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# MLS IDENTIFICATION OF THE VOLTERRA KERNELS OF AUDITORY PATHWAYS

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## 1. NONLINEARITY IN EARS

The Ear can be simplistically considered to perform three functions:

1) Impedance matching between the air outside and the fluid inside,

2) Transduction from acoustic stimulus to nerve response,

3) Compression of the dynamic range of acoustic signals which can be perceived (120 dB) to the dynamic range of nerves (60 dB).

The third or these functions implies nonlinearity. So does the fact that sum and difference tones can be heard when two pure tones at different frequencies are presented. More recently attempts have been made to measure and quantify this nonlinearity. There are several reasons for wanting to be able to do this. Several conficting models have been proposed to model the mechanics of the cochlea. Accurate measurements of nonlinearity could be compared with predicitions from the various theories. There is also clinical evidence to sugest that in certain types of hearing pthology the nonlinear part is the first to be affected, which suggests that the ability to measure nonlinear hearing response would provaide an early indicator of potential hearing loss. The nonlinearity of the ear is strongly allied to the ability to detect pitch and other psychoacoustic effects. Quantitative measures of nonlinear effects might supply insight into these phenomena.

## 2. MEASUREMENTS OF AUDITORY RESPONSE

Quantitative characterisations require quantitative data measurements. This has hitherto mainly been from Brainstem Auditory Evoked Responses (BAER). This is a technique whereby the increased activity of the brainstem in response to signals from the ear is measured from electrodes attached to the head. The measurement of the reaponse to a click has become a standard clinical procedure for the detection of neuromas on the acoustic nerve. The procedure is also useful for screening the hearing of neonates since it does not require the active co-operation of the subject. The resulting signals are weak and require a lot of averaging to acheive an acceptable signal to noise ratio. Input-output measurements of the type characterise the entire auditory *and* neural pathway.

Another audiological measurement which has been more recently developed is the evoked otoacoustic emmission. When the ear is subjected to sound the minute hair cells in the cochlea are stimulated and respond by emitting a sound of their own. This sound can be detected by the use of an ear probe which contains both a loudspeaker and a microphone. This type of measurement doesn't involve the neural pathway and doesn't require the application of electrodes. There is evidence to suggest that the distortion product response of the otoacoustic emmission is related to the hearing threshold.

## 3. NONLINEARITY

Green's theorem tells us that the response of a time-invariant linear system $y(t)$ is the convolution of its input $x(t)$ with its impulse response $h(\tau)$:

$$y(t) = \int_0^\infty h(\tau)x(t-\tau)\,d\tau. \tag{1}$$

A wider class of systems which includes some types of nonlinear system is characterised by the Volterra series:

$$\begin{aligned} y(t) \;=\; & \int_0^\infty h_1(\tau_1)x(t-\tau_1)\,d\tau_1 \\ & + \int_0^\infty\int_0^\infty h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2) \\ & + \cdots. \end{aligned} \tag{2}$$

This series will only converge if the system has fading memory, that is the response to any input will become insignificant if enough time is allowed to elapse. Also the system must not contain any discontinuous or hysteretic elements. Fortunately this sort of representation appears to converge for the types of nonlinearity that occur in the ear. If the symmetric functions $h_n(\tau_1, ..., \tau_n)$ known as the Volterra kernels can be identified then the system will be identified. In practice only a finite number of kernels are identified since the series is assumed to be convergent. Nonetheless a large amount of data is required to measure such a system since the kernels' dimensionality increases with order. Since we will be using digital computers to perform the identification we will attempt a discrete-time identification and hence rewrite the Volterra series as:

$$\begin{aligned} y(n) &= \sum_{i=0}^{\infty} h_1(i)x(n-i) \\ &+\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} h_2(i,j)x(n-i)x(n-j). \end{aligned} \tag{3}$$

Most attempts to identify such systems are an extension of the Gaussian white noise cross-correlation technique for linear systems. Unfortunately, even if $x(n)$ is Gaussian white noise, $x(n-i)x(n-j)$ is generally not, so the terms in the Volterra series are not orthogonal with respect to a Gaussian white noise. For this reason, many authors have examined a related series based on Hermite functionals which achieves orthogonality at the expense of the homogeneity of the kernels [2]. Here we shall follow an alternative approach due to Sutter [1], based on the properties of Maximum Length Sequences (MLS) which we shall now go on to examine.

## 4. MAXIMUM LENGTH SEQUENCES

These sequences are usually generated by feedback shift registers but for our purposes can be better understood in terms of polynomials with mod 2 coefficients. Throughout this section addition will be considered to be mod 2 unless explicitly specified. Consider the polynomial $\alpha^4+\alpha+1\equiv 0$ which is known to be 'primitive'. Because we are working mod 2 we can rewrite this equation as $\alpha^4\equiv\alpha+1$. Using this identity we can rewrite any polynomial of degree higher than three as one of degree three or less. In this way we can develop a table of rewritten versions of the powers of $\alpha$ (table 1).

Every possible polynomial of degree three or less, except 0, occurs in this table. This will happen if and only if the generating polynomial is primitive; in fact, for our purposes, this will serve as a definition of primitiveness. The tabulation of the various powers of $\alpha$ in the right-hand four columns should show how this sequence can be generated by a feedback shift register. Now define a function $C[]$ which takes a polynomial as its argument and returns the coefficient of $\alpha^0$ in that polynomial as its value. It is easy to see that $C[]$ is linear, i.e. $C[f]+C[g]\equiv C[f+g]$. Now form the sequence $b(n)=C[\alpha^n]$ with the aforementioned generating function. This is equivalent to tapping off the rightmost bit of the register. The sum of two relatively delayed sequences will be

$$\begin{aligned} b(n)+b(n-i) &= C[\alpha^n+\alpha^{n-i}] \\ &= C[\alpha^n(1+\alpha^{-i})]. \end{aligned} \tag{4}$$

As long as $i\neq 0$ there will exist $f(i)$ such that $1+\alpha^{-i}\equiv\alpha^{-f(i)}$ (this will be true as long as the generating function is primitive). We can then write

$$
\begin{aligned}
b(n) + b(n-i) &= C[\alpha^n \alpha^{-f(i)}] \\
&= C[\alpha^{n-f(i)}] \\
&= b(n-f(i)),
\end{aligned} \tag{5}
$$

i.e. the mod 2 sum of two relatively delayed sequences is another delayed sequence. The truth table for the mod 2 addition of the elements {0,1} is isomorphic to that for multiplication for the elements {1,-1}. Therefore if we form a sequence $w(n) = 1 - 2b(n)$ then we have a sequence which obeys

$$
w(n)w(n-i) = w(n-f(i)). \tag{6}
$$

The function *f(i)*, which we shall call the offset function will depend on the generating function. For the case of zero relative delay we have $w^2 = 1$.

## 5. IDENTIFICATION

Consider the Volterra series when its input is a sequence such as $w(n)$. We can rewrite the second term as follows:

$$
\begin{aligned}
y_2(n) &= \sum_i \sum_j h_2(i,j) w(n-i) w(n-j) \\
&= w^2(n) * h_2(i,i) + 2w(n-f(1)) h_2(i,i+1) + \cdots,
\end{aligned} \tag{7}
$$

where * denotes convolution. Now the function $h_2(i, i+k)$ is the $k^{th}$ off-diagonal `slice' through $h_2$. This means that when the input to the Volterra series system is a MLS the output is the sum of the convolutions of delayed versions of the input convolved with off-diagonal slices of the kernel. In other words, for this particular input the second order system behaves like a linear system with impulse response:

$$
h_{eq2}(i) = \sum_{k=1}^{\infty} h_2(i - f(k), i + k - f(k)). \tag{8}
$$

The exception to this is $h_2(i,i)$, the main diagonal which contributes a d.c. term to the output since $w^2(n) = 1$. We can identify $h_{eq2}$ by forming the input-output cross-correlation, which will be periodic since $w(n)$ is. If we are fortunate in in the distribution of $f(k)$ then the kernel slices appearing will be sufficiently free from overlap that we can reconstruct an estimate of $h_2(i,j)$ from this cross-correlation record. It can readily be shown that any such estimate will improve as the length of the MLS increases.

So far we have only considered the contribution to the output from the second order term, but terms of all orders will be contributing and will contaminate the estimates. This effect can be reduced by the 'inverse repeat' technique: stimulate the system with $w(n)$ and then with $-w(n)$ and add the results. The resulting output will only contain even ordered terms. If the system converges quickly enough then $h_4$ will be negligible and so $h_2$ can be estimated. Subtracting the results will give the odd-ordered outputs. A longer MLS will generally be required to identify $h_3$ because of its heigher dimensionality, but the principle is the same. Details of procedures for obtaining and manipulating higher-order offset functions can be found in [3]. This method has been applied to visual systems by Sutter [1] and to BAER by Shi and Hecox [4].

## 6. RESULTS

The procedure was simulated in *MATLAB* for a nonlinear filtering algorithm whose Volterra kernels were known. The procedure was found to give the expected results. An analogue interface based on a LSI TMS320C30 DSP board was developed in order to identify real systems. The first system chosen was a nonlinear, battery-powered guitar amplifier which was deliberately designed to distort. The estimated kernel is shown in figure 1. Note that the main diagonal is not present, it cannot be directly estimated via this technique.

The procedure was then applied to the ears of two healthy adult subjects known to have otoacoustic emissions by means of a probe designed by the IHR (Institute of Hearing Research). Each sequence was presented 200 times and the results averaged to improve the signal to noise ratio. The sample rate chosen was 20 kHz. The resultant kernel estimates are plotted in contour form in figures 2 and 3. The main diagonal has been interpolated; whether this is a valid approximation to make would require further study. In order to ensure that the nonlinearity being detected was audiological in origin the procedure was repeated with the probe placed in a cavity designed to approximate the passive acoustic characteristics of a human ear. A small degree of nonlinearity was found, which was eventually traced to the DSP board. The character of this nonlinearity was found to be sufficiently insignificant as not to invalidate the results, see [3] for full details.

## 7. CONCLUSIONS

This study has shown that the MLS measurement of Volterra kernels from otoacoustic emissions is a viable procedure. It is faster and less involved than BAER tests. There are now several avenues of research to be explored based on this work. Micromechanical cochlear models in the form of partial differential equations could be analysed to see what form of Volterra kernels they predict (the calculation of kernels from PDEs has not been as widely explored as from ODEs but is simple enough in principle, although involved in detail

[3]). A more empirical approach which might pay dividends from a health-screening point of view would be to measure the emmission kernels of a large number of subjects and attempt to correlate the results with other aspects of audiological health

If the technique could be sufficiently refined then the resulting kernels could be Fourier-transformed so as to predict sum and difference tone response in a form which could be compared with the results of a direct estimation of combination tone response resulting from the presentation of combinations of pure tones.

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## REFERENCES

[1] E E SUTTER, 'A Practical Nonstochastic Approach to Nonlinear Time-Domain Analysis' in *Advanced Methods of Physiological System Modeling, Vol. 1* pp 303-315, 1989.

[2] M SCHETZEN, 'The Volterra and Weiner Theories of Nonlinear systems' Wiley-Interscience, 1980.

[3] M C M WRIGHT 'Volterra Series Characterisation and Identification of Nonlinear Bioacoustic Systems', PhD Thesis, Southampton University 1993 (to be published).

[4] Y SHI and K E HECOX, 'Nonlinear System Identification by *m*-pulse Sequences: Application to Brainstem Auditory Evoked Responses', *IEEE Trans. Biomed. Eng* 38(9), pp 834-845.

| $\alpha^n$ | $\alpha^3$ | $\alpha^2$ | $\alpha^1$ | $\alpha^0$ |
|---|---|---|---|---|
| $\alpha^1 \equiv \alpha$ | 0 | 0 | 1 | 0 |
| $\alpha^2 \equiv \alpha^2$ | 0 | 1 | 0 | 0 |
| $\alpha^3 \equiv \alpha^3$ | 1 | 0 | 0 | 0 |
| $\alpha^4 \equiv \alpha + 1$ | 0 | 0 | 1 | 1 |
| $\alpha^5 \equiv \alpha^2 + \alpha$ | 0 | 1 | 1 | 0 |
| $\alpha^6 \equiv \alpha^3 + \alpha^2$ | 1 | 1 | 0 | 0 |
| $\alpha^7 \equiv \alpha^3 + \alpha + 1$ | 1 | 0 | 1 | 1 |
| $\alpha^8 \equiv \alpha^2 + 1$ | 0 | 1 | 0 | 1 |
| $\alpha^9 \equiv \alpha^3 + \alpha$ | 1 | 0 | 1 | 0 |
| $\alpha^{10} \equiv \alpha^2 + \alpha + 1$ | 0 | 1 | 1 | 1 |
| $\alpha^{11} \equiv \alpha^3 + \alpha^2 + \alpha$ | 1 | 1 | 1 | 0 |
| $\alpha^{12} \equiv \alpha^3 + \alpha^2 + \alpha + 1$ | 1 | 1 | 1 | 1 |
| $\alpha^{13} \equiv \alpha^3 + \alpha^2 + 1$ | 1 | 1 | 0 | 1 |
| $\alpha^{14} \equiv \alpha^3 + 1$ | 1 | 0 | 0 | 1 |
| $\alpha^{15} \equiv 1$ | 0 | 0 | 0 | 1 |

Table 1 : Mod 2 powers of $\alpha$ given $\alpha^4 + \alpha + 1 \equiv 0$.

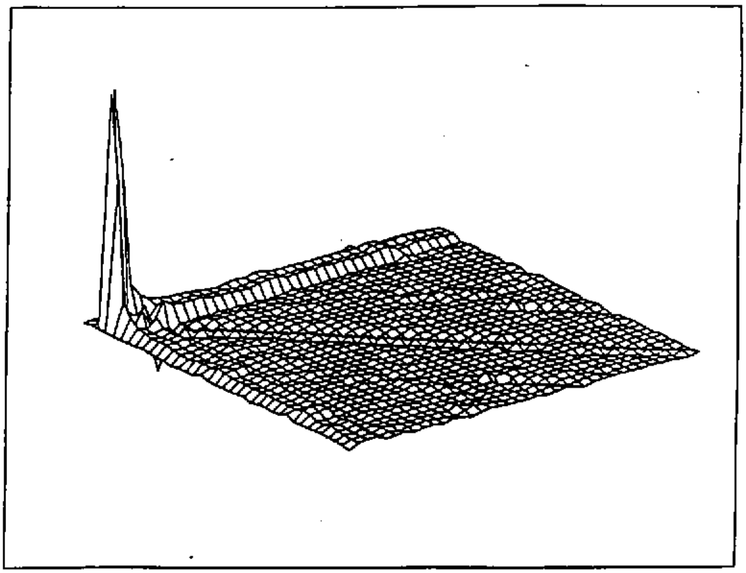

Figure 1 : Estimated second kernel for a nonlinear amplifier

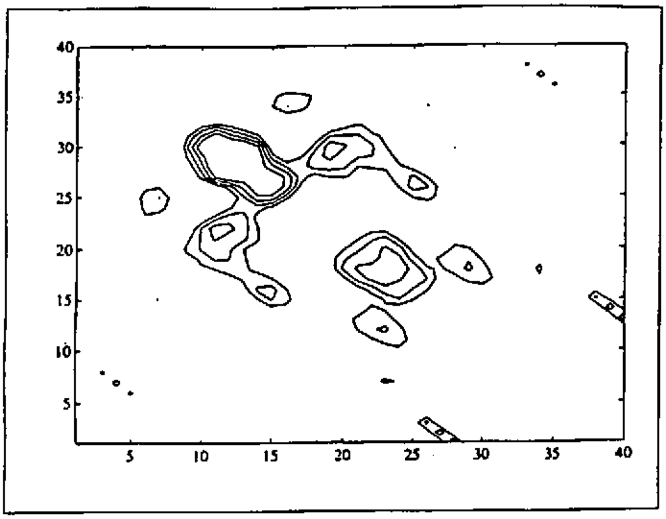


Figure 2 Contour plot of estimated second kernel for the cochlear emissions of subject A.

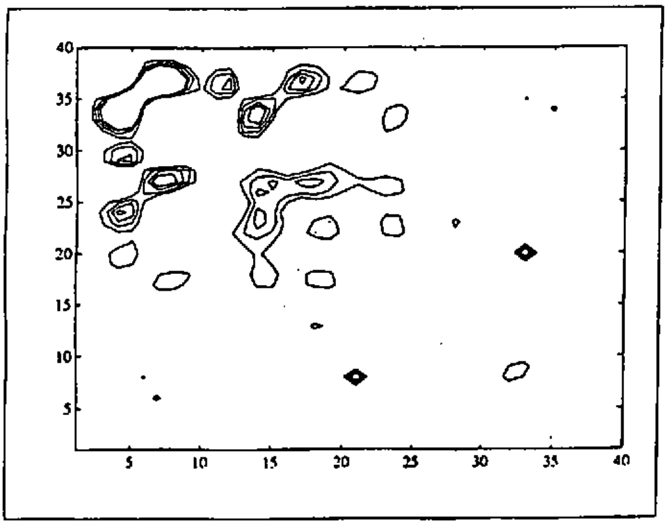


Figure 3 Contour plot of estimated second kernel for the cochlear emissions of subject B.