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WAVE PROPAGATION ALONG PROFILED CLADDING OF BUILDINGS

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ABSTRACT

This paper presents a model to analyse the acoustic properties of profiled cladding which is found on the facade of many buildings. The cladding is modelled as a periodic structure consisting of long narrow individual panels attached to each other. Hamilton's principle is used to calculate the amplitudes of the different wave types (bending, compressional) on the individual panels. The different wave types are coupled to each other at the edges of each panel. This is modelled by suitable boundary conditions. Bloch's theorem is used to describe the periodicity of the cladding. The model predicts the wave numbers and decay rates of structure-borne waves along the cladding and suggests designs of cladding that reduce sound transmission from buildings into the environment.

1. INTRODUCTION

Profiled cladding is found on the facade of many buildings, especially industrial buildings. Since noisy machinery in such buildings is likely to be an environmental nuisance, it is important to have a model for the acoustic properties of cladding which can be used to predict cladding geometries with minimal sound transmission. The work by Windle (1991) shows the measured transmission properties of profiled cladding with different geometries, in terms of frequency spectra of the sound reduction index. While some features of these spectra can be explained in terms of resonances of cladding sections, others cannot be explained without a detailed mathematical model for the structure-borne sound propagation on profiled walls. The aim of this paper is to present such a model. In our model, the cladding is assumed to be made up of narrow sections of infinitely long panels which are connected at right angles. A cross-section is shown in fig. 1.

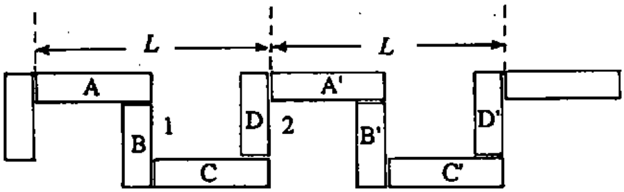


Fig. 1 Cross-section of profiled cladding

The structure is periodic with each repeat (length L), made up of four panels. Our approach is based on Hamilton's principle and Bloch's theorem. Hamilton's principle is used to determine the motion of one repeat, say that made up of panels A, B, C, D; suitable boundary conditions (continuity of displacement and non-rotating connection) are used to describe the connection between adjacent panels. Bloch's theorem is then applied to predict wave propagation in terms of the Bloch attenuation and Bloch propagation wave number which together describe the propagation characteristics of waves crossing many repeats of the profiled cladding.

2. MODEL

The panels A, B, C, D etc. are assumed to be narrow infinitely long Mindlin plates (fig. 2). Only the case where there is no dependence in the z-direction is considered. One-dimensional wave propagation on a Mindlin plate can be described by three independent field quantities:

ξ_M : in-plane compressional motion at central plane of plate (see fig. 2)
η : bending motion of plate
φ : bending angle of plate (see fig. 2)

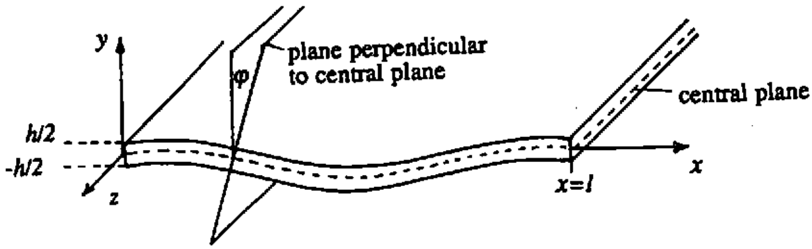


Fig. 2 Geometry of an individual panel in the cladding

The in-plane displacement at any point of the plate (inside it or on the surface) is given by

$$\xi = \xi_M + y\,\varphi\,, \tag{1}$$

and the bending displacement is η. Two different wave types are found on a Mindlin plate for one-dimensional wave propagation: compressional waves giving rise to an in-plane displacement ξ_M and bending waves (propagating and near-field) giving rise to a bending displacement η and bending angle φ,

$$\xi_M = A_1\, e^{ik_D x} + A_2\, e^{-ik_D (x-l)}\,, \tag{2a}$$

$$\eta = A_3\, e^{ik_{Bp} x} + A_4\, e^{-ik_{Bp}(x-l)} + A_5\, e^{k_{Bd}(x-l)} + A_6\, e^{-k_{Bd} x}\,, \tag{2b}$$

$$\varphi = A_7\, e^{ik_{Bp} x} + A_8\, e^{-ik_{Bp}(x-l)} + A_9\, e^{k_{Bd}(x-l)} + A_{10}\, e^{-k_{Bd} x} \tag{2c}$$

These waves are travelling in the positive and negative x-direction with amplitudes A_1, ... A_{10} and the following wave numbers:

k_D for compressional waves,

k_{Bp} for propagating bending waves,
k_{Bd} for the bending wave near field.

It is assumed in (2) that forces act only along the plate edges but not at any other positions between the edges. The above wave numbers are then the wave numbers of a free Mindlin plate which are given by (see Cremer and Heckl 1993 section 2.8.2 for the bending wave numbers)

$$k_D = \frac{\omega}{c_D}, \tag{3a}$$

$$k_{Bp}^2 = \frac{1}{2}\left[\omega^2\left(\frac{1}{c_T^2}+\frac{1}{c_D^2}\right)+\sqrt{\omega^4\left(\frac{1}{c_T^2}-\frac{1}{c_D^2}\right)^2+4\ \omega^2\ \frac{\rho h}{B'}}\ \right], \tag{3b}$$

$$k_{Bd}^2 = \frac{1}{2}\left[\omega^2\left(\frac{1}{c_T^2}+\frac{1}{c_D^2}\right)-\sqrt{\omega^4\left(\frac{1}{c_T^2}-\frac{1}{c_D^2}\right)^2+4\ \omega^2\ \frac{\rho h}{B'}}\ \right], \tag{3c}$$

where ω is the frequency, ρ is the density of the plate material, h is the plate thickness, B' is the bending stiffness of the plate and c_D and c_T are the propagation speeds of compressional and shear waves respectively, given by

$$c_T^2 = \frac{G}{\rho}, \tag{4a}$$

$$c_D^2 = \frac{E}{\rho(1-\mu^2)}. \tag{4b}$$

G is the shear modulus, E is the Young's modulus and μ is the Poisson ratio of the plate. B' is given by

$$B' = \frac{Eh^3}{12(1-\mu^2)}. \tag{5}$$

Hamilton's principle states that a structure moves in such a way that

$$\int_{t_o}^{t_1} (E_{kin} - E_{pot} + W_{ext})\, dt \text{ is a minimum.} \tag{6}$$

E_{kin} and E_{pot} are the kinetic and potential energy of the structure and W_{ext} is the external work applied to the structure. E_{kin} is given by the volume integral over the kinetic energy density,

$$E_{kin}=\frac{1}{2}\int_{x=0}^{l}\int_{y=-h/2}^{h/2}\int_{z=-\infty}^{\infty}\rho\,(\dot{\xi}^{2}+\dot{\eta}^{2})\,dx\,dy\,dz\,, \tag{7a}$$

E_{pot} is given by

$$E_{pot}=\frac{1}{2}\int_{x=0}^{l}\int_{y=-h/2}^{h/2}\int_{z=-\infty}^{\infty}(\sigma_x\,\varepsilon_x+\tau_{xy}\,\gamma_{xy})\,dx\,dy\,dz\,, \tag{7b}$$

and W_{ext} is given by

$$W_{ext}=\frac{1}{2}\,real\left[F_\xi\,\xi_{|x=x_F}+F_\eta\,\eta_{|x=x_F}+M\,\varphi_{|x=x_F}\right] \tag{7c}$$

The dot denotes the time derivative, σ_x and ε_x are the longitudinal stress and strain respectively, τ_{xy} and γ_{xy} are shear stress and strain respectively, F_ξ, F_η are line forces in the in-plane and bending direction acting at $x=x_F$; a moment M exciting motions of φ at $x=x_F$ also contributes to the external work. The stresses and strains are given by Crighton et al. (1992, section 9.2.2) in terms of x-derivatives of the field quantities ξ and η and in terms of the elastic moduli E and G. The fields (2) are now inserted into (6), the integration over x and y is performed analytically and the minimisation is carried out with respect to the unknown amplitutes A_1, ... A_{10}. When differentiating with respect to the amplitudes, which are generally complex, one has to differentiate with respect to their real and imaginary part, and it is convenient to use the notation

$$\frac{\partial}{\partial A_n}=\frac{\partial}{\partial\,(real\ A_n)}+i\,\frac{\partial}{\partial\,(imag\ A_n)},\qquad (n=1,\ldots 10)\,. \tag{8}$$

The setting of these derivatives equal to zero yields a linear set of equations for the unknowns A_1, ... A_{10} which is inhomogeneous with the forces F_ξ, F_η and the moment M on the right hand side. The set of equations can be written symbolically as

$$\left[\;a_{mn}\;\right]\left[\;A_n\;\right]=\left[\;F_A\;\right], \tag{9}$$

where a_{ij} is a 10×10 matrix. The large number of elements make it impossible to list them here. They are expressions depending on material properties such as E, G, ρ, μ, on geometrical characteristics, such as h, l and on the free wave numbers k_D, k_{Bp} and k_{Bd}. The matrix is equal to the complex-conjugate of its transpose. The right hand side is a vector with 10 components which depend linearly on the forces and the moment. Matrix equations analogous to (9) hold for the other panels, e.g. B, C and D.

This method for a single panel can be extended to treat a series of connected panels, say A, B, C, D. A matrix equation such as (9) is written down for each individual panel. The connection between the panels is described by the following conditions, e.g. for the interface between A and B,

$$\left[\xi_M^{(A)} - \frac{h_B}{2} \varphi^{(A)} \right] \Big|_{x=l_A} = \eta^{(B)} \Big|_{y=0} , \tag{10a}$$

(continuity of displacement in the x-direction)

$$\eta^{(A)} \Big|_{x=l_A} = \left[\xi_M^{(B)} + \frac{h_B}{2} \varphi^{(B)} \right] \Big|_{y=0} , \tag{10b}$$

(continuity of displacement in the y-direction)

$$\frac{\partial \eta^{(A)}}{\partial x} \Big|_{x=l_A} = - \varphi^{(B)} \Big|_{y=0} . \tag{10c}$$

(non-rotating connection between the plates)

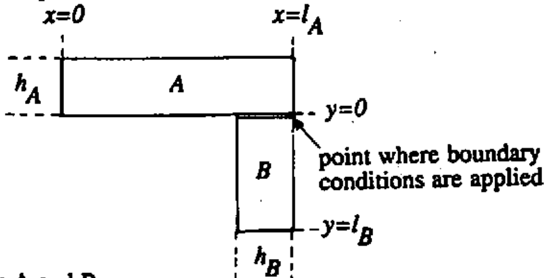
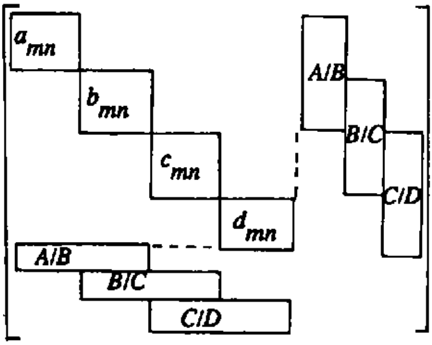


Fig. 3 Connection between panels A and B

The notation is chosen in such a way that ξ always stands for an in-plane displacement and η for a bending displacement. On plate B, ξ is in the y-direction and η in the x-direction. (10a) and (10b) describe the conversion of bending waves into in-plane waves and vice versa at the interface between panels A and B.

Equations (2) and expressions analogous to (2) for the fields on panel B can be inserted into (10a) to (10c) and the three resulting equations added to a set of equations made up of (9) and an equivalent matrix equation for panel B, using Lagrange multipliers. The boundary conditions between plates B/C and C/D yield another three equations each and are added in the same way to a rather larger set of equations. The full set of equations for the amplitudes $A_1, \dots A_{10}$ on panel A, $B_1, \dots B_{10}$ on panel B, $C_1, \dots C_{10}$ on panel C and $D_1, \dots D_{10}$ on panel D can be written symbolically as



$$\cdot \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \\ \lambda_i \end{bmatrix} = \begin{bmatrix} F_A \\ F_B \\ F_C \\ F_D \\ 0 \end{bmatrix} \qquad (11)$$

a_{mn}, b_{mn}, c_{mn} and d_{mn} are square submatrices for plates A, B, C and D and the rectangular sub-matrices stand for the boundary conditions at the interfaces between the plates. The remaining elements of the matrix are zero. The unknowns include the amplitudes of all wave types on the four plates as well as the Lagrange multipliers λ_i ($i=1, \dots 9$). F_A, F_B, F_C and F_D symbolise the inhomogeneous terms produced by the external forces along the plate edges. The matrix has dimension 49×49 and the matrix equation (11) has to be solved numerically.

This procedure is a specialized version of the finite element method. The individual panels constitute the elements, the edges of the panels are the nodal lines and the fields given by equation (2) represent the shape functions or interpolation functions. Here they are the exact solutions for the interior field; this makes it possible to choose elements which are large compared with the scale of the waves.

In order to apply Bloch's theorem, it is necessary to know how the field quantities are related in points that are exactly one period apart. This relation is expected to be a matrix equation with the following structure,

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & & & & & \cdot \\ p_{31} & & & & & \cdot \\ p_{41} & & & & & \cdot \\ p_{51} & & & & & \cdot \\ p_{61} & \cdot & \cdot & \cdot & \cdot & p_{66} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \eta_1 \\ \varphi_1 \\ F_{\xi 1} \\ F_{\eta 1} \\ M_1 \end{bmatrix} = \begin{bmatrix} \xi_2 \\ \eta_2 \\ \varphi_2 \\ F_{\xi 2} \\ F_{\eta 2} \\ M_2 \end{bmatrix}. \quad (12)$$

The field quantities with subscripts 2 refer to point 2 which is one period from point 1; the field quantities in point 1 have subscript 1. The period considered is that consisting of panels A, B, C and D (fig. 1). Point 1 is chosen as the left edge of panel A and point 2 as the left edge of panel A'. It is assumed that forces act at only two lines on this section of the structure: One line is along the left edge of panel A and the other is along the top edge of panel D. While at this stage the forces are considered to be external forces, they will later be interpreted as a simulation of the interaction of the cladding section ABCD with the cladding panels on either side of this section. For a given set of forces $F_{\xi 1}, F_{\eta 1}, M_1, F_{\xi 2}, F_{\eta 2}, M_2$, Hamilton's principle gives all the amplitudes on the four panels which allow one to determine the displacements $\xi_1, \eta_1, \varphi_1, \xi_2, \eta_2, \varphi_2$. If 6 linearly independent sets of forces $F_{\xi 1}^{(i)}, F_{\eta 1}^{(i)}, M_1^{(i)}, F_{\xi 2}^{(i)}, F_{\eta 2}^{(i)}, M_2^{(i)}$ $(i=1, \dots 6)$ are chosen, 6 sets of displacements $\xi_1^{(i)}, \eta_1^{(i)}, \varphi_1^{(i)}, \xi_2^{(i)}, \eta_2^{(i)}, \varphi_2^{(i)}$ are obtained. The rows of the matrix p'_{ij} relating the displacements and forces at the end points of ABCD can now be calculated from the following set of linear equations,

$$\begin{bmatrix} \xi_1^{(1)} & \eta_1^{(1)} & \varphi_1^{(1)} & F_{\xi 1}^{(1)} & F_{\eta 1}^{(1)} & M^{(1)} \\ \xi_1^{(2)} & & & & & \cdot \\ \xi_1^{(3)} & & & & & \cdot \\ \xi_1^{(4)} & & & & & \cdot \\ \xi_1^{(5)} & & & & & \cdot \\ \xi_1^{(6)} & \cdot & \cdot & \cdot & \cdot & M^{(6)} \end{bmatrix} \begin{bmatrix} p'_{11} & p'_{21} & p'_{31} & p'_{41} & p'_{51} & p'_{61} \\ p'_{12} & & & & & \cdot \\ p'_{13} & & & & & \cdot \\ p'_{14} & & & & & \cdot \\ p'_{15} & & & & & \cdot \\ p'_{16} & \cdot & \cdot & \cdot & \cdot & p'_{66} \end{bmatrix} = \begin{bmatrix} \xi_2^{(1)} & \eta_2^{(1)} & \varphi_2^{(1)} & F_{\xi 2}^{(1)} & F_{\eta 2}^{(1)} & M_2^{(1)} \\ \xi_2^{(2)} & & & & & \cdot \\ \xi_2^{(3)} & & & & & \cdot \\ \xi_2^{(4)} & & & & & \cdot \\ \xi_2^{(5)} & & & & & \cdot \\ \xi_2^{(6)} & \cdot & \cdot & \cdot & \cdot & M_2^{(6)} \end{bmatrix}$$

(13)

In order to have a relation between point 1 and the equivalent point 2 on A' which is on the the right hand side of the interface between D and A', the signs of the forces $F_{\xi 2}, F_{\eta 2}, M$, have to be changed according to Newton's second law. The matrix p_{ij} is obtained from the matrix p'_{ij} by changing the signs of the elements of the 4th, 5th and 6th column. It is now possible to apply Bloch's theorem, which requires that

$$\begin{bmatrix} \xi_2 \\ \eta_2 \\ \varphi_2 \\ F_{\xi 2} \\ F_{\eta 2} \\ M_2 \end{bmatrix} = e^{\gamma L} \begin{bmatrix} \xi_1 \\ \eta_1 \\ \varphi_1 \\ F_{\xi 1} \\ F_{\eta 1} \\ M_1 \end{bmatrix}, \tag{14}$$

where γ is the so-called Bloch wave number and L is the length of one period. γ is generally complex,

$$\gamma = \alpha + i\,k\,, \tag{15}$$

where the real part α denotes the Bloch attenuation which is the attenuation of the wave from repeat to repeat. The imaginary part k is the Bloch propagation wave number; it is related to the speed of waves propagating along the cladding.

Substitution for the left hand side of (14) using (12) leads to the following eigenvalue problem,

$$det\,(P - I\,e^{\gamma L}) = 0\,, \tag{16}$$

where P is the matrix with elements p_{ij} and I is a 6×6 unit matrix. If expanded out, (16) would lead to a 6th-order polynomial in $e^{\gamma L}$ whose roots are the eigenvalues. The problem can be simplified by exploiting the symmetry properties of the structure: waves must be able to propagate in either direction along the cladding with the same Bloch wave number, except for the sign. As a consequence, $e^{-\gamma L}$ is also an eigenvalue if $e^{\gamma L}$ is an eigenvalue. After some lengthy manipulations which are not shown here, (16) can be reduced to a cubic equation for $cosh(\gamma L)$. There are three complex roots, from which the three corresponding Bloch wave numbers γ_1, γ_2, γ_3 can be calculated. $-\gamma_1$, $-\gamma_2$, $-\gamma_3$ are also solutions.

3. NUMERICAL RESULTS

Numerical calculations have been performed for steel cladding with

$$\rho = 8000\ \text{kg m}^{-3}\,,$$
$$E = 2 \cdot 10^{11}\ \text{N m}^{-2}\,,$$
$$G = 0.77 \cdot 10^{11}\ \text{N m}^{-2}\,,$$
$$\mu = 0.3\,,$$

for four different geometries. They are listed with the corresponding figure numbers in table 1.

l_A	l_B	l_C	h_A	figure number
0.25	0.15	0.25	0.005	4a
0.25	0.15	0.25	0.004	4b
0.35	0.15	0.15	0.005	4c
0.20	0.15	0.20	0.005	4d

Table 1 Simulated cladding geometries (measurements given in metres)

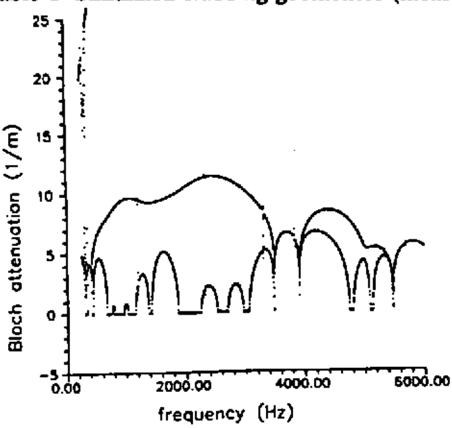


Fig. 4a

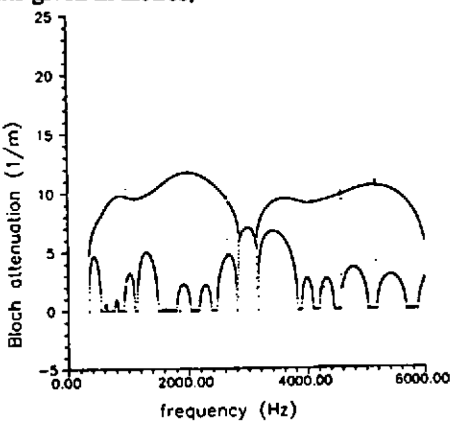


Fig. 4b

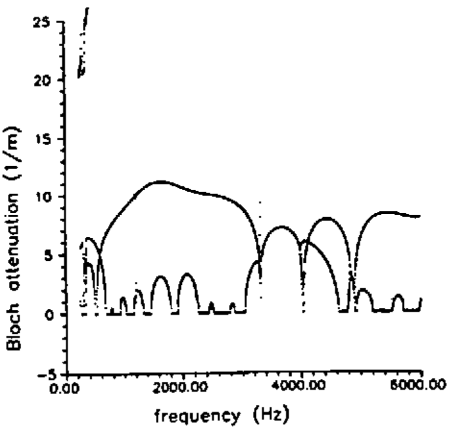


Fig. 4c

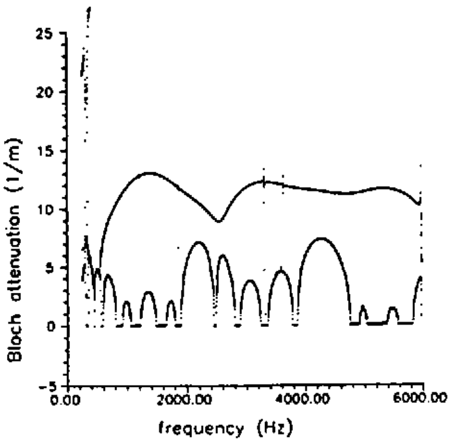


Fig. 4d

l_A, l_B, l_C and l_D denote the width of panels A, B, C and D. l_D was always chosen to be equal to l_B. The quantities h_A, h_B, h_C, h_D are the individual panel thicknesses; they were kept equal to each other. The figures show the real parts α_1, α_2, α_3 of the three solutions for the Bloch wave number as a function of frequency. One of the solutions tends to be well beyond the plotted attenuation range. This corresponds to a rather strongly attenuated wave and is of limited physical interest. Numerical inaccuracies are responsible for the rather disorderly curves at low frequencies (below 500 Hz).

4. DISCUSSION AND OUTLOOK

Waves propagating across a periodic system are expected to show passing and stopping bands in their frequency spectra. When a periodic system is such that it supports waves of more than one Bloch wave number, the meaning of a passing and stopping band will have to be extended:

Passing band: at least one wave propagates with zero attenuation
Stopping band: all waves propagate with non-zero attenuation

The curves shown in figure 4 clearly show such passing and stopping bands. On the whole, the stopping bands tend to dominate; this is most pronounced for the geometry of fig. 4a, where unattenuated sound propagation seems virtually impossible above 3000 Hz. This finding could be exploited to minimise sound transmission through cladding.

Further studies to determine the relative amplitudes of the three Bloch waves are planned. It is possible to extend the model so that it applies to cladding where the angle between adjacent panels is not necessarily a right angle. The treatment of waves oblique to the edges of cladding is possible but needs considerably more analytical and numerical effort.

The predictions of this model can be of significant value in the design of caldding to reduce noise transmission into the environment.

5. REFERENCES

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