separated by an air gap) shows an additional effect in its sound transmission properties because the double wall can resonate like a system of two masses (the panels) separated by a spring (the air gap). The sound transmission is also particularly high at such a resonance.

It is of interest for noise control purposes to find walls with a minimal sound transmission. The aim of this paper is to examine multiple walls of four panels separated by air gaps. The panel thicknesses and the panel spacings will be chosen in such a way that the transmission coefficient is minimised. Since in a practical situation, the sound comes from many different angles and contains a range of frequencies ω, we use an averaged transmission coefficient

(1.2)
$$\bar{\tau} = \int_{\omega} \frac{1}{K(\omega)} \int_{\theta} \tau(\omega,\theta) \sin\theta \, d\theta \, d\omega$$

in our considerations. $K(\omega)$ is a function which describes the A-weighting to take the frequency-dependent sensitivity of the human ear into account. θ is the angle to the surface normal of the wall. The 3-dimensional case with waves from all directions is considered, therefore $\tau(\omega,\theta)$ is weighted with $\sin\theta$. The minimisation will be performed under the constraint that the total thickness of the multiple wall is constant.

2. MODEL

A multiple wall with four panels is considered (see fig. 2.1). The individual panels are made from the same material but may have different thicknesses h_ν (ν = 1, 2, 3, 4). The panels are assumed to be thin compared with the wavelength, so that the velocity is constant across the thickness. The acoustic behaviour of one such panel is given by the impedance

$$Z_\nu = i\omega\varrho_w h_\nu \{ 1 - \sin^4\theta \frac{\omega^2 c_w{}^2}{c_0^4} \frac{h_\nu^2}{12} (1 + i\eta) \}, \tag{2.1}$$

where ϱ_w is the density of the panel material, c_w is the speed of sound of the panel (compressional waves on a plate), c_0 is the speed of sound in the surrounding air, and η is the loss factor of the panel. Z_ν is the ratio between the pressure difference across a panel and the wall velocity, i.e.

$$p_\nu - p_{\nu+1} = Z_\nu u_\nu . \tag{2.2}$$

With the assumption that the gaps have a width d_ν which is also much smaller than the wavelength, a gap can be modelled as a spring between adjacent panels with the stiffness

$$s_\nu = \frac{\varrho_0 c_0^2}{d_\nu \sqrt{1 - ir/(\omega\varrho_0)}} \tag{2.3}$$

per unit area. r is the flow resistance of the gap; it gives rise to losses in the gaps and can be increased by inserting absorbing materials such as mineral wool into the gaps; ϱ_0 is the density of the medium in the gap. Hooke's law relates the velocity difference across a gap with the pressure by

$$\frac{u_{\nu-1} - u_\nu}{i\omega} s_\nu = p_\nu . \tag{2.4}$$

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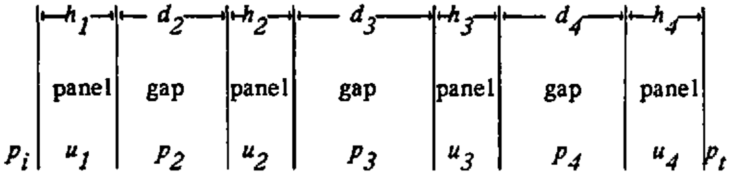

Fig. 2.1 Cross-section through the multiple wall

The motion of the multiple wall as shown in fig. 2.1 is described by a series of equations based on (2.2) and (2.4) for the panels and the gaps.

(2.5) $$2 p_i = (Z_1 + \frac{\varrho_o c_o}{\cos\theta}) u_1 + p_2 ,$$

(2.6) $$\frac{u_1 - u_2}{i\omega} s_2 = p_2 ,$$

(2.7) $$p_2 - p_3 = Z_2 u_2 ,$$

(2.8) $$\frac{u_2 - u_3}{i\omega} s_3 = p_3 ,$$

(2.9) $$p_3 - p_4 = Z_3 u_3 ,$$

(2.10) $$\frac{u_3 - u_4}{i\omega} s_4 = p_4 .$$

(2.11) $$p_4 - p_t = Z_4 u_4 ,$$

(2.12) $$p_t = \frac{\varrho_o c_o}{\cos\theta} u_4 .$$

By successive elimination, this set of equations can be reduced to give an expression for the transmission coefficient τ. The result is rather lengthy and is given in the appendix. τ is a non-linear function of the panel thicknesses and gap widths. The gradient of τ in the space formed by the co-ordinates $(h_1, h_2, h_3, h_4, d_2, d_3, d_4)$ can be calculated analytically, but not the integrals in (1.2) over θ and ω.

3. NUMERICAL TREATMENT

A numerical approach is required to (i) calculate $\bar{\tau}$ from (1.2) which requires integration over θ and ω, and (ii) to find the minimum of $\bar{\tau}$ as a function of the variables h_1, h_2, h_3, h_4, d_2, d_3 and d_4 under the constraints that they are all positive and that their sum is constant.

The integration over θ causes numerical difficulties when coincidence occurs. This happens at frequencies above the critical frequency of coincidence which for a panel of thickness h_ν is

$$(3.1) \qquad \omega_{c\nu} = \frac{\sqrt{12}}{h_\nu} \frac{c_o^2}{c_w} \quad \text{(Heckl and Müller 1975, p. 392).}$$

There is then an angle, the angle of coincidence, where the trace wavelength and bending wavelenth match, giving rise to a extremly sharp peak in τ as a function of θ. Up to four such peaks occur in τ for a multiple wall of four panels. These coincidence angles for a given frequency ω are

$$(3.2) \qquad \sin\theta_{c\nu} = \frac{\sqrt[4]{12}\, c_o}{\sqrt{h_\nu \omega\, c_w}} \quad \text{(Cremer 1975, p. 281).}$$

The numerical integration is done with a particularly small step around these angles to get acceptable accuracy. The integration over ω is unproblematic and is done most efficiently with an adjustable step which rises with ω when the integrand decays to very small values. The function

$$(3.3) \quad K(\omega) = 0.75 + 0.2\left(\frac{f-3000\ \text{Hz}}{3000\ \text{Hz}}\right)^2 + \left(\frac{400\ \text{Hz}}{f}\right)^2 + 3.5\left(\frac{250\ \text{Hz}}{f}\right)^3 + 2\left(\frac{125\ \text{Hz}}{f}\right)^5$$

is used for the A-weighting; f stands for $\omega/(2\pi)$.

The minimisation method is based on a combination of two techniques. The first is a series of one-dimensional minimisations, which provides a starting point for the second technique which is an iteration scheme. The first technique is based on symmetry considerations and the assumption that the optimal multiple wall will be symmetrical with respect to its centre plane; this leaves only three independent variables if the constraint

$$(3.4) \qquad \sum_{\nu=1}^{4} h_\nu + \sum_{\nu=2}^{4} d_\nu = l$$

is taken into account. A series of one-dimensional minimisations with respect to these variables is performed. This is necessary for the second technique which requires a starting point close to the absolute minium. The second technique is an iteration scheme with successive line minimisations in the 6-dimensional space formed by the co-ordinates h_1, h_2, h_3, h_4, d_2, d_3, d_4 under the constraint (3.4). The gradient of $\bar{\tau}$ is calculated, then the minimum is sought along a line in the direction of this gradient. The minimum obtained in this way forms the starting point of the next iteration step. The constraint that all panel thicknesses and gap widths have to be positive is incorporated in the line minimisation.

4. RESULTS

The minimisation was performed for a multiple wall of four glass or aluminium panels.Three different frequency ranges were considered. The loss factor of the panels was varied, and also the flow resistance of the gaps to study the effect of different absorbing materials. The following parameters were used.

$c_o = 340 \text{ m s}^{-1}$	(speed of sound in the air)
$c_w = 5400 \text{ m s}^{-1}$	(speed of sound in the panel (compressional waves))
$\varrho_o = 1.21 \text{ kg m}^{-3}$	(density of air)
$\varrho_w = 2300 \text{ kg m}^{-3}$	(density of the panel material)
$l = 0.15 \text{ m}$	(total thickness of the multiple wall)

The integration over θ was carried out from 0 to 80° because grazing angles close to 90° cause numerical difficulties.

Table 4.1 shows the optimal dimensions for panel thicknesses and spacings for different frequency ranges, loss factors η and flow resistance r. The optimal transmission loss achieved was calculated from

$$TL = 10 \lg \frac{1}{\bar{\tau}} \qquad (4.1)$$

and is listed in the last column of table 4.1.

frequency range in Hz	η	r in $kgm^{-3}s^{-1}$	optimal values in mm of $h_1, h_2, h_3, h_4; d_2, d_3, d_4$	TL in dB
32 ... 2000	10^{-3}	10^4	28, 19, 19, 28; 7, 39, 7	29
100 ... 4000	10^{-3}	10^4	23, 23, 23, 23; 6, 41, 6	33
400 ... 10000	10^{-3}	10^4	11, 7, 7, 11; 38, 38, 38	82
32 ... 2000	0.01	10^4	27, 20, 20, 27; 7, 39, 7	29
32 ... 2000	0.05	10^4	25, 19, 19, 25; 8, 44, 8	29
32 ... 2000	10^{-3}	$4 \cdot 10^4$	25, 19, 19, 25; 8, 44, 8	26
32 ... 2000	10^{-3}	$20 \cdot 10^4$	23, 17, 17, 23; 12,49,12	22

Table 4.1 Optimal geometry of a multiple wall for different frequency ranges, loss factors and flow resistances

5. DISCUSSION

In order to explain the results in the last section, it is necessary to know the frequencies at which the transmission coefficient is particularly high for a plane harmonic wave. These are the resonance frequencies of the multiple wall (seen as a mass-spring system) and the coincidence frequencies of the individual panels.

The resonance frequencies of a multiple wall with the panel thicknesses listed in table 4.1 lie between 50 Hz and 150 Hz. There is not much scope to push this range down to lower frequencies for a wall with a fixed total thickness. To illustrate the problem we consider a double wall, where the resonance frequency (at perpendicular incidence) is given by

(4.2) $$\omega_o = \sqrt{\frac{\varrho_o c_o^2}{d}\left(\frac{1}{\varrho_w h_1} + \frac{1}{\varrho_w h_2}\right)}$$ (Cremer 1975 p. 287);

h_1 and h_2 are the panel thicknesses and d is the panel spacing. If d is increased, h_1 and/or h_2 would have to be decreased in order to keep the total thickness constant. As a consequence, the products $h_1 d$ and $h_2 d$ in (4.2) and thus the resonance frequency can only vary slightly under this constraint.

The optimisation has been performed for a low (32 to 2000 Hz), medium (100 to 4000 Hz) and high (400 to 10000 Hz) frequency range. The low and medium frequency range contain resonance frequencies, whereas the high frequency range does not. As a consequence, the transmission loss for the low and medium range is well below that of the high range (see table 4.1).

In the high frequency range, where resonances are absent, coincidence is the dominant mechanism for sound transmission. A panel with thickness h_ν and an impedance given by (2.1) has a transmission coefficient

$$(4.3)\quad \tau_\nu = \frac{4\,\varrho_o^2 c_o^2}{\left(\varrho_w \frac{\omega^3 h_\nu^3 c_w^2}{12 c_o^4}\,\eta\,\cos\theta\sin^4\theta + 2\varrho_o c_o\right)^2 + \left(\varrho_w \omega h_\nu - \varrho_w \frac{\omega^3 h_\nu^3 c_w^2}{12 c_o^4}\sin^4\theta\right)^2 \cos^2\theta}$$

The first term in the denominator of (4.3) is due to the losses in the panel material, the second term is the radiation damping, the third term represents the mass-controlled losses and the fourth term is due to the bending stiffness of the panel. The sound transmission is strongly dependent on the mass-controlled and stiffness-controlled contributions, which have opposite signs and can cancel each other, yielding coincidence. When this happens, τ_ν is maximal. Two effects are competing when one tries to minimise the averaged (over angle and frequency) transmission coefficient. A decrease of panel thickness increases the critical frequency of coincidence $\omega_{c\nu}$ (see equ. (3.1)), and higher values of $\omega_{c\nu}$ have a smaller effect on the averaged transmission coefficient than low ones. This suggests that the critical frequency of coincidence should be towards the top end of the considered frequency range or above it. This is in conflict with the mass law which suggests that a panel should be as thick as possible for a maximal transmission loss. The fact that the optimal panel thickness for the high frequency range is well below that found for the low and medium frequency range (see table 4.1) indicates that coincidence has been important. The limiting frequency of coincidence for a 0.008 m thick panel is about 1500 Hz.

The influence of loss factor η and flow resistance r was also examined (see table 4.1). Three different factors were considered, $\eta = 10^{-3}$, 0.01 and 0.05. There is no significant effect on the performance of the multiple wall for the low frequency range; the transmission loss achieved is 29 dB for all three values of η. The effect of the flow resistance was studied by choosing $r = 10^4$, $4 \cdot 10^4$ and $20 \cdot 10^4$ kg $m^{-3}s^{-1}$, giving rise to deteriorating transmission losses of 29, 26 and 22 dB in the low frequency range. The optimal value of r is somewhere between 0 and 10^4 kg $m^{-3}s^{-1}$. As r is increased, the optimal gap width tends to increase.

6. REFERENCES

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7. APPENDIX

The transmission coefficient τ of a multiple wall with four panels (impedances Z_1, Z_2, Z_3 and Z_4) and three gaps (stiffnesses s_2, s_3 and s_4) is given by

$$\tau = \left|\frac{f_t}{f_i}\right|^2 ,$$

where

$$f_t = \frac{-2 s_2 s_3}{\omega^2 \left(Z_1 + \frac{\varrho_o c_o}{\cos\theta} + \frac{s_2}{i\omega}\right)}$$

$$f_i = \left(\frac{\cos\theta}{\varrho_o c_o} Z_3 + 1 + \frac{i\omega}{s_4} Z_3 + Z_4 \frac{\cos\theta}{\varrho_o c_o} + Z_3 Z_4 \frac{i\omega}{s_4}\frac{\cos\theta}{\varrho_o c_o} + \frac{s_3}{i\omega}\frac{\cos\theta}{\varrho_o c_o} + \frac{s_3}{s_4} + \frac{s_3}{s_4} Z_4 \frac{\cos\theta}{\varrho_o c_o}\right) \cdot$$

$$\cdot \left(\frac{s_2}{i\omega} + \frac{s_3}{i\omega} + Z_2 + \frac{s_2^2}{\omega^2}\frac{1}{Z_1 + \frac{\varrho_o c_o}{\cos\theta} + \frac{s_2}{i\omega}}\right) + \frac{s_3^2}{\omega^2}\left(\frac{\cos\theta}{\varrho_o c_o} + \frac{i\omega}{s_4} + Z_4 \frac{i\omega}{s_4}\frac{\cos\theta}{\varrho_o c_o}\right)$$