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MEASURING THE ELASTIC AND DAMPING BEHAVIOUR OF FLAT ORTHOTROPIC PLATES

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Vibration problems involving orthotropic plates are quite common, ranging from vibration control in the composite panels of airframe structures or printed-circuit boards to the design of musical instrument soundboards. In all these problems, one cannot get very far without values for the essential physical parameters of the sheet material in question, in particular the elastic and damping constants for the relevant frequency range. In this paper we describe an approach to making these measurements, based on observing the frequencies and damping factors of low vibration modes of plates with free boundaries, which we believe to be the simplest way to achieve the desired accuracy. Free boundaries offer the only straightforward way to realise in practice a set of boundary conditions which correspond well to theoretical assumptions.

For the purposes of this study, an orthotropic plate is one whose elastic behaviour (as described by thin-plate theory) exhibits mirror symmetry with respect to two mutually orthogonal axes in the surface [1,2]. We are interested in bending vibrations of such plates. Plates meeting this description, to a reasonable approximation, arise in a wide variety of ways. In fact, most familiar sheet materials are of this nature - corrugated sheet, plywood, fibre-reinforced composites and conventionally-cut wooden boards all qualify.

Some of the materials listed above can be regarded as sheets cut from a three-dimensional orthotropic solid. It is obvious that a plate cut parallel to one of the symmetry planes of such a solid will have orthotropic symmetry. It is also true, if perhaps rather less obvious, that a flat plate cut from an orthotropic solid in such a way as to contain <u>one</u> of the symmetry axes of the material will be orthotropic in the sense used here.

This latter case is relevant to the study of wooden plates [1,3,4,5,6]. If the curvature of the annual rings of the tree is neglected, then wood can be regarded as an orthotropic solid. Boards are usually sawn parallel to the axis of the tree, thus containing one symmetry axis of the material (the "grain"). A board cut precisely radially will lie in a symmetry plane, and is described as "quarter-cut". More generally, though, the annual rings will pass through the plate at some angle other than normal, a so-called "slab-cut" board. When wood is selected for musical instrument soundboards, quarter-cut boards are always preferred. However, much of the wood sold even for this purpose is not precisely quarter-cut, and an understanding of the resulting variations in elastic and damping properties, which turn out to be sensitive to ring angle [6], is very important for quality control.

The bending vibrations of a thin, flat, orthotropic plate are governed by four independent elastic constants. The most useful way to define these is through the elastic potential energy. For a plate of small thickness h, lying initially in the xy plane and vibrating with a centre-plane transverse displacement $w(x,y)e^{i\omega t}$, the potential energy functional can be written [2,4,7]

$$V = \frac{1}{2} \iint h^3 (D_1 w_{xx}^2 + D_2 w_{xx} w_{yy} + D_3 w_{yy}^2 + D_4 w_{xy}^2)\, dA \tag{1}$$

where subscripts x and y denote partial derivatives, and the double integral is taken over the area of the plate. We will be mainly concerned with plates of uniform thickness and elastic properties, although the expression (1) remains correct for slow variation of these quantities with x and y. The problem of interest is to measure the four elastic constants D_1, D_2, D_3 and D_4, and the four associated damping constants. The paper will describe how the elastic constants may be deduced from measured frequencies of modes of free-edged rectangular plates.

Once we know to reasonable accuracy a particular mode shape and its frequency ω, it is quite easy to calculate the damping of that mode. It is convenient, and probably at least as accurate as any practical measurement technique, to use a "small damping" approximation which assumes that the modal Q-factor is much greater than unity. We first introduce the four quantities η_1 to η_4, defined by

$$\eta_i = \mathrm{Im}(D_i)/\mathrm{Re}(D_i), \quad i=1,2,3,4 \tag{2}$$

(in other words the conventional loss factors associated with the individual complex D's). The reciprocal of the modal Q-factor of any given mode can be shown from Rayleigh's principle to be simply a weighted sum of the four η's [7]. The expression is

$$Q^{-1} = \eta_1 J_1 + \eta_2 J_2 + \eta_3 J_3 + \eta_4 J_4 \ , \tag{3}$$

where

$$J_1 = \frac{D_1 \iint h^3 w_{xx}{}^2 dA}{\omega^2 \iint \rho h w^2 dA}, \qquad J_2 = \frac{D_2 \iint h^3 w_{xx} w_{yy} dA}{\omega^2 \iint \rho h w^2 dA},$$
$$J_3 = \frac{D_3 \iint h^3 w_{yy}{}^2 dA}{\omega^2 \iint \rho h w^2 dA}, \qquad J_4 = \frac{D_4 \iint h^3 w_{xy}{}^2 dA}{\omega^2 \iint \rho h w^2 dA}, \tag{4}$$

so that

$$J_1 + J_2 + J_3 + J_4 = 1 \ . \tag{5}$$

The expressions (4) for the J's simply indicate the partitioning of potential energy, and thus dissipation rate, among the types of motion associated with each of the D's in eq. (1). They may be calculated readily if the vibration mode shape w(x,y) and its (undamped) angular frequency ω are known. It is then quite straightforward to determine at least three of the four η's from measured Q-values for the same modes whose frequencies were used to determine the elastic constants. It turns out that J_2 is frequently so small for these modes that η_2

cannot be determined.

We now give measured results for five different plates which illustrate the range of possible applications of the approach. Under the general heading of composite plates we consider one of plywood, one of glass fibre reinforced circuit board, and one laminated plate consisting of unidirectional carbon fibres bonded to both sides of a light core material. We also study two plates of Norway Spruce (<u>Picea abies</u>), one quarter-cut and one not, both sold as being of suitable quality for use in musical instruments. The measured constants for all five plates are summarised in Table 1. Examples of the detailed measurements and predictions are shown in Tables 2 and 3, for the two wooden plates.

There are two general remarks to be made. First, the simple thin-plate bending theory is quite well supported by all measurements. Both frequencies and damping factors are matched to reasonable accuracy by the best predictions in each case. For the frequency predictions this is perhaps not surprising, although detailed experimental checks on orthotropic plate theory are not very common. For internal damping, this is believed to be the first systematic test of the theory used here, and the level of agreement revealed is broadly satisfactory [8].

The second general remark concerns the large range of parameters and material types covered by the five plates tested. For example, the ratio of bending stiffnesses in the two principal directions, $D_1:D_3$, ranges from almost unity (plate 2) to nearly 70:1 (plate 5). Similarly, the plate materials range from wood (plates 4 and 5), through a traditional composite (plywood, plate 1), to more modern composites (plates 2 and 3). Plate 2 is the familiar material used for printed circuit boards, a resin matrix reinforced by a rectangular array of glass fibres. Plate 3 is an experimental material, arising from preliminary efforts to design a composite material which might match the properties of spruce for musical instrument soundboards. It is a "sandwich" construction, with unidirectional carbon fibres on both outer surfaces enclosing a light core material, balsa wood.

The detailed results for plates 4 and 5 merit a few remarks. Both plates are of Norway spruce (<u>Picea abies</u>), traditionally used for instrument soundboards. Plate 4 is an example of the preferred quarter-cut plate, while plate 5 is very far from being quarter-cut, having a ring angle of about 40°. The two specimens are not closely related, having been obtained through standard commercial channels. But both have a similar ring spacing, and it is not inappropriate to compare the broad trends in the measured results with the theoretical calculations given in ref. [6]. This comparison shows an encouraging level of agreement.

A final small point to note about these two plates is that the damping constants are very similar between the two plates, much more so than the elastic constants. Damping is of great importance in musical instrument design, even more than in other areas of plate vibration analysis, and so further measurements on this topic would be of great value. It is possible that microstructure modelling could shed light on the physics of damping in plates cut in different ways from solid timber, as indeed it might shed light on the relations between the elastic constants [6].

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Plate	a mm	b mm	h mm	ρ kg/m³	D_1 MPa	D_2 MPa	D_3 MPa	D_4 MPa	η_1	η_2	η_3	η_4
1	225	225	6.36	674	986	15	411	233	.0105	-	.0128	.0239
2	150	177	3.23	2081	2840	1200	2560	2470	.0031	.0047	.0033	.0070
3	204	204	1.18	687	5570	560	543	884	.0020	-.0061	.0113	.0154
4	178	178	2.4	415	1320	77	82	227	.0051	-	.0216	.0164
5	190	170	2.26	399	880	24	13	229	.0074	-	.0212	.0139

Table 1. Physical parameters for the five boards tested: (1) plywood plate; (2) glass-fibre-reinforced resin printed circuit board material; (3) unidirectional carbon fibre laminated on both sides of a light core; (4) quarter-cut Norway spruce Picea abies; and (5) Norway spruce cut with a ring angle about 40° to the plate normal. Values for η_2 are quoted only for certain plates, for which it proved possible to make a realistic estimate.

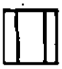
Mode	Measured frequency	Predicted frequency	Measured Q	Predicted Q	J_1	J_2	J_3	J_4
1 	102	102	59	60	.007	-.001	.043	.949
2 	119	119	40	42	.012	-.026	1.012	.001
3 	245	240	57	55	.014	.001	.277	.708
4 	335	328	46	42	.014	-.025	1.008	.003
5 	479	480	140	136	1.001	-.010	.009	.001
6 	522	523	117	120	.843	-.001	.010	.145
	459	460	51	54	.548	-.097	.548	.000
	498	499	81	84	.467	.065	.467	.001

Table 2. Measured and predicted frequencies and Q-factors for plate 4 of Table 1, a quarter-cut plate of Norway spruce of suitable quality for instrument making. The factors J_1, J_2, J_3 and J_4 are also given.

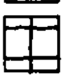
Mode	Measured frequency	Predicted frequency	Measured Q	Predicted Q	J_1	J_2	J_3	J_4
1 	50	50	38	46	.010	-.022	1.011	.001
2 	93	93	65	72	.010	.002	.029	.959
3 	138	138	46	47	.007	-.016	1.008	.002
4 	193	194	69	69	.024	.007	.110	.859
5 	267	270	52	47	.014	-.017	1.001	.002
6 	310	314	73	66	.027	.010	.230	.732
7 	331	331	134	134	.992	.001	.007	.000
8 	391	380	109	109	.752	-.006	.017	.237
	405	404	-	-	-	-	-	-
	437	435	-	-	-	-	-	-

Table 3. Measured and predicted frequencies and Q-factors for plate 5 of Table 1, a plate of Norway spruce cut with a ring angle of about 40°, but sold as being of suitable quality for instrument making. The factors J_1, J_2, J_3 and J_4 are also given. For this particular plate, the ring and X mode Q-factors were not measured, so no damping data are given.