## VISCO-ELASTIC DAMPING IN FIBRE REINFORCED PLASTICS

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#### 1.0 INTRODUCTION

Fibre reinforcement of plastics usually results in very strong and stiff materials, whilst the density is low especially when carbon fibres are employed. Composites of this type offer to the design engineer a choice of fibre configurations and he is therefore free to design components in which the directions of the high stiffness or strength properties are most advantageous. If, however, a high loss plastic is used as the bonding matrix, it is also possible to take into account visco—elastic damping and utilise the effect to control the modes of vibration in components which may be subjected to periodic forces.

In general, materials of high stiffness to density ratio such as ceramics have low internal damping. Loss factors, defined as the ratio of the imaginary to real part of the complex modulus, less than 10<sup>-5</sup> have been reported in ref 1. In this class of material, the stiffness to density ratio is such that the longitudinal wave velocity is about 10 km/s. High loss materials such as rubbers and many plastics usually have comparatively low stiffness to density ratios resulting in longitudinal wave velocities around 2 km/s, but the loss factor may be as high as 2.0 even at room temperature. The high damping is due to molecular relaxation and in consequence is highly dependent on temperature and frequency, typical of all visco-elastic materials. This fact must be taken into account especially when designing components for acoustic applications where a wide range of frequency in their dynamic loading could be encountered.

Some of the more useful properties of ceramics and lossy polymers are combined in fibre reinforced plastics. The contribution from the matrix is the major factor in the determination of the internal loss. The fibres, on the other hand, affect the stiffness and strength without directly affecting the loss although it will be shown later that the loss factor of these composites is dependent on the values of fibre stiffness. The stiffness to density ratios, however, are similar to the ceramics. The longitudinal wave velocity in high modulus carbon fibre is as high as 15 km/s being surpassed only by a few materials, the best known of which is diamond. The resulting longitudinal wave velocity in fully aligned composites can be around 12 km/s which is comparable with the values for ceramics but the internal loss factors associated with the various modes of vibration are more like those for rubbers and high loss polymers. The high velocity and damping can be used with advantage in the design of

acoustic components since the frequencies of any resonances will be much higher than those in similar components made from conventional materials and the higher damping leads to a smoother frequency response.

#### 2.0 ELASTIC PROPERTIES OF FIBRE REINFORCED PLASTICS

The simplest reinforcement system is that in which the fibres are fully aligned and is the only one considered here. The stiffness is greatest along the fibre direction and least in the transverse direction. The elastic properties are determined by the volume ratio  $v_{f^*}$  and the elastic constants of both constitutents. If the fibres are packed into a regular hexagonal array, the elastic stiffness is isotropic in the plane normal to the fibres. The elastic constants of composites of this type have been measured using an ultrasonic pulse technique developed at NPL in which the velocity of longitudinal and shear waves are measured in a wide range of direction in all three principal planes. This is described in ref 2. If the elastic constants along the principal axes are deduced from the measurement of the velocities, it is possible to calculate the rotated or 'effective' elastic stiffness (longitudinal and shear) for any system of axes. Measurements made on high modulus carbon fibre-epoxy resin composites ( $v_f = 0.7$ ) have shown that the tensile stiffness ratio between the fibre and transverse direction is about 30:1. Values of the elastic properties of high modulus and high tensile strength composites for various volume fractions are given in ref 3.

## 3.0 PREDICTED LOSS PROPERTIES OF COMPOSITES

If we assume that the fibres are free from dynamic loss, we can derive approximate expressions for the loss factors of the fully aligned composites. The three cases of particular interest relate to (a) tensile or bending load in the fibre direction, (b) tensile or bending load in the transverse direction and (c) shear in the fibre direction.

#### (a) Tensile loading in fibre direction.

The assumption made in this case is that the fibres and matrix are subjected to equal strain and the stress resulting in each constituent is determined by the values of the two Young's moduli  $E_{1f}$  and  $E_{m}^{*}$ . This is based on Voigt's hypothesis and is approximately true provided we ignore the lateral forces resulting from the differences in the Poisson's ratios of the fibre and matrix. The complex Young's modulus of the matrix  $E_{m}^{*} = E_{m}^{*} + iE_{m}^{*}$ , hence the loss factor is given by:

$$d_{\underline{m}} = \frac{E_{\underline{m}}^{n}}{E_{\underline{m}}^{n}} . \tag{1}$$

The complex Young's modulus in the fibre direction for the composite derived from the stresses in the fibre and matrix is:

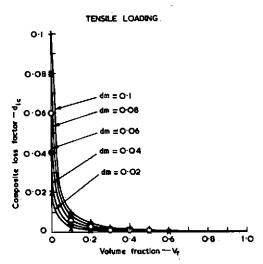
$$E_{1c}^{*} = v_{f} E_{1f} + (1 - v_{f}) E_{m}^{*},$$
 (2)

where  $E_{1f}$  is the Young's modulus of the fibre along its length and  $v_{\mathbf{f}}$  the volume fraction.

The loss factor of the composite, defined as the ratio of the imaginary to real part of Etc is given approximately by:

$$d_{1c} = \frac{E_{1c}^{"}}{E_{1c}^{"}} = \frac{(1 - v_{f}) E_{m}^{"} d_{m}}{v_{f} E_{1f} + (1 - v_{f}) E_{m}^{"}}.$$
 (3)

Since the fibre modulus  $E_{1f}$  occurs in the denominator of equation (3) it follows that the loss factor falls very rapidly with volume fraction as shown in fig 1.



LOSS FACTOR IN COMPOSITES TENSILE LOAD IN FIBRE DIRECTION FULLY ALIGNED TYPE I CARBON FIBRES VOIGT AVERAGE

Fig 1

## (b) Tensile loading in the transverse direction.

When load is applied in the transverse direction the assumption is that the fibre and matrix are both subjected to equal stress and the resulting strains are proportional to the compliances of the fibre and matrix. The modulus based on the Reuss hypothesis is therefore:

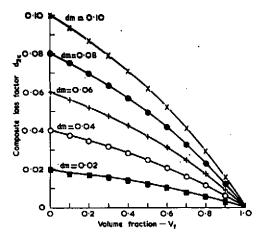
$$\frac{1}{E_{2c}^{*}} = \frac{v_{f}}{E_{2f}} + \frac{(1 - v_{f})}{E_{m}^{*}}, \qquad (4)$$

 $E_{2f}$  is the Young's modulus of the fibre in the transverse direction and in practice is only about one tenth of the longitudinal modulus  $E_{1f}$ .

The composite loss factor in the transverse direction can similarly be deduced from the ratio of the imaginary to real part of equation (4) and is given approximately by:

$$d_{2c} = \frac{(1 - v_f) E_{2f} d_m}{v_f E_m^* + (1 - v_f) E_{2f}}$$
 (5)

and the dependence of  $d_{2c}$  on  $v_f$  and  $d_m$  is shown in fig (2).



TRANSVERSE LOADING

LOSS FACTOR IN COMPOSITES TENSILE LOAD IN TRANSVERSE DIRECTION FULLY ALIGNED TYPE I CARBON FIBRES REUSS AVERAGE

Fig 2

The Voigt and Reuss values given by equations (2) and (4) give upper and lower limits of the longitudinal and transverse moduli respectively. No account is taken of the interaction between the fibre and matrix which could influence the values but they do provide a useful estimate of the composite properties.

## (c) Shear modulus in the fibre direction.

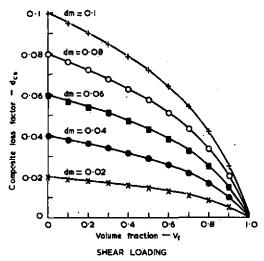
Several models have been used for the derivation of the shear modulus of unidirectional composites. One, due to Hashin and Rosen (ref 4) treats the composite as parallel cylindrical units consisting of a fibre surrounded by a co-axial column of matrix. In order to fill all space, the units are allowed to vary in diameter but are assumed to have a constant fibre fraction equal to that of the composite. The model therefore represents the random packing arrangements in real materials. Values of shear modulus in the fibre direction can be derived from expressions for this model given by:

$$G_{c}^{*} = \frac{G_{m}^{*} \left[G_{f}(1 + v_{f}) + G_{m}^{*}(1 - v_{f})\right]}{G_{f}(1 - v_{f}) + G_{m}^{*}(1 + v_{f})}, \qquad (6)$$

where  $G_f$  is the shear modulus of the fibre along the major axis and  $G_m^* = G_m^* + iG_m^*$ , is the complex shear modulus of the matrix. The shear loss factor  $d_{SC}$  is given approximately by:-

$$d_{sc} = G_{m}^{"} \left[ \frac{1}{G_{m}^{*}} + \frac{(1 - v_{f})}{G_{f}(1 + v_{f}) + G_{m}^{*}(1 - v_{f})} - \frac{(1 + v_{f})}{G_{f}(1 - v_{f}) + G_{m}^{*}(1 + v_{f})} \right]$$
(7)

The dependence of the shear loss factor  $d_{SC}$  with volume fraction is shown in fig (3) which is similar to fig (2) except for the higher values in shear compared with transverse tensile loading for volume fractions around 0.6.



LOSS FACTOR IN COMPOSITES SHEAR LOAD IN FIBRE PLANE IN PRINCIPAL AXIS FULLY ALIGNED TYPE I CARBON FIBRES HASHIN - ROSEN MODEL

Fig 3

#### 4.0 CONCLUSION

The tensile and shear loss factors derived above are for ideal materials in which the fibres are truly bonded to the matrix. Due to lack of adhesion between fibre and matrix the loss factors in real composites will always be greater than those predicted in the equations. If any voids or delamination exist in the mouldings, the effect of these will likewise increase the apparent loss factor. If any solvent is left in the polymer after moulding, the matrix loss factor will be influenced by its presence thus affecting the overall loss in the composite.

The loss values predicted can, however, be used as a guide for the design engineer who may use this data to give him minimum values. He will then have a choice of varying the volume fraction, matrix loss factor and also the type of fibre employed in the design of components where optimum stiffness and damping is a prime consideration.

- References 1 Davies, W.R. The use of resonance methods to determine the elastic constants of ceramics. Ultrasonics \*Ultrasonics for Industry 1969\*Conf. papers. p.11.
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