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AN INVESTIGATION OF ACTIVE VIBRATION ISOLATION SYSTEMS FOR PERIODIC DISTURBANCES

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1. INTRODUCTION

This paper describes the theoretical basis and feasibility of simple one-dimensional active control systems for the isolation of periodic vibrations. Relevant problems in practice include the prevention of transmission of vibrations of a periodic nature from a source, say a reciprocating machine, to a resonant substructure. Such situations are commonplace in many automotive, marine and aerospace applications, where passive isolators do not sufficiently attenuate the transmission of vibrations from the source to the substructure.

Various schemes have been proposed for supplementing passive isolators in order to actively control the transmission of vibrations (see Chaplin [1]). At present, however, no in-depth account of the fundamental characteristics of these one dimensional active isolation systems has been presented. This is of importance when assessing the relative merits of the differing approaches. In this paper two simple one dimensional models are presented in order to show the importance of the position of the secondary actuator in the schemes proposed by Chaplin [1].

This fundamental work has been extended to the implementaion of a multiple parallel active/passive vibration isolation system. This consists of a resiliently mounted raft on a resonant substructure (plate), see Fig. 3. The control system used to dictate the action of the secondary actuators is that described by Elliott *et al* [2,3], and uses the signals from a number of accelerometers placed on the receiving structure as inputs. The output signals produced by the controller are sent to the secondary actuators in order to isolate the vibration of the raft from the plate. The control system is found to perform well when operating at a substructure resonance frequency but an increase in vibration could result when operating at an off resonance frequency.

2. THEORY

Two different one dimensional active isolation systems are shown in Figure 1. With reference to these systems, m_1, k_1 and c_1 refer to the mass, stiffness and damping of the passive isolation system, while m_2, k_2 and c_2 are the effective mass, stiffness and damping of the receiving structure.

These models are very simplified and make several assumptions including the neglect of any shear or bending motion of the receiver produced via the passive isolators. However, the overall performance of these systems is well illustrated by these models.

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In practice an obvious limitation of active control techniques occurs when considerable secondary force generation is required. It is therefore of great importance to be able to predict the magnitude of the secondary force (F_s) required to minimise the motion of the receiver for a given primary force excitation (F_p).

Firstly consider the "parallel" system as shown in Figure 1. The equations of motion for this system may be written as

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = F_p - F_s \qquad (1)$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = F_s \qquad (2)$$

and assuming that the force excitation and response are of harmonic time dependence $e^{j\omega t}$ we may write after some rearrangement

$$(-\omega^2m_1 + j\omega c_1 + k_1)x_1 + (-j\omega c_1 - k_1)x_2 = F_p - F_s \qquad (3)$$

$$(-j\omega c_1 - k_1)x_1 + (-\omega^2m_2 + j\omega c_2 + k_2 + j\omega c_1 + k_1)x_2 = F_s \qquad (4)$$

Now setting $F_s = F_{so}$ the optimal secondary force which will drive x_2, the motion of the receiver to zero, we obtain

$$-(\omega^2m_1 + j\omega c_1 + k_1)x_1 = F_p - F_{so} \qquad (5)$$

and $$(-j\omega c_1 - k_1)x_1 = F_{so}. \qquad (6)$$

These equations may be combined to give the ratio of the secondary force (F_{so}) to the primary force (F_p) thus

$$\frac{F_{so}}{F_p} = \frac{k_1 + j\omega c_1}{\omega^2m_1} = 1 + \frac{2j\xi_1\Omega}{\Omega^2} \qquad (7)$$

This may be written as the modulus and phase of F_{so}/F_p thus:

$$\left|\frac{F_{so}}{F_p}\right| = \frac{1}{\Omega^2}\sqrt{(1 + 4\xi_1{}^2\Omega^2)} \quad \text{and} \quad \angle\frac{F_{so}}{F_p} = \tan^{-1}(2\xi_1\Omega) \qquad (8)$$

where Ω is the non-dimensional frequency ω/ω_1 and $\omega_1 = \sqrt{(k/m)}$ is the natural frequency of the upper mass/spring/damper system and $\xi_1 = c_1/2\omega_1m_1$ is its damping ratio.

Now consider the "opposed" system, also shown in Figure 1. Here the secondary force (F_s) is applied to the opposite side of the receiver than the primary force (F_p). Taking the same analysis as for the parallel system we may write the equations of motion of the two masses thus:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = F_p \qquad (9)$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = -F_S \qquad (10)$$

Again combining these two equations and allowing the optimal condition, that is, $x_2 = 0$ enables an expression for the secondary force required to prevent receiver motion to be obtained. Assuming a harmonic time dependence of the form $e^{i\omega t}$ and substituting gives

$$(-\omega^2m_1 + j\omega c_1 + k_1)x_1 + (-j\omega c_1 - k_1)x_2 = F_P \qquad (11)$$

$$(-j\omega c_1 - k_1)x_1 + (-\omega^2m_2 + j\omega c_2 + k_2 + j\omega c_1 + k_1)x_2 = -F_S \qquad (12)$$

and combining these equations to obtain the ratio of optimal secondary force to primary force gives:

$$\frac{F_{SO}}{F_P} = \frac{k_1 + j\omega c_1}{-\omega^2m_1 + k_1 + j\omega c_1} = \frac{1 + j2\zeta_1\Omega}{1 - \Omega^2 + j2\zeta_1\Omega} \qquad (13)$$

and writing this in terms of modulus and phase yields:

$$\left|\frac{F_{SO}}{F_P}\right| = \frac{1}{(1 - \Omega^2)^2 + (2\zeta_1\Omega)^2} \sqrt{(1 - \Omega^2(2 + 8\zeta_1{}^2) + \Omega^4(1 + 8\zeta_1{}^2}$$

$$+ 16\zeta_1{}^4) + \Omega^6(4\zeta_1{}^2)) \qquad (14)$$

and

$$\angle \frac{F_{SO}}{F_P} = \tan^{-1} - \frac{2\zeta_1\Omega^3}{(1 - \Omega^2)^2 + 4\zeta_1\Omega^2} \qquad (15)$$

Of particular interest is the magnitude of the secondary force required for cancellation as in many practical applications the ability to generate the necessary amplitudes of secondary force is a key to the system's feasibility.

The ratio of the optimal secondary magnitude to the primary magnitude $|F_{SO}|/|F_P|$ is shown for each system plotted, for various values of damping ratio, versus non-dimensional frequency Ω in Figures 2a, b and c. Several points should be noted here. Firstly note that in each case the expression for $|F_{SO}|$ contains characteristics of the upper mass/spring/damper system only. If we consider the case where the excitation frequency is in the region of the resonant frequency of the upper mass/spring/damper, i.e., $\Omega = 1$, then for the parallel system we find, allowing $\zeta_1 \ll 1$, that $|F_{SO}| \approx |F_P|$. However, for the opposed system $|F_{SO}| \approx |F_P|/2\zeta_1$, and thus a much larger secondary force magnitude than primary force magnitude is required at frequencies close to resonance. With the parallel system, however, the primary and secondary magnitudes are approximately equal. This is important in practice where a given secondry force generating capability may be an important design consideration. With the prallel system large values of $|F_{SO}|/|F_P|$ occur at low values of Ω, i.e., $\Omega < 1$. With the opposed system large $|F_{so}|/|F_p|$ occur at the resonant frequency of the upper mass/spring/damper system, i.e., $\Omega = 1$. Therefore depending on the range of frequencies to be cancelled, the stiffnesses of the passive isolators may be designed since the stiffness of the passive isolators will vary the frequency at which $\Omega = 1$. This may also determine which type of active control system is most suitable, as for the parallel system much stiffer passive isolators are acceptable than for the opposed system when isolating "low frequencies".

3. EXPERIMENTAL INVESTIGATION OF A MULTI-CHANNEL PARALLEL PASSIVE/ACTIVE VIBRATION ISOLATION SYSTEM

This experimental work is designed to assess the performance of the control system described by Elliott [2,3] when applied to a complex mechanical vibration system. This vibration system is as follows. A primary force, representing, say, the rotational imbalance of a reciprocating machine, is applied to a thick plate which represents the machinery seating. This thick plate is connected via passive and active isolators in parallel, to a thin resonant base plate representing the receiving structure. The primary force is applied via a coil and magnet shaker and the secondary forces are applied in the same way.

The control system determines the input signals to the secondary force actuators necessary to minimise the sum of the squares of the signals gained via transducers (accelerometers) placed on the receiver. The control system uses an array of adaptive filters through which a reference signal is passed. This corresponds to the sinusoidal signal applied to the primary force generator. The coefficients of these adaptive filters are updated by the control system algorithm [5] in order to perform the minimisation of the sum of the squares of the accelerometer signals.

Three different configurations of the vibration system are investigated (see Figure 4). In each case the control system is applied with four and eight accelerometer inputs. The four accelerometer input signals come from the bases of the passive isolators, and the eight signals come from randomly placing the eight accelerometers over the receiving plate. It is known that the disturbance at the points of application of the pick-up sensors is minimized by the control system. What must be deduced, however, is whether the minimisation at these points implies that a reduction in overall substructure vibration has occurred. The following experiments were undertaken in order to assess this question.

An evenly spaced grid was placed on the plate and a total of $N = 35$ equally spaced points created (see [4] for full details). A measure of the acceleration value at each of the 35 nodes was taken and recorded both before and after the control was initiated.

The control system effectiveness was defined as

$$E_N = 10 \log_{10} \frac{\sum_{n=1}^{n=N} |V_n|^2 \text{ (before control)}}{\sum_{n=1}^{n=N} |V_n|^2 \text{ (after control)}} \text{ dB}$$

where V_n is the velocity at the n'th nodal position.

This value was derived for a number of different frequencies corresponding to both base plate resonant and anti-resonant conditions. The full details of these experiments can be found in reference [4], and a summary of the principal results can be seen in Fig. 4.

4. CONCLUSIONS

When the receiving structure (plate) was excited at a resonance, the control system was effective in producing reductions in the space averaged vibrational level in the plate of between 8 and 16 dB (see Fig. 4). There also seemed to be little difference in minimising the acceleration at four positions (at the base of the mounting points) to minimising the acceleration at eight positions distributed over the plate surface. When the plate was excited off resonance, however, there was considerable advantage to be gained from minimising the acceleration at eight positions. This produced no reduction in level although a four sensor system led to an increase in plate vibration.

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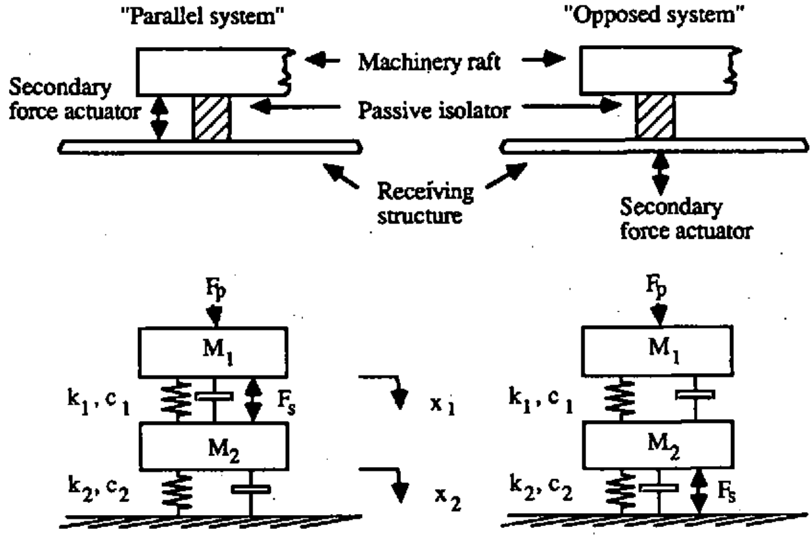

Figure 1. Sketches of isolation systems using secondary force actuators in "parallel" with a passive element or simply providing an "opposing" force input in order to cancel the force applied by the passive isolator to the receiving structure.

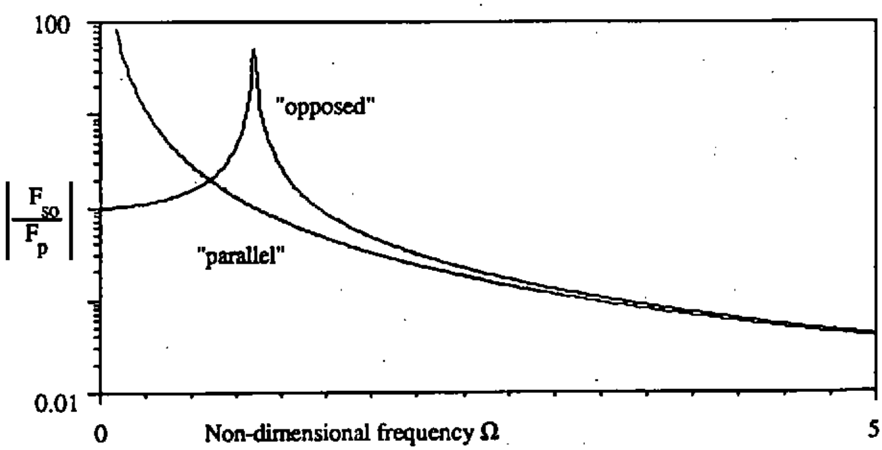

Figure 2a. Comparison of the magnitude of the secondary force required for the "parallel" and "opposed" systems for ζ_1=0.01

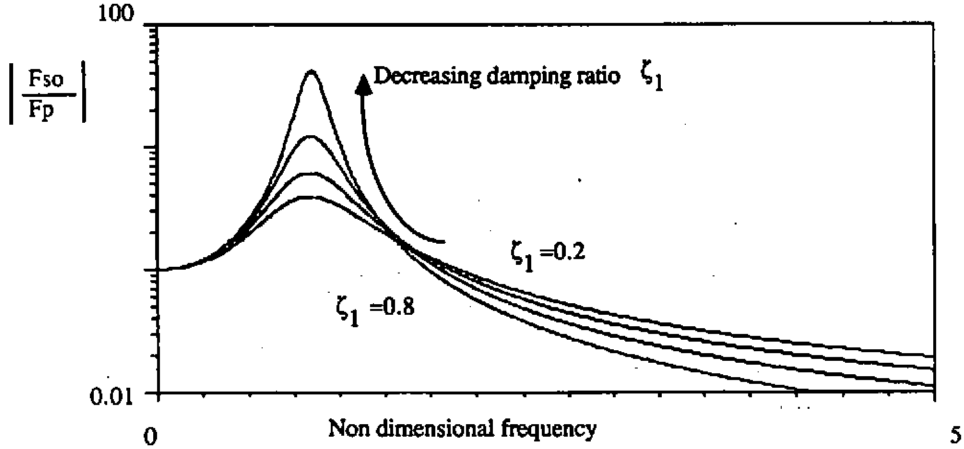

Figure 2b. Magnitude of Fso / Fp for various values of damping ratio (zeta)

Opposed system

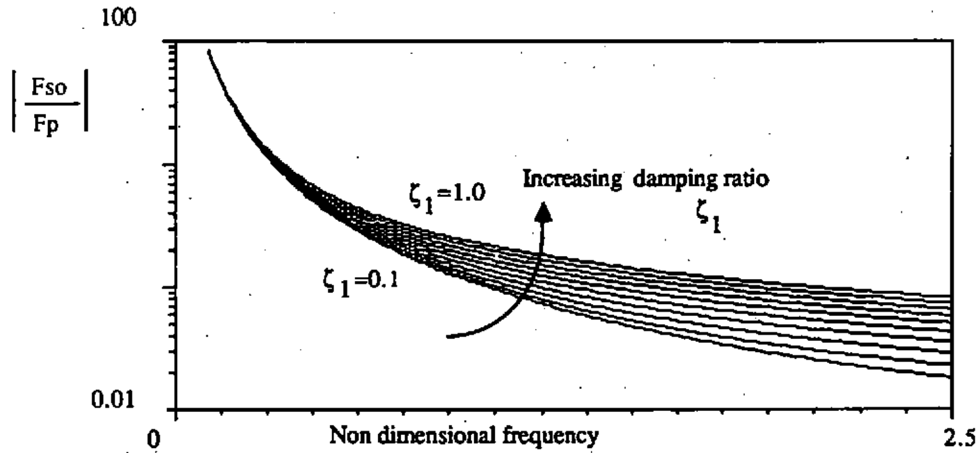

Figure 2c. Magnitude of Fso / Fp for various values of damping ratio (zeta)

Parallel system

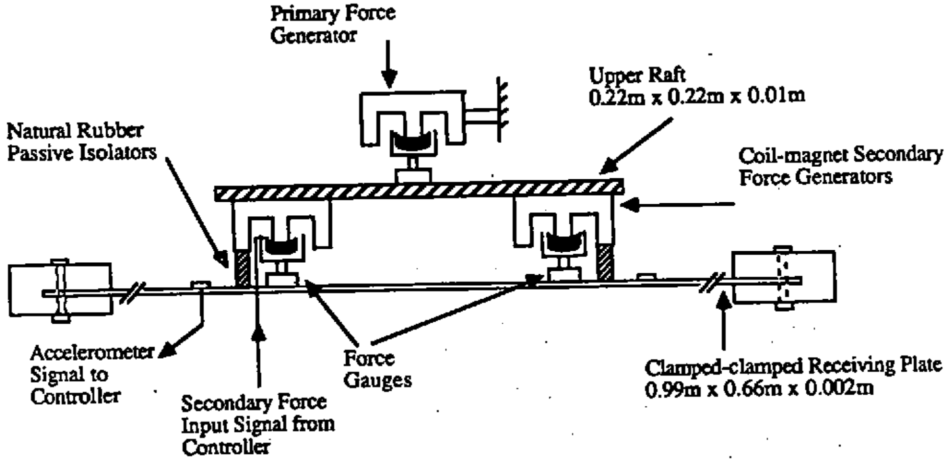

Figure 3. Schematic diagram of experimental rig (side view)

Frequency (Hz)	Receiver plate response	Configuration (see below)	No. of accelerometers used by controller	Effectiveness E_N dB
70	Resonant	(a)	4	16.6
70	"	(a)	8	17.8
105	Resonant	(b)	4	14.7
103	"	(b)	8	8.9
95	Off-resonant	(c)	4	8.7
95	"	(c)	8	8.9
98	Anti-resonant	(a)	4	-8.0
98	"	(a)	8	0.8

Configurations : (plan view)

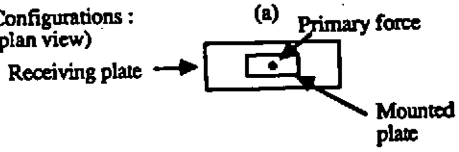

Figure 4. Results for the control system effectiveness for various experimental configurations and frequencies