SOUND PROPAGATION IN A FAST BREEDER REACTOR

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ABSTRACT

The sound propagation in the core of a liquid metal cooled fast breeder reactor is examined. The reactor core is a complicated anisotropic medium. It consists of hexagonal tubes which form a honeycomb-like structure immersed in a liquid and filled with densly packed fuel rods. This medium is modelled in the following way. The hexagonal tubes are replaced by parallel walls which are connected by angular plates in regular intervals. This structure is immersed in liquid, and embedded in the liquid is a large number of small masses to represent the fuel rods. The periodicity of this medium allows Bloch's theorem to be used. The speed and attenuation of sound depends on various parameters, (e.g. angle of incidence, frequency, mass of the tubes), and their influence is predicted. The scattering of sound at the angles of the hexagonal tubes is also discussed. Information about the sound propagation is important for acoustic fault location techniques which are based on the transit times of sound signals.

1. INTRODUCTION

The core of a liquid metal cooled fast breeder reactor is a complicated anisotropic medium. It consists of hexagonal tubes (sub-assemblies) which form a honeycomb-like structure immersed in a liquid and filled with densly packed fuel rods (fig.1).

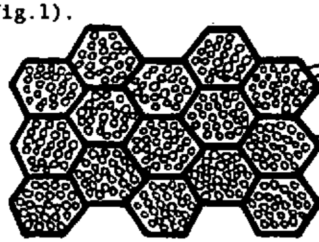

Fig. 1 Cross-section through the reactor core

Our aim is to model the sound transmission properties of this medium. The medium has the following characteristics:

(1) Inhomogeneous medium inside the sub-assemblies,
(2) complicated hexagonal geometry,
(3) interaction between liquid and structure,
(4) periodicity.

We consider these points and make the following assumptions.

(1) The fuel rods are represented by a large number of small masses distributed evenly throughout the liquid. The medium composed of liquid and masses is isotropic. The speed of sound is different from that in pure liquid. This will be discussed in section 2.1.

(2) An accurate calculation of the sound field in the hexagonal geometry of the reactor core can only be done with great numerical effort, e.g by the finite element method. We simplify the problem by modelling the sub-assemblies as walls connected by angled plates. A cross-section of the model is shown in fig. 2.

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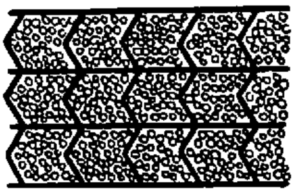


Fig. 2 Cross-section through the model for a group of sub-assemblies

We shall calculate the sound scattered by the joints between walls and adjacent plates in section 2.2. It will turn out that the scattering is neglegibly small.
(3) The motion of each wall is described by the bending wave equation with a force term caused by the action of the springs. In the liquid between the plates we have plane waves which are partly reflected and partly transmitted when they hit a wall. At an interface between a wall and a liquid layer we assume the usual boundary conditions of continuity of velocity and the momentum balance.
(4) The periodicity of the medium allows Bloch's theorem [1] to be applied. A full description of the model for the periodic medium is given in section 2.3. In section 3 we shall present and discuss some numerical results.

2. ANALYSIS OF THE MODEL

2.1 Speed of sound in the medium inside the sub-assemblies

Liquids with suspended particles can show an unusual acoustic behaviour. The calculation by Crighton et al. [2] of the low frequency sound speed of a bubbly liquid also holds for a liquid with immersed little masses. The speed of sound in this medium is given by

$$\frac{1}{c_{\ell m}^2} = \frac{(1-v)}{c_\ell^2} + \frac{v^2}{c_m^2} + v(1-v)\,\frac{\rho_\ell^2 c_\ell^2 + \rho_m^2 c_m^2}{\rho_\ell \rho_m c_\ell^2 c_m^2}, \qquad (1)$$

where c_ℓ and c_m are the speed of sound in the liquid and in the metal from which the masses are made, respectively. ρ_ℓ and ρ_m are the density of the liquid and the masses respectively, and v is the volume concentration of the masses. This result is valid for low frequencies where the wavelength is much larger than the diameter of the masses.
For $v=0.28$, $c_\ell=1465$ m/s (water), $c_m=5000$ m/s (steel), $\rho_\ell=1000$ kg/m³, $\rho_m=8000$ kg/m³ we get $c_{\ell m}=1000$ m/s. $c_{\ell m}$ is lower than c_ℓ because the inertia of the masses prevents sound waves from propagating with the same speed as in pure liquid.
Throughout this paper we give the numerical results for water instead of liquid sodium because water is more suitable for future experimental checks of our model.

2.2 Sound scattering at the joints

In order to estimate the sound scattered at the joints we look at two adjacent connected walls and the liquid layer inbetween, separated from the rest of the medium as shown in fig. 3.

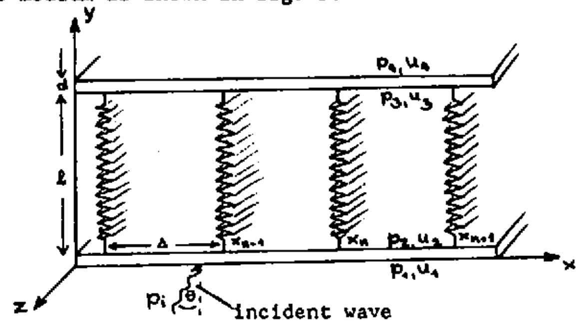


Fig. 3 Cross-section through two walls connected by springs in regular intervals

The pressures on either side of both walls are denoted by p_1, p_2, p_3, p_4, and the y components of the velocities by u_1, u_2, u_3, u_4 (fig. 3). The walls are assumed to be thin compared with the wavelength so that the velocity is the same on both sides of a wall, $u_1=u_2$, $u_3=u_4$. There is no restriction on the thickness of the liquid layer. The motion of the walls can be described by the bending wave equation,

$$\frac{\partial^4 u_1}{\partial x^4} - k_B^4 u_1 = - \frac{i\omega}{B} (p_1 - p_2) - \frac{i\omega}{B} \sum_n F_{1n} \delta(x-x_n) , \qquad (2a)$$

$$\frac{\partial^4 u_3}{\partial x^4} - k_B^4 u_3 = - \frac{i\omega}{B} (p_3 - p_4) - \frac{i\omega}{B} \sum_n F_{2n} \delta(x-x_n) . \qquad (2b)$$

B is the bending stiffness (per unit length in z direction) which is, for a plate of thickness d and Young's modulus E, approximately (see [3], p.280)

$$B = \frac{Ed^3}{12} . \qquad (3)$$

k_B is the bending wave number, it is given by

$$k_B^4 = \frac{\omega^2 m}{B} , \qquad (4)$$

where ω is the angular frequency and m the mass per unit area of the wall. F_{1n} and F_{2n} are the forces exerted on the two walls by the springs and δ is the δ-function. According to Hooke's law,

$$F_{1n} = - F_{2n} = \frac{D}{i\omega} (u_2(x_n) - u_1(x_n)) , \qquad (5)$$

where D is the stiffness of one of the springs per unit length in z direction.

The time factor is $e^{-i\omega t}$ and it is omitted throughout.
Fourier transform of (2) with respect to x yields

$$(k_x^4 - k_B^4)\,\hat{u}_1 = -\frac{i\omega}{B}(\hat{p}_1 - \hat{p}_2) - \frac{i\omega}{B}\sum_n F_{1n}\,e^{-ik_x x_n}, \tag{6a}$$

$$(k_x^4 - k_B^4)\,\hat{u}_3 = -\frac{i\omega}{B}(\hat{p}_3 - \hat{p}_4) - \frac{i\omega}{B}\sum_n F_{2n}\,e^{-ik_x x_n}, \tag{6b}$$

where k_x is the x component of the wave number vector.
In the medium between the walls we have backward and forward travelling waves,

$$p(x,y) = \frac{1}{2\pi}\int (\hat{p}_+ e^{ik_y y} + \hat{p}_- e^{-ik_y y})\,e^{ik_x x}\,dk_x\,, \tag{7}$$

$$u(x,y) = \frac{1}{2\pi}\int (\hat{u}_+ e^{ik_y y} + \hat{u}_- e^{-ik_y y})\,e^{ik_x x}\,dk_x\,, \tag{8}$$

where

$$k_y = \begin{cases} \sqrt{\dfrac{\omega^2}{c_{\ell m}^2} - k_x^2} & \text{for } k_x < \dfrac{\omega}{c_{\ell m}} \\[2ex] \sqrt{-\dfrac{\omega^2}{c_{\ell m}^2} + k_x^2} & \text{for } k_x > \dfrac{\omega}{c_{\ell m}}\,. \end{cases} \tag{9}$$

p and u are solutions of the two-dimensional wave equation.
Relationships between the pressures p_1, p_2, p_3, p_4 at the wall sufaces and the wall velocities u_1, u_3 are obtained from the usual boundary conditions of continuity of velocity and validity of the momentum balance at the interfaces between liquid and wall. The results are

$$\hat{p}_1(k_x) = 2\,\tilde{p}_i\,\delta(k_i - k_x) - \frac{\omega\rho_{\ell m}}{k_y}\hat{u}_1(k_x)\,, \tag{10a}$$

$$\hat{p}_2(k_x) = \frac{\omega\rho_{\ell m}}{k_y}\,\frac{2\,\hat{u}_3(k_x) - \hat{u}_1(k_x)\,(e^{ik_y\ell} + e^{-ik_y\ell})}{e^{ik_y\ell} - e^{-ik_y\ell}}\,, \tag{10b}$$

$$\hat{p}_3(k_x) = \frac{\omega\rho_{\ell m}}{k_y}\,\frac{-2\,\hat{u}_1(k_x) + \hat{u}_3(k_x)\,(e^{ik_y\ell} + e^{-ik_y\ell})}{e^{ik_y\ell} - e^{-ik_y\ell}}\,, \tag{10c}$$

$$\hat{p}_4(k_x) = \frac{\omega\rho_{\ell m}}{k_y}\hat{u}_3(k_x)\,, \tag{10d}$$

where $\tilde{p}_i$ and k_i are the amplitude and the x component of the wave number respectively of the incident wave. $\rho_{\ell m}$ is the density of the medium between the plates. If (10) and (5) are inserted into the bending wave equation (6) a set of coupled equations for $\hat{u}_1(k_x)$, $\hat{u}_3(k_x)$, $u_1(x)$ and $u_3(x)$ is obtained. It is possible to decouple these equations and obtain explicit equations for $\hat{u}_1(k_x)$ and $\hat{u}_3(k_x)$ with Stephanishen's approach [4, appendix] who

analysed a single periodically supported plate. The manipulations and the results are rather lengthy so we omit them here and only present the numerical results.
We calculated the wave number spectrum for steel walls connected by springs with the stiffness of an angled plate (angle 120 degrees) which represents two adjacent sides of a hexagonal tube. The medium between the walls is that described in section 2.1 and the dimensions are ℓ=0.07 m, d=0.003 m, Δ=0.07 m. The spectrum of the transmitted wave has non-zero components at $k_i \pm \frac{2\pi n}{\Delta}$ (n integer) which (for n≠0) are at least a factor of 10^3 smaller than the component k_i. This is the case for all frequencies, even for critical ones, e.g. where the bending wavelength coincides with the distance between adjacent springs. These results show that the waves scattered at the joints are unimportant.

2.3 Model for the periodic medium
We now consider an infinitely extended layered medium, a section of which is shown in fig. 4.

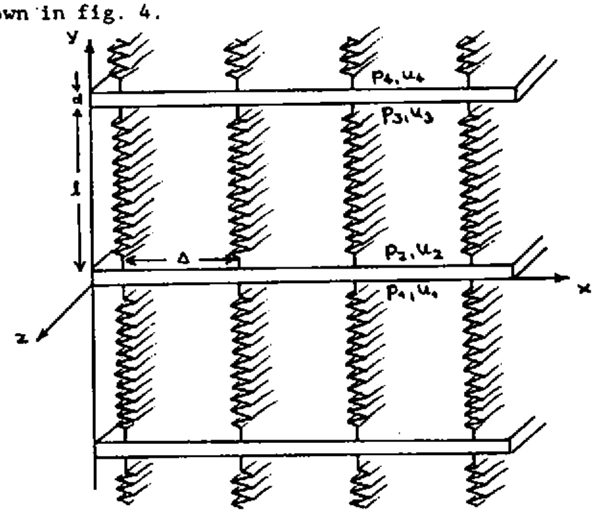


Fig. 4 Cross-section through the periodic medium

We neglect the scattering at the joints so that we need to deal with only one wave number.
The pressure and velocity field in a liquid layer, e.g. the one between y=d and y=ℓ+d, is composed of a backward and forward travelling wave,

$$p(y) = A\, e^{ik_y(y-\ell-d)} + B\, e^{-ik_y(y-\ell-d)} , \quad (11)$$

$$u(y) = A \frac{\cos\theta}{\rho_{\ell m} c_{\ell m}} e^{ik_y(y-\ell-d)} - B \frac{\cos\theta}{\rho_{\ell m} c_{\ell m}} e^{-ik_y(y-\ell-d)} . \quad (12)$$

The x dependence $e^{ik_x x}$ is omitted throughout this section. (11) and (12) can be used to relate the field quantities at the two wall surfaces y=d and y=ℓ+d,

$$p_2 = a_{11}\, p_3 + a_{12}\, u_3 \quad , \tag{13a}$$

$$u_2 = a_{21}\, p_3 + a_{22}\, u_3 \quad , \tag{13b}$$

with

$$a_{11} = \cos k_y \ell \quad , \tag{14a}$$

$$a_{12} = -i\, \frac{\rho_{\ell m} c_{\ell m}}{\cos\,\theta} \sin k_y \ell \quad , \tag{14b}$$

$$a_{21} = -i\, \frac{\cos\theta}{\rho_{\ell m} c_{\ell m}} \sin k_y \ell \quad , \tag{14c}$$

$$a_{22} = \cos k_y \ell \quad . \tag{14d}$$

This matrix notation is very useful, because a relation like (13a/b) with the same matrix coefficients holds for the field quantities on either side of any liquid layer.
As in section 2.2, the wall motion is described by the bending wave equation. For the wall between y=0 and y=d, we can use equation (2) with a modified force term,

$$\frac{\partial^4 u_1}{\partial x^4} - k_B^4\, u_1 = -\frac{i\omega}{B}\,(p_1 - p_2) - \frac{i\omega}{B}\,\frac{F_{01} - F_{23}}{S} \quad , \tag{15}$$

where

$$F_{01} = \frac{D}{i\omega}\,(u_1 - u_0) \qquad \text{and} \qquad F_{23} = \frac{D}{i\omega}\,(u_3 - u_2) \tag{16}$$

are the forces exerted by the springs in the adjacent liquid layers. $\frac{D}{S}$ is the stiffness per unit area in the xz plane. With $u_1=u_2$, (15) can also be written in matrix notation,

$$p_1 = b_{11}\, p_2 + b_{12}\, u_2 \quad , \tag{17a}$$

$$u_1 = b_{21}\, p_2 + b_{22}\, u_2 \quad , \tag{17b}$$

with

$$b_{11} = 1 \quad , \tag{18a}$$

$$b_{12} = \frac{2\,\frac{D}{i\omega S}\,(a_{11} - 1) + Z}{1 - \frac{D}{i\omega S}\, a_{21}} \quad ; \tag{18b}$$

$$b_{21} = 0 \quad , \tag{18c}$$

$$b_{22} = 1 \quad . \tag{18d}$$

Z is the impedance of the wall [3, p.281].
We now apply a form of Bloch's theorem [1] which relates field quantities at points that are exactly one period apart,

$$p_1 = e^{\gamma}\, p_3 \quad , \tag{19a}$$

$$u_1 = e^{\gamma}\, u_3 \quad . \tag{19b}$$

The real and imaginary part of γ are related to the speed of sound and the attenuation in the composite medium. By combining (13), (17) and (19), we get a homogeneous set of two equations, and γ can be determined from the condition that the determinant has to be zero. The solution is

$$\gamma = \ln \frac{(c_{11} + c_{22}) \pm \sqrt{(c_{11} + c_{22})^2 - 4}}{2} , \qquad (20)$$

where c_{11} and c_{22} are the diagonal elements of the matrix that is obtained when multiplying the matrices b_{ij} and a_{jk}.
The speed of sound can be found to be

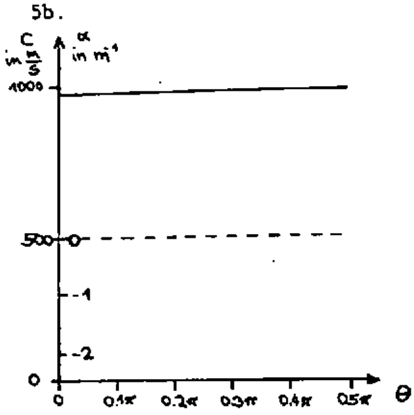
$$c = \frac{1}{\sqrt{\frac{\sin^2\theta}{c_{\ell m}^2} + \frac{(\text{Imag}\gamma)^2}{\omega^2(\ell+d)^2}}} , \qquad (21)$$

and the spatial attenuation

$$\alpha = \frac{\text{Real } \gamma}{\ell+d} . \qquad (22)$$

3. NUMERICAL RESULTS AND DISCUSSION

Equations (21) and (22) were evaluated numerically for the geometry and material properties given in section 2.1 and 2.2. Our calculations show that for low frequencies, waves can propagate through the composite medium in any direction without attenuation. At higher frequencies, there are ranges of angles where propagation can take place (passing bands) and others where the attenuation is so high, that there is an exponentially decaying near field (stopping bands). Examples are shown in figures 5a and 5b.



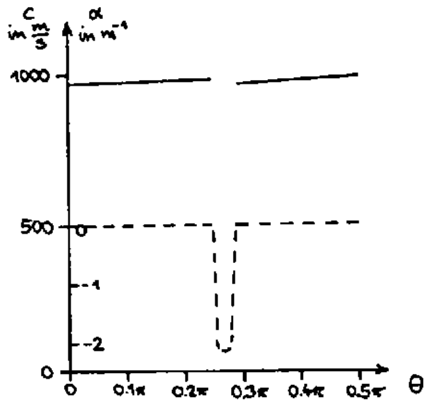


Fig. 5 Speed of sound (solid line) and attenuation (broken line) as a function of the propagation angle
(a) frequency 3 kHz (b) frequency 10 kHz

The speed of sound in the composite medium is considerably smaller than that in the pure liquid (c_ℓ) and also smaller than that in the mixture of liquid and masses ($c_{\ell m}$). This is because the walls also represent extra mass which further slows down the sound waves. The stiffness added by the angled plates is expected to increase the speed of sound, but this effect is very small for the geometry considered.

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ACKNOWLEDGMENTS

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