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# APERIODICITY IN BOWED-STRING MOTION

M.E.McINTYRE, R.T.SCHUMACHER and J.WOODHOUSE

UNIVERSITY OF CAMBRIDGE, CARNEGIE-MELLON UNIVERSITY, and
TOPEXPRESS LTD, CAMBRIDGE

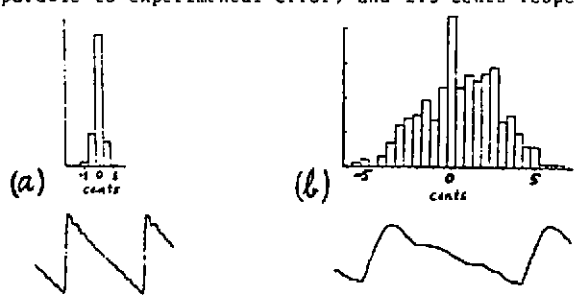
An investigation into the causes of aperiodicity in bowed-string oscillations is made using a combination of simple experiments, mathematical analysis, and computer simulation, and the status of theoretical models which idealise the behaviour as exactly periodic is thus assessed.† Three distinct sources of departure from a strictly periodic, Helmholtz type of motion are identified (§§1,2,3 below). The third is the most important musically, and depends on differential slipping of the bow hairs during the nominal sticking phase of the Helmholtz cycle, a phenomenon not previously studied.

## 1. Flyback jitter.

'Flyback jitter', or cycle-by-cycle variation in the period of the sawtooth waveform of the transverse force exerted by the string on the bridge, reflects random variations in the timing of the round trip of the Helmholtz corner or propagating velocity jump. It can be measured very accurately from the zero-crossings in the steep flyback in the digitized waveform. The interesting suggestion has been made [4] that flyback jitter might be intrinsic to the basic nonlinear dynamics of the bowed string. Superficially this seems consistent with the fact that certain simple mathematical models predict instability of the Helmholtz motion, as pointed out some time ago by Friedlander [5].

Careful measurements of flyback jitter were made under a variety of conditions. Two examples of waveforms and the correponding statistical distributions of period lengths are shown in Fig. 1, for opposite extremes of string thickness and length. Normal bowing by hand was used, with bow force not far above minimum in the cases shown, and great care was exercised to keep conditions steady. Fig. 1a is for a very thin (0.007 inch diam.) 'rocket-wire' string on a laboratory monochord 450mm long; Fig. 1b is for a violin G string stopped at 715 Hz (near the end of the fingerboard). The standard deviations of these distributions are 0.04 cents (comparable to experimental error) and 2.3 cents respectively, where

FIG. 1




†Details are to appear in Acustica later this year [1]; preliminary results were mentioned in two earlier publications [2],[3].

APERIODICITY IN BOWED-STRING MOTION

a cent is a hundredth of a semitone or 0.0006 of a period. (For comparison, the standard deviation for just-audible Gaussian jitter in pulse trains presented via headphones is typically of the order of 5 cents [6].) Many other cases were tried, bow force being varied as well as string length and thickness.

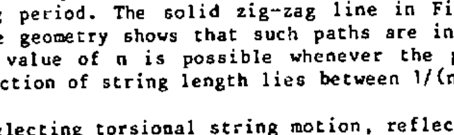
The observed parameter dependence of the jitter supports neither the suggestion that such jitter is intrinsic to the basic nonlinear dynamics, nor that it has any connection with Friedlander's instability. Rather, the results consistently support the hypothesis that flyback jitter arises principally from small, externally-imposed irregularities, such as the uneven distribution of rosin on the bow-hair. Such irregularities can be expected to produce jitter roughly proportional to the flyback time $\epsilon$, or characteristic time of the velocity jump at the Helmholtz 'corner': $\epsilon$ or some fraction of it represents the available latitude for variations in the timing of the transitions from sticking to slipping and vice versa. It tends to scale with string thickness. A parameter dependence similar to that in the observations was exhibited by computer simulations with a model having finite $\epsilon$ (using the efficient computational scheme described in references [7], [8]) when and only when the friction characteristics were randomly varied from time-step to time-step.

2. Subharmonics and Friedlander's instability

The second source of departures from Helmholtz periodicity is the excitation of subharmonic perturbations. These do turn out to have a connection, in a certain sense, with Friedlander's instability.

Consider a space-time diagram such as Fig. 2, for a string carrying a Helmholtz motion. The path of the Helmholtz corner is shown by the dashed zig-zag line. The state of the bow-string contact is indicated by the heavy, horizontal line segments, which represent sticking, and the intervening 'windows' which represent slipping. The term 'window' is physically apt because, as was pointed out some time ago in Schelleng's important paper [9], the bow during slipping is nearly transparent to any extra disturbance incident upon it, owing to the near-constancy of the coefficient of sliding friction. During sticking the bow acts, by contrast, as a reflecting barrier to small disturbances.

FIG. 2.




One consequence (which occasionally has audible results as we shall see) is that there exist periodic paths whose periods are integral multiples n of the fundamental string period. The solid zig-zag line in Fig. 2 gives an example with n=4. A little geometry shows that such paths are indeed exactly periodic, and that a given value of n is possible whenever the position $\beta$ of the bow expressed as a fraction of string length lies between $1/(n-1)$ and $1/(n+1)$.

In most models neglecting torsional string motion, reflection from the sticking bow is perfect; for the real bowed string it is much less so as has been clearly pointed out by Cremer [8], [10]. If losses including the scattering into torsional modes are sufficiently low, negative resistance at the bow during slipping [9] can cause perturbations propagating along subharmonic paths to become self-excited; this, in fact, is the physical mechanism behind

Friedlander's instability. A complete analysis has been carried out for a simple bowed-string model (the 'Raman model' described e.g. in reference [11]). It involves keeping track of partial reflections at windows, and therefore consideration of a set of interlocking subharmonic paths all of the same order. This analysis does give quantitative agreement with the exponential growth rates and stability thresholds predicted by Friedlander's original method and its extension to the Raman model. The results have been further checked by computer simulations.

The subharmonic mechanism appears to be more important for understanding the behaviour of idealised models neglecting torsional string motion, than for understanding that of real strings under normal playing conditions. Losses to torsional modes seem more than enough to prevent self-excited subharmonic perturbations in practice. Nevertheless, the existence of the subharmonic paths means that some transients arising from stray disturbances to the Helmholtz motion tend to exhibit subharmonic quasi-periodicities over limited times, and a faint, muffled subharmonic of the note being played can sometimes be heard especially if pitch is high, bow force large and bow speed small. An example of the string velocity observed near the bowing point is shown in Fig. 3, obtained

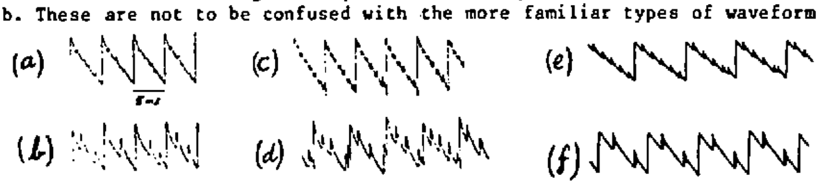
FIG. 3.

from a violin E string stopped at 1730 Hz and bowed near $\beta = 1/3$ with an ordinary bow. A third subharmonic (577 Hz) was audible. The pattern showed little persistence, let alone regularity, over longer times.

## 3. Audible noise

The obvious noise which builds up behind the musical note when a violinist tries to play more and more loudly near the bridge turns out to be due to another mechanism altogether. Subharmonics are usually inaudible under these conditions, and flyback jitter is usually observed to be well below the nominal audibility threshold even when the noise is very obvious to the ear. Observations of the bridge-force waveform have consistently shown, however, that when the noise is prominent so too are the irregular 'spikes' two examples of which are shown in Fig. 4a,b. These are not to be confused with the more familiar types of waveform

FIG. 4.




perturbation noted by Helmholtz, Raman and Schelleng [9] and which may predominate when the bow is further from the bridge. The average timing and amplitude of the spikes are sensitive to bow force, the second trace being for larger bow force (and louder noise relative to the musical note). Similar spikes appear in the waveform, accompanied by similar-sounding noise, when the string is bowed with a smoothed, rosined stick having two parallel ridges separated by a distance of the same order as the width of the bow hair, both ridges being in contact with the string (Fig. 4c,d). They disappear when only one ridge is in contact. This indicates that the phenomenon is connected with the finite width of the ribbon of bow hair; and the 'spikiness' suggests at once that differential slipping is the cause.

This hypothesis is shown to be self-consistent and to fit the observed facts (a) by an order-of-magnitude analysis of the frictional forces required to prevent differential slipping, which shows clearly that it must occur, and (b) by computer simulations for the case of two rigid 'bow hairs' (representing most closely the experiment with the two-ridged stick) again using the efficient numerical model introduced in reference [7].

Results from two of the simulations are shown in Fig. 4e,f, the lower trace again corresponding to the larger bow force. The qualitative variation with bow force of the timing of the spikes is well reproduced. As can be seen from further details presented in [1], the spikes in the simulations are unquestionably caused by differential slipping events, in most of which the 'hair' nearest the bridge slips while the other sticks.

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[1] McIntyre, M.E., R.T. Schumacher and J. Woodhouse 1981 Aperiodicity in bowed-string motion Acustica 50, in press.

[2] McIntyre, M.E., R.T. Schumacher and J. Woodhouse 1977 New results on the bowed string. Catg. Acoust. Soc. Newsl. 28, 27-31.

[3] McIntyre, M.E., J. Woodhouse 1978 The acoustics of stringed musical instruments. Interdisciplinary Science Reviews 3, 157-173.

[4] Cremer, L. 1974 Der Einfluss des 'Bogendrucks' auf die selbsterregten Schwingungen der gestrichenen Saite. Acustica 30, 119-136.

[5] Friedlander, F.G. 1953 On the oscillations of a bowed string. Proc. Cambr. Philos. Soc. 49, 516-530.

[6] e.g. Cardozo, B.L. and R.J. Ritsma 1968. On the perception of imperfect periodicity. IEEE Trans. on Audio & Electroacoust. AU-16, 159-164.

[7] McIntyre, M.E., J. Woodhouse 1979 On the fundamentals of bowed-string dynamics. Acustica 43, 93-108.

[8] Cremer, L. 1981 Physik der Geige. Stuttgart, S. Hirzel (in press).

[9] Schelleng, J.C. 1973 The bowed string and the player. J. Acoust. Soc. Amer., 53, 26-41.

[10] Cremer, L. 1979 The absorption of 'secondary correction waves' in the excitation of a string by bowing. Catg. Acoust. Soc. Newsl. 31, 12-16.

[11] Schumacher, R.T. 1979 Self-sustained oscillations of the bowed string. Acustica, 43, 109-120.