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ACOUSTIC OPTIMISATION OF PROFILED CLADDING OF BUILDINGS

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1. INTRODUCTION

Profiled plates or claddings are commonly used as a facade of industrial buildings. Their structural advantages are in their stiffness and in their low weight. However, from a noise-control point of view, their benefit is doubtful. The aim of this paper is to present a detailed mathematical model for the acoustic behaviour of profiled plates. This will give a physical insight into phenomena, such as wave diffraction, resonances, trace matching of free plate waves and air-borne waves, which can occur when a sound wave impinges on a profiled plate. The model may also be used to give guidelines on how to optimize the design of cladding for noise-control purposes.

The profiled plate is modelled as a structure which is made of narrow sections of infinitely long panels which are connected at right angles. The structure is periodic with each repeat (length L) consisting of four panels. A cross-section is shown in figure 1.

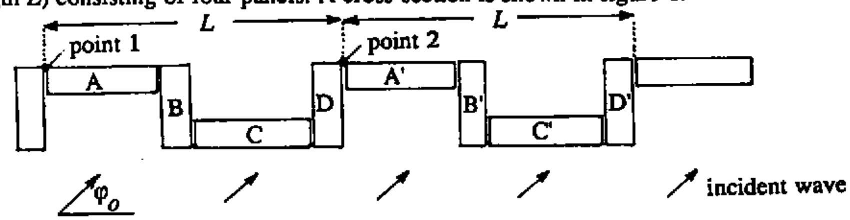

Figure 1 Cross-section of profiled cladding

There are two aspects to the model. The first is sound transmission if the plate is excited by an air-borne wave (section 2.1). The second aspect is structure-borne waves propagating freely (i.e. without external excitation) along the cladding.

2. MODEL

The panels A, B, C and D are considered as narrow, infinitely long Mindlin plates. Only the two-dimensional case, where there is no dependence in the z-direction, is treated here. Panel A has width l_A and thickness h_A; analogous notation is for panels B, C and D. The free waves that can propagate on each panel are free bending waves and free compressional waves. Their wave numbers are k_{Bp} (propagating bending wave), k_{Bd} (exponentially decaying bending wave near-field), and k_D (compressional wave). Each of the four panels can have a different set of free wave numbers. They are marked by superscripts.

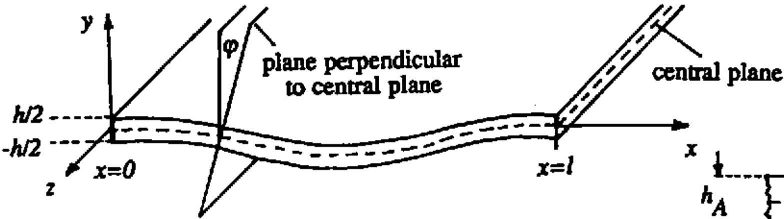

Figure 2 Geometry of an individual panel

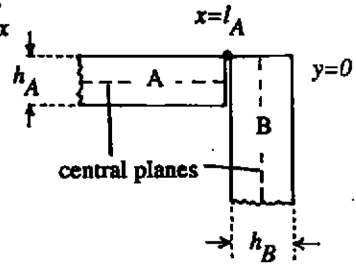

Figure 3 Connection between panels A and B

There are three independent field quantities:

- ξ_M: in-plane compressional motion at central plane of the plate
- η: bending motion of the plate
- φ: bending angle of the plate (see figure 2).

The connection between the panels is described by boundary conditions. For the connection between panels A and B (see figure 3) they are

$$\left[\xi_M^{(A)} + \frac{h_A}{2} \varphi^{(A)} \right] \Big|_{x=l_A} = \eta^{(B)} \Big|_{y=0} \tag{1a}$$

(continuity of displacement in the x-direction),

$$\eta^{(A)} \Big|_{x=l_A} = - \left[\xi_M^{(B)} - \frac{h_B}{2} \varphi^{(B)} \right] \Big|_{y=0} \tag{1b}$$

(continuity of displacement in the y-direction), and

$$- \varphi^{(A)} \Big|_{x=l_A} = \frac{\partial \eta^{(B)}}{\partial y} \Big|_{y=0} \tag{1c}$$

(non-rotating connection between the panels). The notation is chosen in such a way that ξ always stands for an in-plane displacement and η for a bending displacement. On panel B, ξ is in the y-direction and η in the x-direction. The first two boundary conditions (1a) and (1b) describe the conversion of bending waves into in-plane waves and vice versa at the interface between panels A and B.

2.1. Sound transmission across the cladding

The profiled plate is surrounded by air and excited across its whole width by a plane air-borne sound wave with pressure

$$p(x,y) = p_o\, e^{ik_o x \cos\varphi_o + ik_o y \sin\varphi_o} \tag{2}$$

This pressure is harmonic with angular frequency ω, wave number k_o and angle φ_o between the propagation direction and the panel A (see figure 1). The influence of the surrounding air on the motion of the cladding can be neglected, since air is very light compared with the cladding material, which is typically steel. One thus avoids treating the fully coupled problem which would be required for significant fluid loading; instead the problem can be tackled in two steps. (i) Structure-borne waves are generated on the profiled plate by fluctuating forces which in turn are generated by the incident and rigidly reflected wave. (ii) As a consequence, the cladding radiates sound into the surrounding air.

Step (i)
The incident wave excites bending waves with wave numbers k_{ex} on the individual panels. There are also free waves on each panel; these are excited by the forces that the adjacent panels exert at the panel edges. The total wave field on panel A is

$$\xi_M^{(A)} = A_1\, e^{ik_D^{(A)}x} + A_2\, e^{-ik_D^{(A)}(x-l_A)}, \tag{3a}$$

$$\eta^{(A)} = A_3\, e^{ik_{Bp}^{(A)}x} + A_4\, e^{-ik_{Bp}^{(A)}(x-l_A)} + A_5\, e^{k_{Bd}^{(A)}(x-l_A)} + A_6\, e^{-k_{Bd}^{(A)}x} + A_7\, e^{ik_{ex}^{(A)}x}, \tag{3b}$$

$$\varphi^{(A)} = A_8\, e^{ik_{Bp}^{(A)}x} + A_9\, e^{-ik_{Bp}^{(A)}(x-l_A)} + A_{10}\, e^{k_{Bd}^{(A)}(x-l_A)} + A_{11}\, e^{-k_{Bd}^{(A)}x} + A_{12}\, e^{ik_{ex}^{(A)}x}. \tag{3c}$$

The amplitudes $A_1, \dots A_{12}$ are unknown at this stage. Analogous expressions hold for the other three panels B, C and D of the repeat with amplitudes $B_1, \dots B_{12}$, $C_1, \dots C_{12}$, and $D_1, \dots D_{12}$.

The boundary conditions (1) hold for the connection of panels A/B, and similar equations hold for the connection of panels B/C and panels C/D. There is also a periodicity condition which relates the field quantities ξ_M, η, φ at the two points 1 and 2 (see figure 1) which are exactly one period apart,

$$\left[\xi_M^{(A)}, \eta^{(A)}, \varphi^{(A)}\right]\Big|_{\text{point 1}} = e^{-ik_o L \cos\varphi_o} \left[\xi_M^{(A')}, \eta^{(A')}, \varphi^{(A')}\right]\Big|_{\text{point 2}} \tag{4}$$

This says that the field at points that are exactly one period apart only differ by the phase factor $e^{-ik_o L \cos\varphi_o}$ which is imposed by the incident wave.

Hamilton's principle is now applied to the repeat ABCD and combined with the boundary conditions (1) and the periodicity condition (4). This is described in detail by Heckl (1993), and leads to a matrix equation for the amplitudes $A_1, \dots A_{12}, \dots D_1, \dots D_{12}$. Once these amplitudes are known, the motion of the repeat ABCD can be calculated at every point.

Step (ii)
The radiated pressure p is given by the Helmholtz/Kirchhoff integral equation. The integration over the total cladding surface can be reduced to an integration over only one of the repeats, say ABCD (see Heckl 1993), and the result is

$$p(\underline{r}) = \varepsilon \sum_{\nu=-\infty}^{\infty} \int_{S_{ABCD}} \left[p(\underline{r}') \frac{\partial G_\nu(\underline{r},\underline{r}')}{\partial n'} + G_\nu(\underline{r},\underline{r}')\, \rho_o \omega^2 \eta(\underline{r}') \right] dS' , \tag{5a}$$

with

$$G_\nu(\underline{r},\underline{r}') = \frac{i}{4} \frac{2}{k_o L} \frac{1}{\sin\varphi_\nu} e^{ik_o(x-x')\cos\varphi_\nu + ik_o|y-y'|\sin\varphi_\nu} \tag{5b}$$

$\underline{r} = (x,y)$ is the observer point, $\underline{r}' = (x',y')$ is a source point, $\frac{\partial}{\partial n'}$ is the derivative in the direction of the surface normal, and $\varepsilon = 1$ or 2, depending on whether $\underline{r}$ is on the surface or away from it. The 'scattering angles' φ_ν are defined by

$$\cos\varphi_\nu = \cos\varphi_o + \frac{2\pi\nu}{k_o L}, \tag{6a}$$

$$\sin\varphi_\nu = \begin{cases} \sqrt{1 - \cos^2\varphi} & \text{for } |\cos\varphi_\nu| \le 1 \\ i\sqrt{\cos^2\varphi_\nu - 1} & \text{for } |\cos\varphi_\nu| > 1 \end{cases} \tag{6b}$$

Equation (5a) represents a superposition of plane waves with discrete angles φ_ν. These waves are propagating for ν-values with $|\cos\varphi_\nu| \le 1$; ν-values with $|\cos\varphi_\nu| > 1$ represent a near-field which decays exponentially with the distance from the cladding. An equation analogous to (5) is found in the theory of optical diffraction gratings; the cladding is therefore an 'acoustic diffraction grating'. Equation (5) can be evaluated by the boundary element method. This gives the pressure field at any point of the surrounding air on the transmitted-wave side of the cladding.

The radiation efficiency is given by

$$\sigma = \frac{Real\left[\; i \int_{S_{ABCD}} p\; \eta^{*}\; dS \;\right]}{\rho_o\; c_o\; \omega \int_{S_{ABCD}} |\eta|^2\; dS}\,, \qquad (7)$$

where ρ_o and c_o are respectively the density and speed of sound in the surrounding air.

2.2. Free waves on the cladding
This section deals with free waves on the cladding as a whole, as opposed to free bending and compressional waves on individual panels. These free waves are waves which can exist on the cladding without being maintained by external forces, such as the incident sound wave in section 2.1.

The total wave field is similar to (3) and given by

$$\xi_M^{(A)} = \overline{A}_1\, e^{ik_D^{(A)}x} + \overline{A}_2\, e^{-ik_D^{(A)}(x-l_A)}\,, \qquad (8a)$$

$$\eta^{(A)} = \overline{A}_3\, e^{ik_{Bp}^{(A)}x} + \overline{A}_4\, e^{-ik_{Bp}^{(A)}(x-l_A)} + \overline{A}_5\, e^{k_{Bd}^{(A)}(x-l_A)} + \overline{A}_6\, e^{-k_{Bd}^{(A)}x}\,, \qquad (8b)$$

$$\varphi^{(A)} = \overline{A}_7\, e^{ik_{Bp}^{(A)}x} + \overline{A}_8\, e^{-ik_{Bp}^{(A)}(x-l_A)} + \overline{A}_9\, e^{k_{Bd}^{(A)}(x-l_A)} + \overline{A}_{10}\, e^{-k_{Bd}^{(A)}x}\,, \qquad (8c)$$

with amplitudes $\overline{A}_1, \dots \overline{A}_{10}$. Expressions analogous to (8a,b,c) hold for the other panels B, C, D of the repeat with unknown amplitudes $\overline{B}_1, \dots \overline{B}_{10}, \overline{C}_1, \dots \overline{C}_{10}, \overline{D}_1, \dots \overline{D}_{10}$.

Free waves in a periodic system obey Bloch's theorem, which states that all field quantities at two points exactly one period apart are related by the same factor. For the case considered here, this can be written as

$$\left[\xi_M, \eta, \varphi, F_\xi, F_\eta, M\right]\Big|_{\text{point 1}} = e^{\gamma L} \left[\xi_M, \eta, \varphi, F_\xi, F_\eta, M\right]\Big|_{\text{point 2}}\,, \qquad (9)$$

where F_x, F_y are internal forces and M is a moment exerted by the panels on either side of the repeat ABCD. The factor relating the field quantities at the two points is commonly denoted by $e^{\gamma L}$. L is the length of one period. γ is the 'Bloch wave number'; it is generally complex,

$$\gamma = \alpha + ik\,, \tag{10}$$

where the real part α denotes the 'Bloch attenuation' which is the attenuation of the wave from repeat to repeat. The imaginary part k is the 'Bloch propagation wave number'; it is a measure of the speed of waves propagating along the cladding. γ has to be calculated numerically from an eigenvalue problem with a 6×6 matrix. There are three independent solutions, γ_1, γ_2, γ_3; the other three solutions are $-\gamma_1$, $-\gamma_2$, $-\gamma_3$ which correspond to waves with the same attenuation and propagation speed as γ_1, γ_2, γ_3, but with the opposite propagation direction.

3. NUMERICAL RESULTS AND DISCUSSION

Numerical calculations have been performed for steel cladding with

$$\rho = 8000 \text{ kg m}^{-3}\,,$$
$$E = 2 \cdot 10^{11} \text{ N m}^{-2}\,,$$
$$G = 0.77 \cdot 10^{11} \text{ N m}^{-2}\,,$$
$$\mu = 0.3\,,$$
loss factor: 0.01.

The thickness of the individual panels is 4 mm. The lengths of the panels A and C are $l_A = l_C = 0.25$ m. Claddings with two different depths were considered, one was a shallow cladding with $l_B = l_D = 0.03$ m (figure 4) and the other was a deep cladding with $l_B = l_D = 0.15$ m (figure 5). The figures show spectra in the frequeny range from 0 to 3000 Hz. Figures 4a,b and 5a,b refer to the sound transmission across the cladding which is hit by an incident wave at angle $\varphi_0 = 0.9\,\frac{\pi}{2}$ (nearly perpendicular); figures 4c,d and 5c,d refer to free waves on the cladding. The average squared velocity of panels A (solid line) and B (dashed line) is shown in figures 4a and 5a. Figures 4b and 5b show the radiation efficiency of the cladding. Resonances are clearly seen as sharp minima in the spectrum of the radiation efficiency and sharp peaks in the velocity spectra. It is impossible to identify these peaks as resonances of individual panels because of the coupling between the panels. Figures 4c and 5c show the three independent solutions obtained for the Bloch attenuation. There are frequency bands where the attenuation is zero. These are called 'passing bands' because waves can propagate unattenuated at these frequencies. There are also frequency bands where the Bloch attenuation is positive for all three solutions. These are 'stopping bands'; only non-propagating waves exist at such frequencies. The 'pin-pin frequencies', i.e. frequencies where the length L of a repeat is an integer multiple of the free bending wavelength of an equivalent flat plate (4 mm thick), are marked by arrows. The Bloch propagation wave numbers in the passing bands are shown in figures 4d and 5d. At this stage, these wave numbers are determined uniquely modulo 2π. One can graphically construct from these curve sections a monotonically rising curve (with discontinuities) and compare it with the spectrum of the wave number of an air-borne wave. Coincidence, analogous to the coincidence in a flat plate, occurs if the two wave number curves intersect. For a 4 mm thick flat plate, the critical frequency of coincidence is just above 3000 Hz. Preliminary studies for the case of the shallow cladding indicate that the critical frequency of coincidence is around 2900 Hz. There are several peaks in the spectrum of the radiation efficiency of the deep cladding (see figure 5b). Some of these peaks are not due to resonances because there are no corresponding features at those frequencies in the velocity spectra (see figure 5a). They could be caused by coincidence effects. Simulations with a number of different configurations still need to be done.

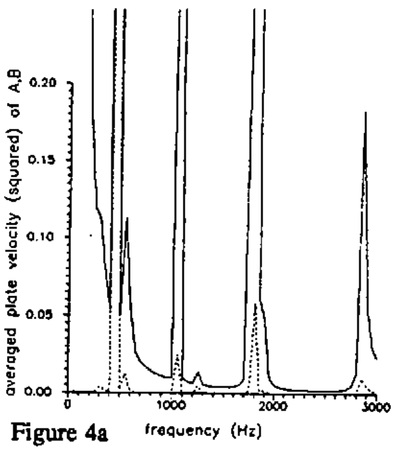

Figure 4a

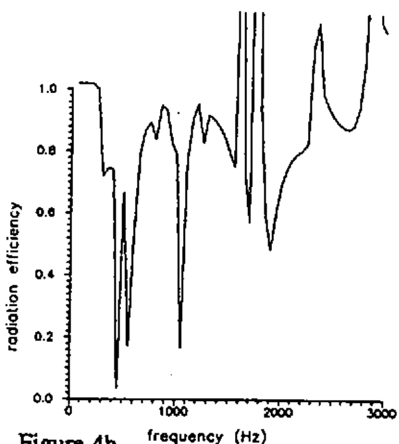

Figure 4b

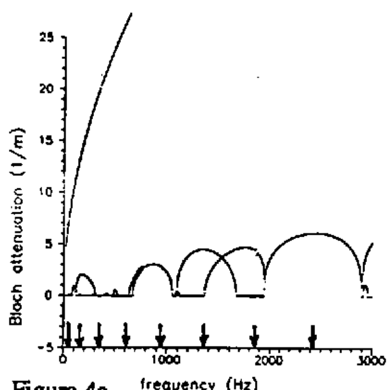

Figure 4c

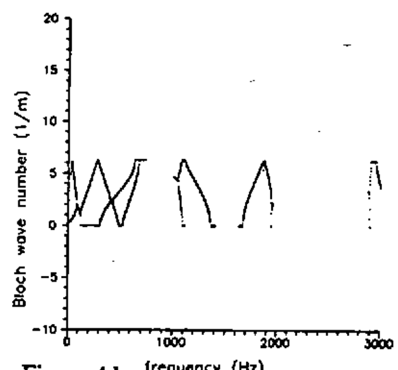

Figure 4d

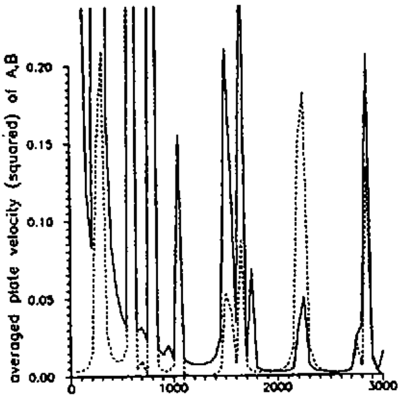

Figure 5a frequency (Hz)

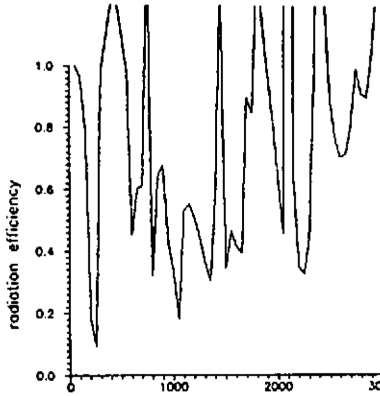

Figure 5b frequency (Hz)

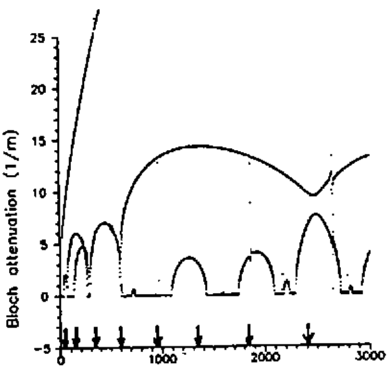

Figure 5c frequency (Hz)

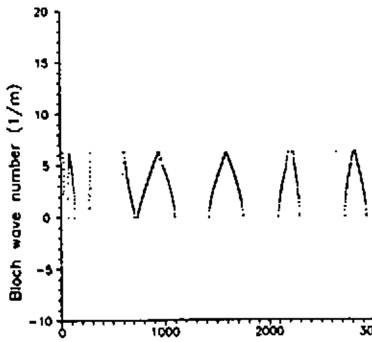

Figure 5d frequency (Hz)

4. REFERENCES

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