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A 3-D OCEAN-ACOUSTIC PARABOLIC PROPAGATION MODEL IN A VARIABLE ENVIRONMENT

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1. INTRODUCTION

In the ocean the variations of sound velocity, due to temperature, salinity and pressure, are usually weak, though essential for sound propagation. But, in the presence of eddies or currents, sudden changes with range and azimuth of the velocity can occure at certain depths. Horizontal variations of the sound speed cannot be neglected any longer, leading to the use of three-dimensional models.

In underwater acoustics, the acoustic pressure p is usually taken as the solution of a wave equation in cylindrical coordinates (r, Z, θ) where r stands for the (horizontal) range, Z for the (vertical) depth and θ for the azimuth (in an horizontal plane). The boundary condition at the free surface, i.e. in $Z = 0$, is a release condition on the pressure : $p = 0$. At the bottom $Z = H$, the situation is much more complicated. The simple case of a perfectly reflective bottom leads to a Neumann boundary condition $\frac{\partial p}{\partial \upsilon} = 0$ (normal derivative).

But usually several sedimental layers have to be taken into account, and the bottom is not flat. At infinity an outgoing radiation condition is usually assumed. It is incorporated, by introducing the acoustic field v as the envelop of the pressure p by the formula :

$$p(r, \theta, Z) = H_0^{(1)}(k_0 r)\, v(r, \theta, Z) \qquad (1)$$

where $H_0^{(1)}$ denotes the Hankel function of order 0 of the first kind.

The parabolic approximation has been popularized in the underwater acoustics community by F.D. Tappert under the form:

$$\frac{\partial v}{\partial r} = \frac{i}{2k_0}\left(\frac{\partial^2 v}{\partial Z^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}\right) + i\frac{k_0}{2}(n^2 - 1)\, v \qquad (2)$$

in the simplest case (linear approximation). The backscattered waves are not taken into account into the model. In (2) a reference constant sound speed c_0 being fixed, the reference wave number k_0 and the refraction index $n(r, \theta, Z)$ are defined by :

$$k_0 = \frac{2\pi f}{c_0} \quad \text{and} \quad n^2 = \frac{c_0^2}{c^2}\left(1 + i\,\frac{\beta}{27.2875}\right) \qquad (3)$$

where f stands for the source frequency, β being the attenuation coefficient in water or sediments and $i = \sqrt{-1}$.

More sophisticated models can be built, allowing larger apertures of the source. An example is the quadratic model of B. Grandvuillemin developed at C.E.R.D.S.M. :

$$\frac{\partial v}{\partial r} = \frac{i}{2k_0}\left(-\frac{1}{4k_0^2}\frac{\partial^4 v}{\partial Z^4} + \left(1 - \frac{n^2-1}{4}\right)\frac{\partial^2 v}{\partial Z^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}\right)$$
$$+ i\frac{k_0}{2}\left((n^2-1) - \frac{1}{4}(n^2-1)^2 - \frac{1}{4k_0^2}\frac{\partial^2}{\partial Z^2}(n^2-1)\right)v \qquad (4)$$

For the sake of simplicity, we will restrict ourselves to the linear model (2).

2. BOTTOM MODELISATION AND THERMIC FRONTS

The environment is defined by a range r running from 0 to R_{max}, an azimuth θ running from 0 to 2π, and a variable depth Z running from 0 to $H = Z_{max}(r, \theta)$. The bottom is made of homothetic sedimental layers, each of them having its own density ρ and attenuation β. The transmission conditions at an interface between medium 1 and medium 2 for the acoustic pressure are known to be :

$$p_1 = p_2 \qquad \frac{1}{\rho_1}\frac{\partial p_1}{\partial \upsilon} = \frac{1}{\rho_2}\frac{\partial p_2}{\partial \upsilon} \qquad (5)$$

In the case of non horizontal interfaces, condition (5) will lead to unusual transmission conditions on the acoustic field v, involving v and a transverse derivative of v.

Now, in order to release the standard cylindrical-symetry hypothesis, and allow the treatment of sloping ocean bottoms, we use an affine mapping which sends the variable interval $]0, Z_{max}(r, \theta)[$ onto the reference interval]0, 1[, that is :

$$z = Z\omega(r, \theta), \qquad \omega(r, \theta) = \frac{1}{Z_{max}(r, \theta)} \qquad (6)$$

Setting $\quad u(r, \theta, z) = v(r, \theta, Z) = v\left(r, \theta, \frac{z}{w(r, \theta)}\right)$

we can write equation (2) in terms of u and z ; we obtain :

$$\frac{\partial u}{\partial r} = ia(r, \theta, z)\,u + \left(-\frac{z}{\omega}\frac{\partial \omega}{\partial r} - \frac{ib}{r^2}\left(\frac{\partial \omega}{\partial \theta}\right)^2\frac{z}{\omega^2} + \frac{ib}{r^2}\frac{z}{\omega}\frac{\partial^2 \omega}{\partial \theta^2} + \frac{ib}{r^2}\frac{1}{\omega}\frac{\partial \omega}{\partial \theta}\right)\frac{\partial u}{\partial z}$$
$$+ 2\frac{ib}{r^2}\frac{z}{\omega}\frac{\partial \omega}{\partial \theta}\frac{\partial^2 u}{\partial \theta \partial z} + ib\left(\left(\omega^2 + \frac{z^2}{r^2\omega^2}\left(\frac{\partial \omega}{\partial \theta}\right)^2\right)\frac{\partial^2 u}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) \qquad (7)$$

with

$$a(r, \theta, z) = \frac{k_0}{2}\left(n^2\left(r, \theta, \frac{z}{\omega}\right) - 1\right), \qquad b = \frac{1}{2k_0} \qquad (8)$$

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Let $\vec{\upsilon} = (\upsilon_r, \upsilon_\theta, \upsilon_Z)$ be the normal unitary vector at the bottom ; the boundary conditions are easily derived in r = 0 and on the surface z = 0 :

$$u(0, \theta, z) = u_0(\theta, z) \qquad u(r, \theta, 0) = 0 \tag{9}$$

At the bottom, z = 1, we obtain from (5) :

$$\left(\frac{\partial u}{\partial r} + \frac{z}{\omega}\frac{\partial \omega}{\partial r}\frac{\partial u}{\partial z} + i k_0 u\right)\upsilon_r + \frac{1}{r}\left(\frac{\partial u}{\partial \theta} + \frac{z}{\omega}\frac{\partial \omega}{\partial \theta}\frac{\partial u}{\partial z}\right)\upsilon_\theta + \omega\frac{\partial u}{\partial z}\upsilon_z = 0 \tag{10}$$

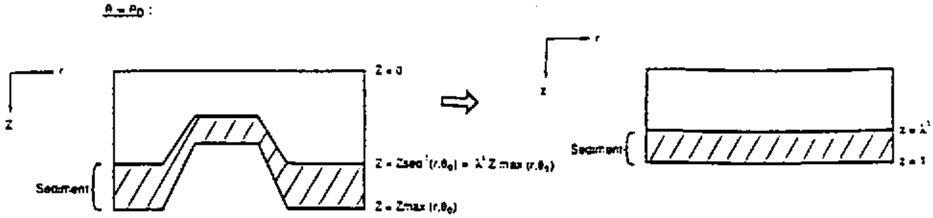
Periodicity in θ is moreover assumed.

Let us now consider Q sedimental layers. We shall denote by $Z = Zsed^q(r, \theta)$ the equation of the interface between layers (q - 1) and q, layer 0 being the water layer, and make the following homotheticity hypothesis :

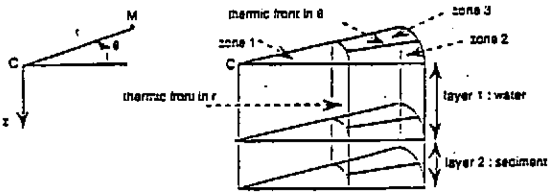
$\exists$ Q constants λ^q, $q = 1 ... Q$, $0 < \lambda_1 < \lambda_2 < ... < \lambda_Q < 1$

$$\text{such that } Zsed^q(r, \theta) = \lambda^q Z_{max}(r, \theta) \tag{11}$$

Under hypothesis (11) our change of variable (6) will transform the Q interfaces $Z = Zsed^q(r, \theta)$ into horizontal planes $z = \lambda^q$, $q = 1 ... Q$, parallel to the bottom $z = 1$.



To take thermic fronts in range and azimuth into account, the environment is split up into different zones. In each zone the sound speed is a function of Z only and is assumed to be continuous in each layer. Each layer of a zone has its own attenuation.



On each interface (thermic front in range or azimuth, interface between layers) we have to derive transmission conditions in the new variables (r, θ, z).

3. INTERFACE CONDITIONS

As usual, the equations concerning the different interfaces are obtained using the transmission conditions (5), the parabolic equation (7) written in each of the two media, as well as Taylor developments for $u(r, \theta, z)$ in the vicinity of the interface. We thus need to derive from (5) the appropriate transmission conditions on u.

A normal direction $\vec{\upsilon} \neq \vec{0}$ along a smooth surface (σ) with equation

$$(\sigma): \quad \zeta(r, \theta, Z) = 0 \tag{12}$$

is given by $\vec{\upsilon} = \left(\frac{\partial \zeta}{\partial r}(r, \theta, Z)\ ,\ \frac{1}{r}\frac{\partial \zeta}{\partial \theta}(r, \theta, Z)\ ,\ \frac{\partial \zeta}{\partial Z}(r, \theta, Z)\ \right)^T$

so that the normal derivative of p in terms of u given by (10) writes, setting $\vec{\nabla}_H = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}\right)^T$

$$\frac{\partial u}{\partial \upsilon} = \vec{\nabla}_H \zeta . \vec{\nabla}_H u + \left(\frac{z}{\omega}\vec{\nabla}_H \omega . \vec{\nabla}_H \zeta + \omega^2 \frac{\partial \zeta}{\partial z}\right)\frac{\partial u}{\partial z} + i k_0 \frac{\partial \zeta}{\partial r} u \tag{13}$$

We check that, in the case of a flat interface, when $\zeta(r, \theta, Z) = Z - C$ (C constant)

$$\vec{\nabla}_H \zeta = \vec{0} \quad \text{and} \quad \frac{\partial \zeta}{\partial z} = \frac{1}{\omega} \quad \text{implie} \quad \frac{\partial u}{\partial \upsilon} = \omega\frac{\partial u}{\partial z} = \frac{\partial v}{\partial Z}$$

In the general case, the transmission equations (5) across the interface (σ) are straightforward from (13):

$$u_1(r, \theta, z) = u_2(r, \theta, z) \quad \text{on } (\sigma) \tag{14}$$

$$\frac{1}{\rho_1}\left[\vec{\nabla}_H \zeta\left(\vec{\nabla}_H u_1 + \frac{z}{\omega}\vec{\nabla}_H \omega \frac{\partial u_1}{\partial z}\right) + \omega^2 \frac{\partial \zeta}{\partial z}\frac{\partial u_1}{\partial z} + i k_0 \frac{\partial \zeta}{\partial r} u_1\right] =$$

$$\frac{1}{\rho_2}\left[\vec{\nabla}_H \zeta\left(\vec{\nabla}_H u_2 + \frac{z}{\omega}\vec{\nabla}_H \omega \frac{\partial u_2}{\partial z}\right) + \omega^2 \frac{\partial \zeta}{\partial z}\frac{\partial u_2}{\partial z} + i k_0 \frac{\partial \zeta}{\partial r} u_2\right] \quad \text{on } (\sigma) \tag{15}$$

Tangential derivation of (14) combined with (15) leads in the case where $\frac{\partial \zeta}{\partial z} \neq 0$ to the two relations:

$$\vec{D} u_1 = \vec{D} u_2 \quad \text{on } (\sigma) \tag{16}$$

$$\frac{1}{\rho_1}\left[\vec{\nabla}_H \zeta . \vec{D} u_1 + |\overrightarrow{grad}\, \zeta|^2 \frac{\partial u_1}{\partial z} + i k_0 \frac{\partial \zeta}{\partial r} u_1\right] =$$

$$\frac{1}{\rho_2}\left[\vec{\nabla}_H \zeta . \vec{D} u_2 + |\overrightarrow{grad}\, \zeta|^2 \frac{\partial u_2}{\partial z} + i k_0 \frac{\partial \zeta}{\partial r} u_2\right] \quad \text{on } (\sigma) \tag{17}$$

where $\overrightarrow{grad} = \left(\vec{\nabla}_H, \frac{\partial}{\partial Z}\right)^T$ and $\vec{D} = \frac{\partial \zeta}{\partial z}\vec{\nabla}_H + \left(\frac{z}{\omega}\vec{\nabla}_H \omega \frac{\partial \zeta}{\partial z} - \vec{\nabla}_H \zeta\right)\frac{\partial}{\partial z}$

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In the case of a flat interface $\zeta(r,\theta,Z) = Z - C$ (C constant), $\frac{\partial \zeta}{\partial z} = \frac{1}{\omega}$, $\vec{\nabla}_H \zeta = \vec{0}$, $|\,\overrightarrow{\mathrm{grad}}\,\zeta\,| = 1$

and we recover exactly by (17) the usual condition : $\frac{1}{\rho_1}\frac{\partial u_1}{\partial z} = \frac{1}{\rho_2}\frac{\partial u_2}{\partial z}$

4. PARABOLIC EQUATION SATISFIED BY THE FIELD U ON THE INTERFACE BETWEEN TWO SEDIMENTAL LAYERS

Under hypothesis (11), the equation of the interface can be chosen as :

$$\zeta(r,\theta,Z) = \omega(r,\theta)\,Z - \lambda = z - \lambda = 0 \tag{18}$$

and thus $\frac{\partial \zeta}{\partial z} = 1 \quad \vec{\nabla}_H \zeta = \frac{\lambda}{\omega}\vec{\nabla}_H \omega$. This implies : $\vec{D} = \vec{\nabla}_H$ and (14), (16), (17) read :

$$u_1(r,\theta,\lambda) = u_2(r,\theta,\lambda) \qquad \vec{\nabla}_H\, u_1(r,\theta,\lambda) = \vec{\nabla}_H\, u_2(r,\theta,\lambda) \tag{19}$$

$$\frac{1}{\rho_1}\frac{\partial u_1}{\partial z} - \frac{1}{\rho_2}\frac{\partial u_2}{\partial z} = -\left(\frac{\lambda^2}{\omega^2}|\vec{\nabla}_H \omega|^2 + \omega^2\right)^{1}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)\left(\frac{\lambda}{\omega}\vec{\nabla}_H \omega . \vec{\nabla}_H u\right) \text{ on } z = \lambda \tag{20}$$

Moreover, equation (7) holds in each medium, k = 1, 2:

$$\frac{\partial u_k}{\partial r} = ia_k(r,\theta,z)\,u_k + (\alpha(r,\theta,z) + i\beta(r,\theta,z))\frac{\partial u_k}{\partial z} + i\gamma(r,\theta,z)\frac{\partial^2 u_k}{\partial\theta\partial z} + i\delta(r,\theta,z)\frac{\partial^2 u_k}{\partial z^2} + \frac{ib}{r^2}\frac{\partial^2 u_k}{\partial\theta^2}$$

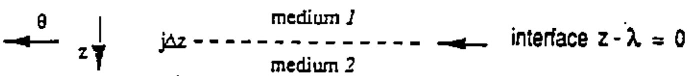
$$a_k = \frac{k_0}{2}\left(n^2(r,\theta,\frac{z}{\omega}) - 1\right) \qquad b = \frac{i}{2k_0} \tag{21}$$

$$\alpha(r,\theta,z) = -\frac{z}{\omega}\frac{\partial\omega}{\partial r} \qquad \beta(r,\theta,z) = \frac{b}{r^2}\left(-\frac{z}{\omega^2}\left(\frac{\partial\omega}{\partial\theta}\right)^2 + \frac{z}{\omega}\frac{\partial^2\omega}{\partial\theta^2} + \frac{1}{\omega}\frac{\partial\omega}{\partial\theta}\right)$$

$$\gamma(r,\theta,z) = \frac{2b}{r^2} + \frac{z}{\omega}\frac{\partial\omega}{\partial\theta} \qquad \delta(r,\theta,z) = b\left(\omega^2 + \frac{z^2}{r^2\omega^2}\left(\frac{\partial\omega}{\partial\theta}\right)^2\right)$$

Δr, $\Delta\theta$, Δz denoting the steps in range, azimuth and depth we use the following notation:

$u(n\Delta r, l\Delta\theta, j\Delta z) = u^n_{l,j}$.



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We use Taylor-formulae to evaluate the second derivatives $\frac{\partial^2 u_k}{\partial z^2}$ and $\frac{\partial^2 u_k}{\partial\theta\partial z}$, $k = 1, 2,$ on (σ).

Setting the results in (21) according to $k = 1$ or 2 we get, with (19):

$$\left(\frac{\partial u}{\partial r}\right)^n_{l,j} - i(a_1 u)^n_{l,j} - \frac{i\gamma^n_{l,j}}{2\Delta\theta\Delta z}\left(u^n_{l-1,j-1} - u^n_{l+1,j-1}\right) - \frac{2i\delta^n_{l,j}}{(\Delta z)^2}\left(u^n_{l,j-1} - u^n_{l,j}\right) - \frac{i\gamma^n_{l,j}}{\Delta z}\left(\frac{\partial u}{\partial\theta}\right)^n_{l,j}$$
$$- \frac{ib}{(n\Delta r)^2}\left(\frac{\partial^2 u}{\partial\theta^2}\right)^n_{l,j} = \left(\alpha + i\beta + \frac{2i\delta}{\Delta z}\right)^n_{l,j}\left(\frac{\partial u_1}{\partial z}\right)^n_{l,j} \qquad (22)$$

and

$$\left(\frac{\partial u}{\partial r}\right)^n_{l,j} - i(a_2 u)^n_{l,j} + \frac{i\gamma^n_{l,j}}{2\Delta\theta\Delta z}\left(u^n_{l+1,j+1} - u^n_{l-1,j+1}\right) - \frac{2i\delta^n_{l,j}}{(\Delta z)^2}\left(u^n_{l,j+1} - u^n_{l,j}\right) + \frac{i\gamma^n_{l,j}}{\Delta z}\left(\frac{\partial u}{\partial\theta}\right)^n_{l,j}$$
$$- \frac{ib}{(n\Delta r)^2}\left(\frac{\partial^2 u}{\partial\theta^2}\right)^n_{l,j} = \left(\alpha + i\beta - \frac{2i\delta}{\Delta z}\right)^n_{l,j}\left(\frac{\partial u_2}{\partial z}\right)^n_{l,j} \qquad (23)$$

We are now ready to use the transmission condition(20) to obtain :

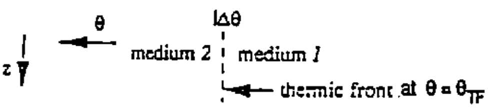
$$\left(\frac{\partial u}{\partial r}\right)^n_{l,j} = C\, u^n_{l,j} + d\left(u^n_{l-1,j-1} - u^n_{l+1,j-1}\right) + e\left(u^n_{l+1,j+1} - u^n_{l-1,j+1}\right) + f\, u^n_{l,j-1}$$
$$+ g\, u^n_{l,j+1} + s\left(\frac{\partial u}{\partial\theta}\right)^n_{l,j} + t\left(\frac{\partial^2 u}{\partial\theta^2}\right)^n_{l,j} \qquad (24)$$

the coefficients depending on b, ρ_1, ρ_2; discrete values of a_1, a_2, ω; steps Δr, $\Delta\theta$, Δz.

5. EQUATION ON A THERMIC FRONT

The following equations have been established in [1] in two simple cases.

5.1 Thermic front at $\theta = \theta_{TF}$



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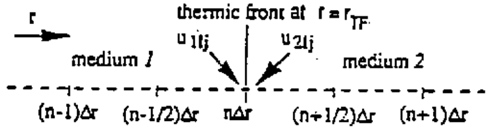
We assume that, around $\theta = \theta_{TF}$, $\dfrac{\partial \omega}{\partial \theta} = 0$ (horizontal in θ)

A Taylor development in θ around θ_{TF} and the interface conditions lead to :

$$\left(\frac{\partial u}{\partial r}\right)^n_{l,j} = ib\omega^2\left(\frac{\partial^2 u}{\partial z^2}\right)^n_{l,j} - \left(\frac{i\Delta z}{\omega}\,\frac{\partial \omega}{\partial r}\,\frac{\partial u}{\partial z}\right)^n_{l,j} + \left(\frac{a_1 + a_2}{2} - \frac{2ib}{(n\Delta r)^2\,(\Delta\theta)^2}\right)u^n_{l,j}$$
$$+\frac{ib}{(n\Delta r)^2\,(\Delta\theta)^2}\left(u^n_{l,j-1} + u^n_{l,j+1}\right) \tag{25}$$

5.2 Thermic front at $r = r_{TF}$



We assume that, around $r = r_{TF}$, $\dfrac{\partial \omega}{\partial r} = 0$ (horizontal in r)

We obtain

$$u^{n+1/2} = -u^{n-1/2} + 2u^n + \frac{\Delta r}{4}\left(\frac{\partial u^{n+1/2}}{\partial r} - \frac{\partial u^{n-1/2}}{\partial r}\right) \tag{26}$$

where $\left(\dfrac{\partial u^{n-1/2}}{\partial r}\right)$ and $\left(\dfrac{\partial u^{n+1/2}}{\partial r}\right)$ are solutions of :

$$\frac{\partial u}{\partial r} = iau + \frac{ib}{r^2\omega}\left(-\frac{z}{\omega}\left(\frac{\partial \omega}{\partial \theta}\right)^2 + z\,\frac{\partial^2 \omega}{\partial \theta^2} + \frac{\partial \omega}{\partial \theta}\right)\frac{\partial u}{\partial z} + \frac{2ib}{r^2}\,\frac{z}{\omega}\,\frac{\partial \omega}{\partial \theta}\,\frac{\partial^2 u}{\partial \theta \partial z}$$
$$+ ib\left(\omega^2 + \frac{z^2}{r^2\omega^2}\left(\frac{\partial \omega}{\partial \theta}\right)^2\right)\frac{\partial^2 u}{\partial z^2} + \frac{ib}{r^2}\,\frac{\partial^2 \omega}{\partial \theta^2} \tag{27}$$

in which the coefficient a is relative to medium 1 (resp. medium 2) and all the terms are written at range $n - \frac{1}{2}$ (resp. $n + \frac{1}{2}$).

6. NUMERICAL SOLUTION OF THE PROBLEM

6.1 Apart from the interfaces

We discretize equation (7) by the alternating direction method : to progress of one step in range, two half steps are needed. In the first half step, implicit treatment of the terms in u and $\frac{\partial^2 u}{\partial z^2}$ is achieved, the others being treated explicitly. In the second half-step, the second derivative $\frac{\partial^2 u}{\partial \theta^2}$ is the only implicit term.

6.2 Initial and boundary conditions

Discretization of (9) is straight forward. Discretization of (10) leads to a relation of the form:

$$u^{n+1/2}_{l,N+1} = f\left(u^{n+1/2}_{l,N},\ u^{n+1/2}_{l,N-1},\ u^{n}_{l+1,N},\ u^{n}_{l-1,N},\ u^{n}_{l,N}\right) \quad \text{where } N \text{ is such that } N\Delta z = 1$$

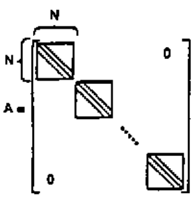
6.3 Matricial system

Therefore, knowing the field $(u^n_{l,j})$ at range $n\Delta r$, one can deduce the sound field at range $(n+1)\Delta r$ after solving two linear systems :

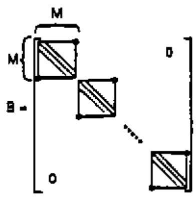
$$A u^{n+1/2} = \bar{B} u^n + F \qquad \bar{A} = 2I - A$$

$$B u^{n+1} = \bar{A} u^{n+1/2} + G \qquad \bar{B} = 2I - B$$

where F and G are two vectors depending only on previously calculated values of the sound field and *A and B* are two block diagonal matrices, each block being itself a tridiagonal matrix or almost one. With indexes in azimuth and depth running from 1 to M and 1 to N, each of those two big matricial systems can be solved using M or N tridiagonal or almost tridiagonal smaller linear systems. These last systems can be solved in parallel.



M blocks



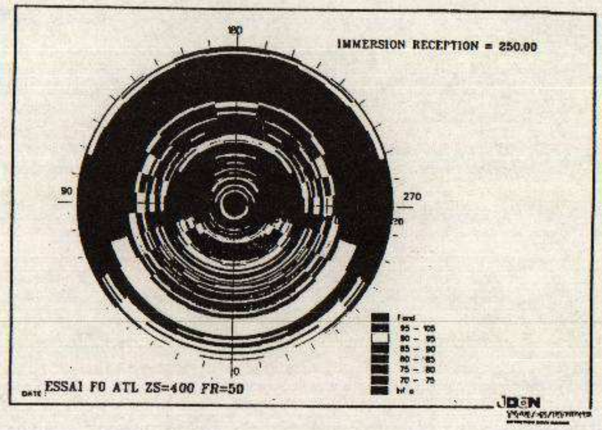
N blocks

6.4 Sedimental layers or thermic fronts

For each range step, solving the problem of sedimental layers or thermic fronts in θ only implies a modification of the coefficients for the row (s) of the matrix A corresponding to the indexes l and j related to the azimuth and the depth of the interface. For a thermic front in the r-direction the method consists in calculating a new starting field $u^{n+1/2}$ in medium 2 from equation (26). In (26) $\frac{\partial u^{n+1/2}}{\partial r}$ $\left(\text{resp.}\frac{\partial u^{n-1/2}}{\partial r}\right)$ is calculated from equation (27).

6.6 Numerical results

The code is vectorised on a CONVEX computer. The results obtained for flat ocean bottoms in the presence of thermic fronts are encouraging [2]. At the present time sloping ocean bottoms are being implemented.



7. CONCLUSION

We are developping a three-dimensional propagation loss model for the case of sloping ocean bottoms, assuming no cylindrical symetry of the geometry of the bottom. The numerical analysis of the full problem has been performed. Some special cases are giving encouraging results. Implementation of the general problem is under development.

8. REFERENCES

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