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Volume Velocity Sensors for Active Control

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1 Introduction

Active control systems are generally designed to minimize some idealized cost function, such as total acoustic potential energy or radiated sound power. In practice, measured error signals are used to monitor the system and to calculate approximations to the idealized cost function which the system is attempting to minimize. To actively control the radiation of sound power from a plate, we must have some method of directly measuring or inferring this quantity; either by using far field microphones surrounding the plate or by using vibration transducers attached to the plate itself. In this paper we are concerned with developing methods of estimating sound power radiation using vibration transducers attached to the plate.

The sound power radiation from a plate or a beam is a function of the velocity distribution over the surface of that plate or beam. The relationship is not a simple one and certain velocity distributions or modes will radiate sound more efficiently than others [1]. What is important for our control purposes, is to be able to detect the velocity distributions that radiate sound well.

The acoustic power radiated from a structure can be expressed in terms of the vector of its structural mode amplitudes $\boldsymbol{a}$ and a matrix of radiation resistances $\boldsymbol{M}$, as described for example by Baumann [2].

$$W = \boldsymbol{a}^H \boldsymbol{M} \boldsymbol{a} \tag{1}$$

where the superscript H denotes the Hermitian transpose and $\boldsymbol{M}$ is a real, symmetric, positive definite matrix. The diagonal terms in $\boldsymbol{M}$ represent the self radiation terms and the off diagonal terms represent the mutual radiation terms, since structural modes do *not* radiate sound independently. The radiation efficiencies of the lower order structural modes of a rectangular simply supported panel are proportional to the elements of the matrix $\boldsymbol{M}$ and are illustrated in Figure 1, as a function of kl (the wavenumber times the largest panel dimension) [8]. However, since $\boldsymbol{M}$ is real and symmetric it has an eigenvalue/eigenvector decomposition which can be written as,

$$\boldsymbol{M} = \boldsymbol{P}^T \boldsymbol{\Omega} \boldsymbol{P} \tag{2}$$

where $\boldsymbol{P}$ is an orthogonal matrix of eigenvectors and $\boldsymbol{\Omega}$ is a diagonal matrix whose elements, Ω_n are the eigenvalues of $\boldsymbol{M}$, which are real and positive. From equation 1 we can now write,

$$W = \boldsymbol{a}^H \boldsymbol{M} \boldsymbol{a} = \boldsymbol{a}^H \boldsymbol{P}^T \boldsymbol{\Omega} \boldsymbol{P} \boldsymbol{a} \tag{3}$$

If we define $\boldsymbol{b} = \boldsymbol{P}\boldsymbol{a}$, which is a vector of transformed 'radiation mode' amplitudes, the power output can now be expressed as,

$$W = \boldsymbol{b}^H \boldsymbol{\Omega} \boldsymbol{b} = \sum_{n=1}^{N} \Omega_n \mid b_n \mid^2 \tag{4}$$

where the latter expression is a consequence of $\boldsymbol{\Omega}$ being diagonal. These 'radiation modes' [8] radiate sound *independently* with a radiation efficiency proportional to Ω_n. The radiation efficiencies of the first three radiation modes of a rectangular panel are shown as a function of kl in Figure 2. Because these modes radiate sound independently there are no mutual radiation terms. The first radiation mode, which is the dominant radiator of sound at low frequencies, is a piston type mode. Measuring the first 'radiation' mode is therefore equivalent to measuring the net volume displacement of the surface which is proportional to its *volume velocity* for a pure tone excitation. It is also important to note that the radiation mode shapes and radiation efficiencies are only dependent on the dimensions of the structure and are independent of any other structural properties or its boundary conditions.

We can attempt to measure the motion of a two dimensional surface by using arrays of transducers, much as a microphone array can be used in the estimation of the total acoustic potential energy present in an enclosure. The space average velocity of the plate could, for example, be calculated by adding together the instantaneous signals from a large number of such point transducers, assuming they were all perfectly matched. Alternatively we can use 'distributed' sensors. These are transducers that do not act just at a single point but along a length or over an area. The sensor detects a net change along that length or over that area and hence these transducers are also termed 'integrating sensors'. There are currently three types of integrating sensors which are commonly available. Strain gauges, which act on the principle that a strained piece of wire changes its electrical resistance; optical fibres, which allow us to measure the change in the path length of a beam of light and piezoelectric materials, which produce a voltage which is proportional to the strain they experience. This paper looks specifically at the use of piezoelectric materials as distributed sensors.

2 Theory

2.1 Piezoelectrics

Piezoelectric materials [11] are ones that produce an electric charge when undergoing a strain. They also exhibit the reciprocal property of changing their dimensions when subjected to an electric field. These properties allow us to use piezoelectric materials as sensors and actuators. PVDF (PolyVinyliDene Fluoride) is a piezoelectric material which can be manufactured into wires and films (films are commonly available as thin as $8\mu m$). The output of a piece of piezoelectric film will be a function of strain in two dimensions ie. a change in area. The short circuited charge output of a film bonded to a thin isotropic plate is given by Lee [4] in IEEE standard notation [12] as,

$$q = -\int_0^{l_y}\int_0^{l_x} h(x,y)\left[e_{31}Z^{xx} + e_{32}Z^{yy} + 2e_{36}Z^{xy}\right]dxdy \tag{5}$$

where the superscripts denote partial differentials with respect to that superscript, $h(x,y)$ is the separation between the film and the neutral axis (Figure 3), and e_{ij} are the stress/charge constants for relating charge across the i-axis due to stress along the j-axis. This result assumes that there is no stretching or compression of the plate's mid-plane. By measuring the film's response using a charge amplifier we can satisfy the above assumption that the film is short circuited. The value of e_{36} as given by AUTOCHEM [11] is zero and hence the charge response has two components which can be split as follows,

$$q_x = -e_{31}\int_0^{l_y}\int_0^{l_x} h(x,y)Z^{xx}dxdy \tag{6}$$

$$q_y = -e_{32}\int_0^{l_y}\int_0^{l_x} h(x,y)Z^{yy}dxdy \tag{7}$$

where q_x is the closed circuit charge output due to bending in the x-direction and q_y is the closed circuit charge output due to bending in the y-direction. Let us first consider q_x. If we integrate the inner integral in equation 6 by parts, we will arrive at,

$$q_x = e_{31}\int_0^{l_y}\left[-[h(x,y)Z^x]_0^{l_x} + [h^x(x,y)Z]_0^{l_x} - \int_0^{l_x} h^{xx}(x,y)Zdx\right]dy \tag{8}$$

We can consider the function $h(x,y)$ to be the spatially varying sensitivity of the film, which in principle can be arbitrarily defined at the design stage. We define $h(x,y)$ to be quadratic in the x-direction such that $h(x,y)$ equals zero when $x=0$ or $x=l_x$ and to be independent of y (Figure 4), so that

$$h(x,y) = \frac{4h_0}{l_x}\left[l_x x - x^2\right] \tag{9}$$

The concept of quadratically weighting the sensitivity of a one-dimensional sensor has been explored by Rex [10] and here we examine the extension of this work to two dimensional sensors. By substituting equation 9 into equation 8 we arrive at an expression for the closed circuit charge output due to bending in the x-direction with a quadratically weighted sensitivity.

$$q_x = \frac{8h_0e_{31}}{l_x}\int_0^{l_y}\int_0^{l_x} Z(x,y)dxdy - 4h_0e_{31}\int_0^{l_y}[Z(l_x,y)+Z(0,y)]dy \tag{10}$$

The charge output is a function of the integrated displacement of the plate (our desired result) minus the net displacement along the edges $x=0$ and $x=l_x$. If the edges are not fixed additional transducers will be needed to measure the total displacement, but by assuming fixed edges we can simplify the problem to one where only one transducer is required.

This result only considers the x component of the charge output in (equation 6). The output due to bending in the y-direction (equation 7) does *not* give the desired result and is a function of the gradients at the edges and corners of the plate (Appendix A). In general we must therefore find a method of eliminating the y component of the charge output, as discussed in the next section.

2.2 Cancellation of cross sensitivity

The piezoelectric stress/charge constants can be calculated from the piezoelectric strain/charge given by ATOCHEM [11] as,

$$e_{31} = 5.2 \times 10^{-2}(N/Vm) \qquad e_{32} = 2.1 \times 10^{-2}(N/Vm) \tag{11}$$

We can effectively reduce the contribution of q_y (equation 7) to zero by layering two sheets of piezofilm on top of one another. If two separate sheets of piezofilm are attached to the panel with their polarity in different directions so that the first sheet's piezoelectric stress constant e_{31} aligns with the x-axis of the panel, and the other sheet's constant e_{31} aligns with the y-axis of the panel, and the spatial sensitivity *with respect to the panel* remains the same on both sheets, we can subtract a portion of one sheet's output from the other sheet's output to effectively cancel the sensitivity in the y-direction (Figure 5). If the charge output of the first sheet is given by,

$$q_1 = q_x + q_y \tag{12}$$

then the output of the second sheet is given by,

$$q_2 = q_x \frac{e_{32}}{e_{31}} + q_y \frac{e_{31}}{e_{32}} \tag{13}$$

We can multiply q_2 by e_{32}/e_{31} (which is approximately 0.4 for the constants quoted above) and subtract it from q_1 we arrive at,

$$q = q_1 - \frac{q_2 e_{32}}{e_{31}} = q_x \left(1 - \left(\frac{e_{32}}{e_{31}}\right)^2\right) \tag{14}$$

where q is the total output and is completely independent of q_y. Using the piezostrain constants given earlier we can substitute into the above equation to arrive at, $q = 0.84q_x$ and therefore there is little loss in sensitivity. As long as the sensors are well positioned over one another and are well bonded to the vibrating surface, the sensitivity in the y-direction can be effectively canceled.

3 Spatial weighting

The design of a two dimensional volume velocity sensor requires us to quadratically weight the sensitivity of a piezoelectric film which is sensitive to bending in only one direction. Spatial weighting could be achieved by varying the distance between the film and the neutral axis of the plate or by *doping* or mixing PZT powder with the PVDF matrix in different proportions across the film [3]. To accomplish these tasks after the structure and film have been manufactured is difficult. We will therefore suggest an alternative method of achieving a spatially weighted sensor.

Piezoelectric film is covered with a thin layer of metal which acts as an electrode. The piezoelectric material by its nature is very resistive and hence the electrical output will be due purely to areas that are covered by the metal electrode. The electrode can be etched away to produce shaped strips which effectively change the area over which we sense. We are therefore attempting to produce a spatial weighting by reducing the electrode covered area where we need low sensitivity, and leaving the electrode cover where we desire a high sensitivity. This is in effect keeping $h(x,y)$ constant and instead varying the area over which we integrate. Two basic designs for etching are briefly discussed in this paper; (i) the use of rectangular strips to produce a quadratic density and (ii) quadratically shaped strips. They are more fully described in [9].

The use of rectangular strips to produce a quadratic density is illustrated in Figure 6. If we divide the sensor into a number of rectangular strips which we can turn on (leave electrode present) or off (etch electrode away), we can approximate a quadratic weighting by making the density of strips 'turned on' vary quadratically. Choosing a configuration of rectangular strips is a difficult process since the number of possible configurations is astronomical (2^N when N is the number of strips used). By assigning a probability of being 'on' to each strip and varying that probability quadratically across the sensor, we can use a computer 'random' function to generate large numbers of likely configurations. The most accurate of these configurations is then selected.

The use of quadratically shaped strips is illustrated in (Figure 7). If each strip has a sufficiently small width it will not experience a large change in displacement from one side to the other (in the y-direction), and its output will thus approximate a quadratically weighted line integral of the strain in the x-direction. The sum of the outputs of all the quadratic strips will thus approximate equation 10. It is therefore necessary to produce a larger number of thinner strips to more accurately measure higher order modes.

To assess how effective these two methods are, we must be able to test the accuracy of a given design and also explore to some extent the susceptibility of the design to errors in implementation. We can then compare the results for a variety of different configurations to select the best method. We will define a given design's accuracy in terms of it's ability to sense the net volume displacement, of the structural modes of a simply supported plate, up to and including the seventh order modes. The error in sensing is considered to be the root mean squared deviation of the sensor's response from the ideal response. The sensor's output is normalized so that the response to the first order mode is exact [9] since we are interested the relative response of the sensor and not its absolute response. This method of testing was intended to be general and to give an overall idea of how accurate a given sensor would be in a variety of situations.

4 Results

There are three major factors which will effect the accuracy of a given sensor design; (i) the configuration chosen (ie. the number of strips used or which strips are turned on), (ii) the accuracy with which we can etch or implement our desired design and (iii) the conditions under which we expect our sensor to operate. Since we do not want, at this point, to make any assumptions about the system we are measuring, we will only consider the effects of the first two factors on the sensor's accuracy.

To gain some idea as to how susceptible the rectangular strip sensor (shown in Figure 6) is to errors in implementation, we will test the effect of introducing a random error ϵ into the width of a given strip. We will define ϵ as,

$$\epsilon = v \times r \tag{15}$$

where ϵ is the product of a normal random variable r, which has mean zero and a standard deviation of unity, and an error factor v which we can vary to test different levels of inaccuracy. The error factor v is considered to be a fraction of the sensors entire length so that the testing is independent of the sensors dimensions. The resulting sensing errors due to implementation errors will have their own probability distribution due to the random nature of the testing. The results presented here are an estimate of the average error over twenty tests [9].

The average value of the sensing error is plotted as a function of implementation error for three different configurations using 96, 150 and 240 rectangular strips in Figure 8. The three configurations were selected from three sets of five hundred thousand randomly generated configurations. The results show that with near perfect implementation our sensor accuracy increases with the number of strips used. It would also seem that configurations with fewer strips are slightly less susceptible to errors in implementation.

Six different configurations, using various numbers of quadratic strips (Figure 7), were tested to determine their accuracy and their susceptibility to errors in implementation (Figure 9). In this case the widths of each strip were randomly varied by a distance given by equation 15 at 90 points along each quadratically weighted strip [9]. The sensing error using this configuration are seen to be considerably smaller than with the rectangular strips, and to be less sensitive to implementation errors. When the implementation is poor, the errors seem to be fairly independent of the number of strips used. For example, with implementation errors larger than 10^{-4} there are only slight gains to be made from having more than 15 quadratic strips.

5 Discussion and conclusions

The objective of this paper is to demonstrate that two dimensional volume velocity sensors can be produced. Volume velocity sensing requires us to gather information over an area which suggests that integrating sensors will be more efficient than point sensors. The use of point sensors would require large numbers of well matched sensors and the ability to process the output from each sensor. An integrating sensor can potentially give a single output which is a function of the strain averaged over every point on the surface of the structure. This simplifies the electronics necessary, makes the sensor more easy to implement and produces an output which is potentially more useful in a control system. By quadratically weighting the sensitivity of the sensor across its x-axis and cancelling its sensitivity to bending along the y-axis, we have found that potentially the volume velocity of the structure due to bending can be directly measured.

The quadratic weighting of sensitivity can be achieved during manufacture by spatially varying the distance between the piezoelectric material and the neutral axis of the structure or by doping. We have shown that by shaping the sensors into quadratic strips or by using rectangular strips to produce a quadratic density we can also approximate a quadratic sensitivity. For general applications, the use of quadratically shaped strips appears to be the more accurate method of approximating a quadratic sensitivity. The quadratic shapes are also exactly defined mathematically and finding a suitable configuration is a somewhat easier task than with the use of rectangular strips.

The objective of the error testing was to determine how robust our sensor design was to practical inaccuracies, and to gain an understanding of the relationships between design, implementation error and sensing error. There are other approaches to error testing which are as equally valid as the approach taken here, but until specific assumptions about the structural system are made, it was thought best to keep the investigation as general as possible.

Volume velocity sensors represent a step forward in the control of acoustic radiation from structures. The approach to controlling acoustic radiation from panels has generally been to identify the dominant structural modes and attempt to cancel them [3] [5] [7]. This therefore requires specific design of modal sensors for every application. It is more efficient to consider the radiation of sound in terms of *radiation modes* since they make no assumptions about the structure other than its geometry. The first radiation mode, whose amplitude is proportional to the volume velocity, will be the dominant radiator of sound at frequencies with wavelengths larger than the structure considered. In cases where the panel is 'floppy', there is potentially a high structural modal density, although the basic mechanism for radiation is still the production of volume velocity. To approach the active control of sound radiation by attempting to control the structural modes, would be very complicated and involve the use of many sensors and actuators. By using a volume velocity sensor this problem requires only one sensor and one actuator. Sensors to measure the higher order radiation modes could also be implemented to either increase the useful frequency range of the system or to increase the attenuation possible [9].

There is also the potential of using piezoelectric films to actuate structures [3] [6]. Other types of actuators are generally difficult to install and are obtrusive. By using piezoelectric films as both actuators and sensors these control components could be integrated into the structure. If we have a film on one side of a panel acting as a sensor, and another identical one on the other side acting as an actuator, the transfer response between then will exhibit minimum phase characteristics. This property of integrated transducers may be useful in reducing the adaptation time of feedforward controllers, and in the design of feedback controllers.

A Change in area due to strain in the y-direction

If the spatial sensitivity is given by,

$$h(x,y) = \frac{4h_0}{l_x}\left[l_x x - x^2\right] = h(x) \tag{16}$$

where h is purely a function of x. The y-component of the charge output (equation 7) is given by,

$$q_y = -e_{32}\int_0^{l_x}\int_0^{l_y} h(x) Z^{yy} dy dx \tag{17}$$

By substituting the spatial sensitivity (equation 16 into the above equation and integrating by parts, we can arrive at the output due to bending in the y-direction.

$$q_y = \frac{4h_0e_{32}}{l_x}\left[[(l_x - 2x)Q(x)]_0^{l_x} + 2\int_0^{l_x} Q(x)dx\right] \tag{18}$$

where $Q(x)$ is befined to be,

$$Q(x) = \int\int [Z^y(x, l_y) - Z^y(x, 0)]\,dxdx \tag{19}$$

The charge output due to strain in the y-direction is therefore a function of integrated gradients at the plate's edges $y = 0$ and $y = l_x$ and at its corners.

References

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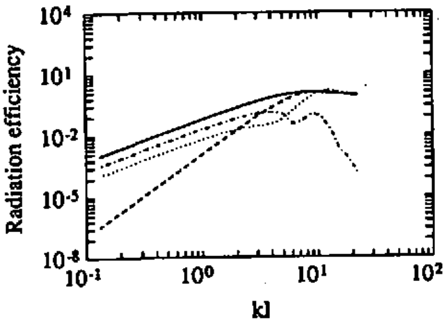

Figure 1: The radiation efficiencies of the lower order structural modes of a simply supported plate. The self-radiation terms for the (1,1), (2,1) and (3,1) modes are shown as (—), (· ·) and (···). The mutual-radiation term between the (1,1) and (3,1) modes is given by (-·-).

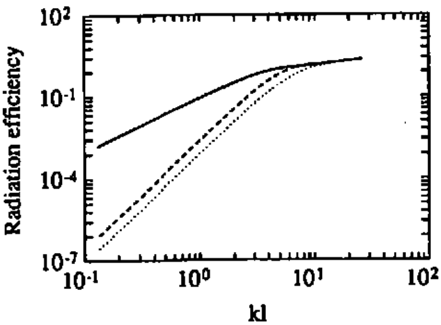

Figure 2: The radiation efficiencies of the first 3 *radiation* modes which radiate sound independently.

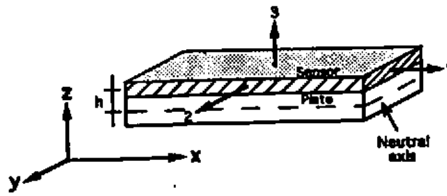

Figure 3: The piezoelectric sensor fixed to a plate such that the 1-axis of the sensor corresponds to the x-axis of the plate.

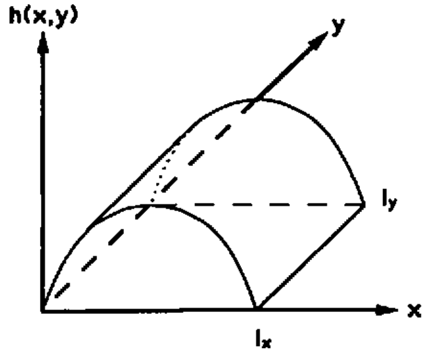

Figure 4: The spatial variation in sensitivity for a two dimensional sensor.

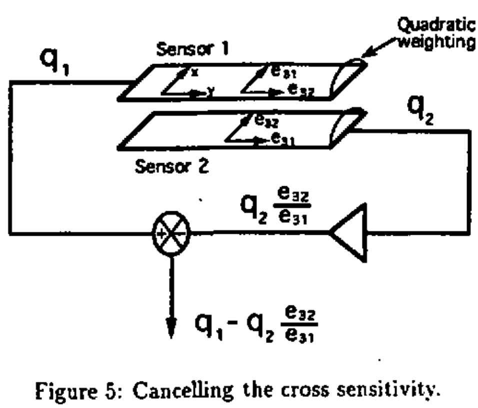

Figure 5: Cancelling the cross sensitivity.

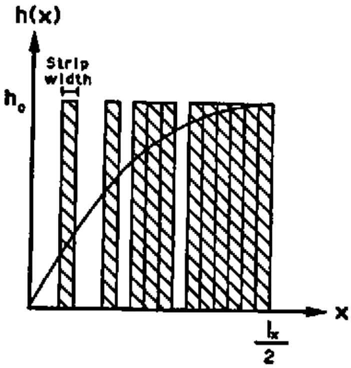

Figure 6: Rectangular strips to produce a quadratic density.

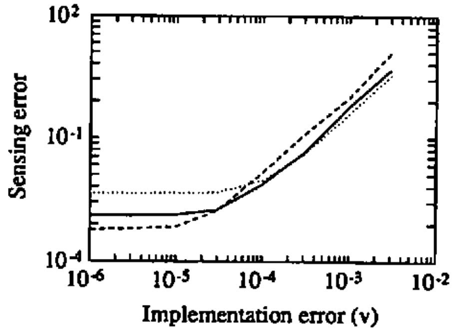

Figure 8: The sensing error as the implementation error is varied for three configurations using 96 (···), 150 (—) and 240 (- -) rectangular strips.

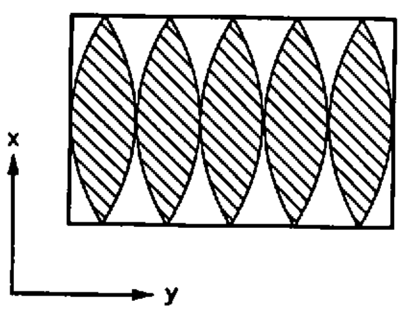

Figure 7: Quadratically shaped strips.

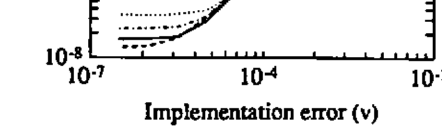

Figure 9: The sensing error vs implementation error for six configurations using 15 (top graph), 30, 45, 60, 75 and 90 (bottom) quadratically shaped strips.