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A PARAMETRIC STUDY OF THE BOWED STRING: THE VIOLINIST'S MENAGERIE

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There are many ways to make an unsatisfactory noise on a violin. Some of these require extreme actions of one kind or another from the player, but there are a few which even the best players slip into from time to time when playing near the limits of the normal performance regime. Indeed, these undesirable regimes determine those limits. We explore this "rogue's gallery" (of bowed-string motions of direct interest to musicians) using simple theory complemented by observations and numerical simulations of bowed string motion, in an attempt to understand under what conditions the usual regime for steady playing is accessible.

This usual regime is the Helmholtz motion [1,2,3], in which at any given instant the string lies in two more or less straight pieces separated by a sharp corner ("the Helmholtz corner"). This corner shuttles around the visible envelope of the string motion at the wave speed of the string, alternately triggering the onset of slipping and sticking as it passes the bow. Thus there is one period of sticking and one of slipping in each cycle.

A major reason for this study is that it might shed light on the harder problem of how the precise conditions under which the Helmholtz motion is possible vary among different violins. While the layman commonly supposes that violins are chosen solely on the basis of their sound qualities, the player may be at least as concerned about differences in "feel" which make one instrument more "docile" than another. Among the many things implied by such use of words is surely a difference between instruments in the range of bowing parameters for which normal steady playing is possible. In any case, the tolerance problem for steady playing is the simplest problem for scientific study and forms a necessary first step in a more complete study.

We build upon the well-known work on this problem by Raman [4] and Schelleng [5]. We take as our starting point the last-named's diagrammatic representation of bowing tolerance. During steady bowing the player controls three parameters: bow speed v_b, normal force f_b and position of the bow on the string, which we describe by the parameter β denoting the distance of the bowed point from the bridge as a fraction of total string length. Schelleng held v_b constant, and plotted a first approximation to the region of the f_b-β plane in which the Helmholtz motion could exist.

Schelleng considered two other types of motion to which the Helmholtz motion could give way. He calculated a minimum bow force based on transition to motion with two slip periods per cycle (the "double-slip" motion), and a maximum bow force where the Helmholtz corner is no longer strong enough to initiate slipping when it passes the bow. Above this maximum force, motion may be aperiodic ("raucous" motion), or it may be more or less periodic with a period substantially greater than the string's natural period.

A competent player will not stray into, or even close to, the raucous regime. His maximum usable bow force is usually determined by the need to avoid one of two other undesirable deviations from the Helmholtz motion with the natural string period. When β is not too small (i.e. playing with the bow not too near the bridge), the limit is determined by the string playing unacceptably flat, as a result of an effect of frictional hysteresis discussed by us previously [3,6].

When playing nearer the bridge, a different effect sets the limit on bow force. As a result of the finite width of the ribbon of bow hair in contact with the string, some of the hairs start to slip during the nominal sticking period of the Helmholtz motion. These slips tend not to be accurately periodic, and give rise to a component of audible noise accompanying the note being played. Depending on the musical context, this eventually reaches an unacceptable level, so determining the maximum bow force [7,8].

A quite different member of our menagerie is encountered when playing well away from the bridge (sul tasto), at a point on the string close to a simple subdivision of the length. The midpoint is the most extreme case, but is unusual in practice. The one-third, one-quarter and one-fifth points are more commonly encountered, and progressively less troublesome. What usually happens near one of these points is the onset of a totally different oscillation regime, described collectively by Lawergren [9] as "S-motion". The S-motion regimes form an interesting and important subset of the "higher types" classified by Raman in his monumental work on the bowed string. An audible characteristic of S-motion is the very strong presence in the note of the Nth harmonic, when playing close to the $\beta=1/N$ point. S-motion is sometimes used for colouristic effects in sul tasto playing.

It should be noted that S-motion does not occur exactly at the 1/N points: at those points, a different set of higher types is obtained which correspond simply to removing from the Helmholtz motion every Nth Fourier component. These motions, known as "Helmholtz's crumples", were observed by Helmholtz himself, but are not very important in practice since they require extremely accurate placement of the bow at the 1/N point (and a light bow force).

The final (and more roguish) character we need to include in the gallery is another motion which is one of Raman's higher types, which we have christened the "double flyback" motion. So far as we are aware, specific attention has not been drawn to this motion in the past [6]. The motion contains two slip periods per cycle, in close succession. This distinguishes it from what we have called the double slip motion above, where the two slips are roughly equally spaced. It is an entirely different oscillation regime from the double slip motion. In Raman's classification (by the number of "corners" propagating on the string), it is of the third type rather than the second type. The reason for the name will become clear below when an example of the motion will be given.

Raman's catalogue of higher types contains many other possible motions of a bowed string, a good number of which have been observed [10]. However, from the point of view of the player, and thus for our present purpose, the list given above seems substantially complete. Only under rather rare circumstances do competent players who are trying to produce the Helmholtz motion slip into any regime we have not mentioned. The only candidate known to us for addition to the menagerie is the E-string "whistle" to which certain instruments are prone, but we have never had access to a sufficiently repeatable example to pin down what motion is involved. We would be most interested to obtain access to an instrument which suffers seriously from this problem, to fill this gap in current knowledge. This empirical observation that our menagerie is now substantially complete is the result both of watching oscillograms of bridge force during much playing, and of extensive experiments with the computer simulations which we have described previously [3,7,11].

The various desirable and undesirable regimes of oscillation described above are illustrated in Figure 1. These all show waveforms of transverse force exerted by the string on the bridge, observed by means of a piezoelectric transducer in the string notch. They were all obtained on the same open violin G string, bowed by hand with a conventional bow, and the time scale is the same in each case. The bridge force waveform for the Helmholtz motion is approximately a sawtooth. Since this is a real string, the rapid flyback (as the Helmholtz corner reflects from the bridge) is rather rounded. This is shown as Fig. 1(a). The other inmates of the menagerie appear in the other figures, with the exception of flattening, which even when clearly audible is virtually indistinguishable from the Helmholtz motion in such a small picture. Figure 1(e) shows a "double-flyback motion". The bridge-force waveform exhibits a pair of closely-spaced flybacks.

Schelleng's original study of tolerance considered the motions shown in Figs. 1(b) and 1(f). We are suggesting that a more refined version of his study should allow for the motions shown in Figs. 1(b)-(e) plus the flattening effect. Figure 1(f) is rarely relevant in practice. This middle road between allowing for Schelleng's two motions and Raman's vast collection seems not to have been pursued before. It provides a framework for a more complete study of the practical limits on bowing parameters for

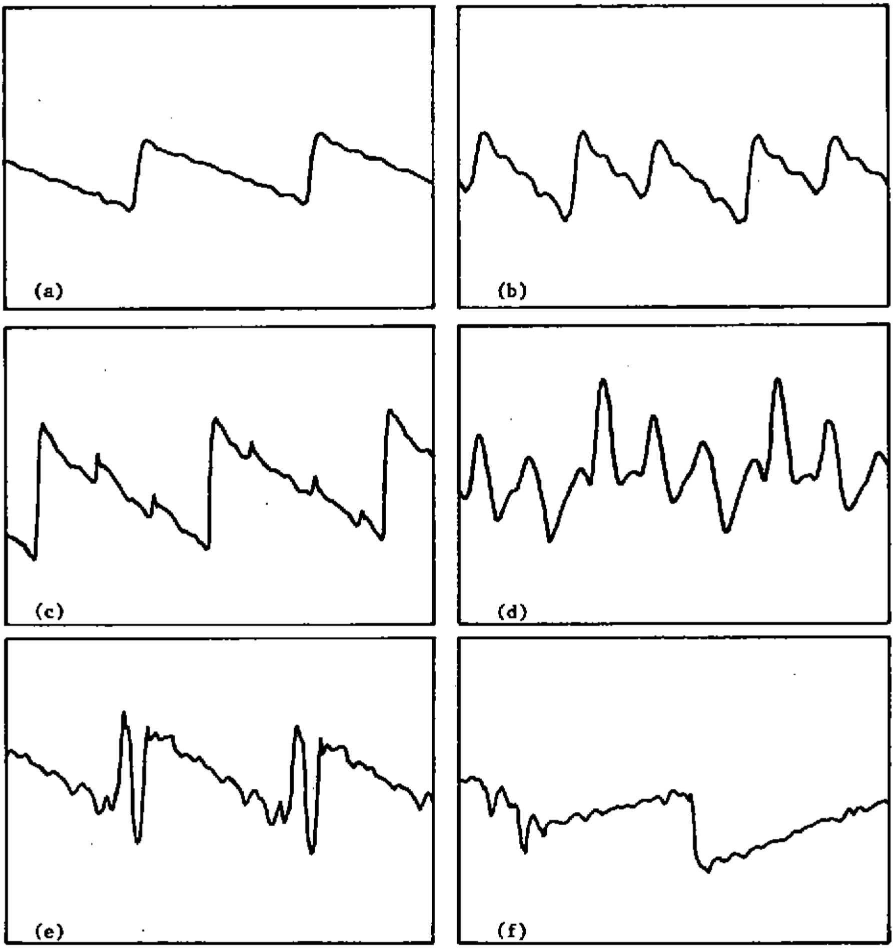

Figure 1. Steady oscillation regimes of a bowed violin G string. (a) Helmholtz motion. (b) Double-slip motion. (c) Spikes. (d) S-motion for $\beta=1/4$. (e) Double-flyback motion. (f) Slightly raucous motion.

steady playing. Such a study is still under way, and some general findings only are reported here.

Figure 2 shows roughly where most of the oscillation regimes discussed above fall in Schelleng's diagram. The two slanting lines represent Schelleng's maximum and minimum bow forces for the Helmholtz motion. These vary like β^{-1} and β^{-2} respectively, giving rise to the two different slopes in the log-log graph. The regions of spikes and flattening fall inside Schelleng's region, and vertical stripes surrounding the points $\beta=1/N$ for low values of N indicate where S-motion is encountered. Schelleng's lines are defined by transition to double-slip motion and raucous motion respectively, so these appear outside his tolerance region.

Double-flyback motion is not indicated at all on Fig. 2, and to explain why we need to examine another aspect of the behaviour of the different oscillation regimes as a function of bow force. Suppose we fix β at some typical, moderately small, value. Figure 3 then indicates the relationship between Helmholtz motion, double-slip motion and double-flyback motion. The first vertical bar indicates the Schelleng tolerance range for the Helmholtz motion at this β. Alongside is the corresponding range for the double-slip motion. If we start with a Helmholtz motion and slowly reduce the bow force, a transition to double-slip motion occurs at Schelleng's minimum force as indicated by the arrow labelled "decreasing F_b". However, if we now increase the force again we do not immediately revert to Helmholtz motion. The bars overlap, and we have a hysteresis of regimes. The transition back to Helmholtz motion occurs at a far higher force, indicated by the arrow labelled "increasing F_b".

The right-hand bar in Fig. 3 shows the tolerance range for the double-flyback motion. It is virtually identical to the Helmholtz range. This means that there is never a forced transition from Helmholtz to double-flyback or vice versa in the way we have just discussed for the double-slip motion. This is the reason that the double-flyback motion was not indicated in Fig. 2: its tolerance region is almost exactly coextensive with the Helmholtz region.

This behaviour of the double-flyback motion has both its good and its bad aspects, from the point of view of the player. Once a Helmholtz motion is established, gradual changes to bow force will not cause an unwanted transition to double-flyback motion. However, if the note is started with the wrong kind of transient so that double-flyback motion is established, then no small adjustment can change it to a Helmholtz motion. A new transient is required. Since the sound of the double-flyback motion is rather unpleasant, it is fortunate that most simple transients give rise to the Helmholtz motion rather than the double-flyback motion.

In summary, we have identified a small subset of the large collection of possible steady motions of a bowed string which

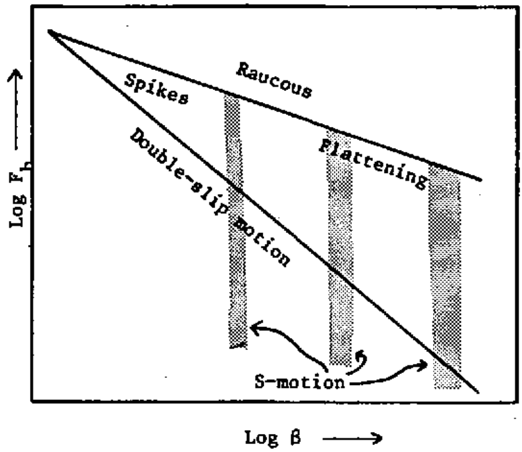

Figure 2. Schelleng's tolerance diagram showing the approximate positions of some of the regimes discussed.

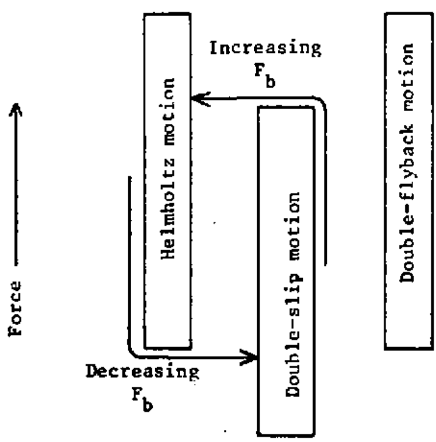

Figure 3. Force tolerance ranges for three oscillation régimes at fixed β, illustrating regime hysteresis.

seems to include most regimes of interest to the player. We have indicated where these occur in parameter space, using Schelleng's well-known diagram of the F_b-β plane. We have also drawn attention to the importance of hysteretic behaviour of the different oscillation regimes. A more detailed analysis of these effects should give a good basis of understanding of the parameter ranges in which a musically-acceptable Helmholtz motion can be sustained. It can also yield other incidental results of some interest, such as a more realistic criterion for the occurrence of wolf notes, based on an assumption of slowly-varying alternation between Helmholtz and double-slip motions.

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