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DYNAMIC ANALYSIS OF SHALLOW SHELLS WITH A DOUBLY CURVED TRIANGULAR FINITE ELEMENT

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A dynamic analysis capability for shallow shell structures based on an arbitrarily curved triangular shell finite element is developed. The shallow shell approximation is used for two reasons. Firstly there are enough practical problems which satisfy the shallow shell assumption to warrant their treatment as a separate class, and secondly, this approximation greatly reduces the complexity of the theory and programming required. This work is the first application to dynamics of a new curved finite element, which proved to be superior to all earlier elements in static applications (Ref.1). The present developments include an extension to shells of variable thickness.

The formulation of this shell finite element is briefly as follows. The shallow shell theory of Novozhilov is used, and the shell surface is defined by a quadratic function of the base plane Cartesian coordinates. The displacement function for the normal deflection, w, of the shell is taken as a quintic polynomial (21 terms) in the coordinates in the base plane. Three constraints are placed on the polynomial to ensure that the normal derivative varies cubically along each edge. The tangential displacements u and v for the shell are each expressed as cubic polynomials (10 terms each), and the generalized coordinates are taken to be u and v and their first derivatives at each element vertex, plus u and v at the centroid. The shell thickness is assumed to vary linearly over the triangular area of the element, although this thickness variation may be extended to higher order with little effort due to the closed form nature of the matrix component solutions. The stiffness and consistent mass matrices for the element are then obtained from calculations of strain and kinetic energies, respectively. A major advance is achieved in that the matrix components are obtained in closed form relative to the polynomial coefficients and are then easily transformed to generalized coordinate notation in local or global coordinates by simple matrix multiplication.

Most practical shell vibrations involve predominantly normal motion, and hence it is customary to neglect tangential inertia. This assumption allows the tangential displacements u and v at the element's centroid to be condensed out of the final elemental stiffness matrix by minimizing the strain energy with respect to them. Hence, the final element has 36 degrees of freedom and is completely conforming. The element contains an exact representation of all six required rigid body modes and has an asymptotic strain energy convergence rate of N^{-6} where N is the number of elements per side of a shell. The same asymptotic convergence rate should also hold for vibration eigenvalue predictions. Finally, once the master matrices are assembled for a particular finite element representation of a shell structure, the assumption of neglecting tangential inertia further allows all tangential degrees of freedom to be condensed out, thus greatly reducing the eigenvalue problem sizes.

The foregoing method was used to analyze the vibrations for several shallow shell applications. The first application was to a shallow spherical cap "freely supported" on a square base. This problem is a convenient test case because it has an exact solution, thus allowing the eigenvalue convergence to be studied in detail. Using symmetry, one quarter of the shell was analyzed with finite element grids of one, two and three elements along an edge. Eigenvalue accuracies of the order of 0.01 per cent and convergence approaching a rate of N^{-5} were obtained from these calculations. This convergence was somewhat less than the predicted asymptotic rate of N^{-6}. However, the above mentioned accuracies are quite sufficient for most applications.

The next two applications were to experimental fan blade models. These models were constructed by rolling sheet steel into a cylindrical shape and then welding one curved edge to a massive steel block. The first model was of uniform thickness, while the second had a thickness varying linearly in the curved direction. The vibrations of these models were predicted with finite element grids of two, three and four elements per edge and these predictions were verified experimentally. Excellent accuracy and rapid convergence were obtained in both cases for up to twenty-five modes. Good comparison of predicted and measured mode shapes was also obtained.

The next application was to an experimental model of a spherical dish antenna. The circular model was spun formed to a spherical radius and then firmly clamped at its centre. Several finite element predictions of the vibrations were obtained by analysing one quadrant of the dish, and the results were verified experimentally. Some small discrepancies between theory and experiment were noted at the lower frequencies and seemed to be associated with inaccuracies in the shell model curvatures.

The final application considered was to a clamped cylindrical shell panel for which experimental results were available elsewhere (Ref. 2). Symmetry conditions were again invoked on one quarter of the shell, and eigenvalue predictions were obtained with several finite element grids. A comparison of the natural frequencies for this shell as predicted by various workers using different methods is given in Table 1 along with the experimental results (reproduced from Reference 2). Here ERR stands for an extended Rayleigh-Ritz method (Ref. 3); FET is the present triangular finite element using a 3×3 grid, FER is from a 4×6 grid of rectangular finite elements (Ref. 4) and K is a Kantorovich method (Ref. 5). The sizes of the eigenvalue problems solved, which provide a measure of the computing efforts required, are given in the heading of Table 1 for the different symmetry cases, sym.-sym.; sym.-antisym. and antisym.-antisym. A comparison of these numbers indicates that the present method is the most efficient. The ERR method shows up second best, but it has the distinct disadvantage that each new problem must be reprogrammed, whereas the present finite element method may be used for arbitrary shallow shells with quite general boundary shapes and edge conditions.

Finally, it may be noted that the complete details of this work are available in Reference 6.

References:

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Table 1 - COMPARISON OF VARIOUS PREDICTION METHODS FOR CLAMPED CYLINDRICAL PANEL

Mode	m,n*	Experi-mental (Hz.)	Theoretical ERR 55,55,55	FET 49,42,37	FRR 77,77,77	K
1	1,2	814	870	870	890	890
2	1,3	940	958	958	973	966
3	1,3	1260	1288	1288	1311	1295
4	2,1	1306	1364	1363	1371	1375
5	2,2	1452	1440	1440	1454	1450
6	2,3	1802	1753	1756	1775	1745
7	1,4	(1735 1770)	1795	1780	1816	
8	3,1	2100	2057	2056	2068	
9	3,2	2225	2220	2222	2234	
10	2,4	2280	2300	2295	2319	

*m,n are the numbers of half waves in the straight and curved directions, respectively.