# A NEW METHOD FOR ANALYSIS OF NOISE TRANSMISSION IN COMPLEX STRUCTURES

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#### ABSTRACT

We describe the principles and practice of a method for determining the contribution of individual noise paths in systems where many paths are active. The method does not usually require any modifications to the system, or any detailed knowledge of its dynamics. The method does, however, require a thorough appreciation of its principles.

We describe a hardware system which can analyse over a hundred noise paths simultaneously.

### INTRODUCTION

The control of acoustic noise and vibration transmission from power plants and other machinery is becoming of increasing importance to their operators. In the civil field, areas of particular concern include the following:

- transmission of noise from a vehicle's power plant to the pilot and passengers (cars; helicopters; propeller-driven aircraft; cruise ships and power yachts);
- transmission of noise from factory plant to nearby residential areas;
- overboard transmission of engine noise from ships (seismic survey vessels towing hydrophone arrays; fisheries survey vessels).

In the military area, noise transmission makes for increased detectability by hostile forces; this applies particularly to:

- overboard transmission of acoustic noise from machinery on submarines;
- · airborne noise from land vehicles used by land forces.

The most direct solution to a noise problem is to prevent the source from producing it in the first place: careful balancing of the shaft in rotating turbomachinery, for example, can lead to significant reductions. In many cases this is not possible: reciprocating engines, for example, inevitably produce out-of-balance forces at multiples of shaft turning frequency.

The next step in such cases is to look at ways of reducing transmission from the source to the environment. For example, in the case of a marine diesel, mounts, air intake and exhaust, cooling water pipes and airborne transmission may all be significant paths. Treating all possible paths will, of course, maximise one's chances of success, but is likely to be very expensive.

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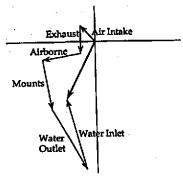


Figure 1 Hypothetical phasor plot of contributions to the radiated noise of a diesel engine

Since all of these paths are excited by a common source, the vibrations will interfere coherently in the region where reduction is required. This process may be represented by a phasor plot. In Figure 1, each vector represents the amplitude and phase (relative to the rotation of the diesel's shaft) of the noise transmitted via one group of paths. Their sum will almost always be less than the sum of their individual amplitudes because of their different phases. In this example, if noise transmitted via the water inlet were controlled without otherwise altering the dynamics of the system, the transmitted noise would actually increase; by contrast, if airborne noise and the mounts were both treated then a very large reduction would be obtained. An engineer with no prior knowledge of this phasor diagram might simply attack the most obvious path, say the mounts in this case, and would then be doomed to achieving no more than a few dB reduction at best.

It should be borne in mind that if the noise source excites the paths at several harmonics of the fundamental frequency, the phasor diagrams at each harmonic may be different. This may arise because of frequency dependence in the paths; to take a simple example, suppose that the cooling water pipes transmit noise via fluid pressure, and that they are of different length. If the diesel engine excites both in the same way, then at certain frequencies the length difference will be equal to half a wavelength and their transmissions will interfere destructively; at intermediate frequencies it will be a whole number of wavelengths and their contributions will be in phase.

In addition to this, the source excitations may change with frequency; for example, the phases of the vibrations at each mount may change because of modal vibrations of the engine casing.

In what follows we present a method of analysis which allows phasor plots such as Figure 1 to be produced for the excitation frequencies of systems of practical interest. We discuss its applicability in different circumstances.

#### PRINCIPLES OF THE METHOD

We divide the overall system conceptually into two elements: the noise source (shown as E in Figure 2) and the substructure (S in Figure 2). The two communicate via a finite number of acoustic and/or mechanical degrees of freedom of connection across the boundary between them. These connections (p) lie on the noise paths which are of interest.

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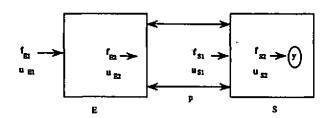


Figure 2

We instrument the connections with noise sensors. There must be sufficient sensors to observe all degrees of freedom of connections, and they must be sited so that any degree of freedom can in principle be resolved by suitably combining sensor signals. We also instrument the region (y) within the substructure where noise reduction is required. We may require only one sensor (if y is compact on a wavelength scale), or many.

We then apply a series of forcings to E, in various directions and at various points. These result in different patterns of excitations of the sensors at (p) and at (y). Given a sufficient number of forcings, we can conceive of solving a set of simultaneous equations to determine the path strengths from sensor excitations at p to those at y. It may seem odd to talk of sensor signals ratios as "path strengths", but we shall see that in this special case they have the requisite characteristics.

The final step in the method is to activate the noise source E: run the engine, pump or whatever at the relevant condition. This provides measurements of the sensor signals at p, which we multiply by the calculated path strengths to obtain the estimated path contributions to noise at p. The sum of these contributions may be compared with the observed noise signals at y as a check on accuracy.

To express this procedure more formally, we use the following notation. Let E be the mobility matrix of the noise source, relating forcings at our excitation points and at its points of contact with E to velocities at these contact points. Let E be the mobility matrix of the substructure, relating forcings at its points of contact with E to velocities at those points and in the reduction region E. Let E and E be the vectors of forcings on E and E, and let E be the velocities. Thus

$$\mathbf{u}_{\mathbf{E}} = \mathbf{E}\mathbf{f}_{\mathbf{E}}$$

$$\mathbf{u}_{\mathbf{S}} = \mathbf{S}\mathbf{f}_{\mathbf{S}}$$
(1)

We partition the matrices and vectors to separate out our external forcings  $(\mathbf{f}_{E1})$  of E from those arising from contact with S,  $(\mathbf{f}_{E2}, \mathbf{f}_{S1})$ . Similarly we wish to separate out velocities at our forcing points  $(\mathbf{u}_{E1})$  and velocities in the region y  $(\mathbf{u}_{S2})$  from those at the contact points between E and S,

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 $(\mathbf{u}_{p2}, \mathbf{u}_{S1})$ . We write

$$\begin{pmatrix} \mathbf{u}_{E1} \\ \mathbf{u}_{E2} \end{pmatrix} = \begin{pmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{f}_{E1} \\ \mathbf{f}_{E2} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{u}_{S1} \\ \mathbf{u}_{S2} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{f}_{S1} \\ \mathbf{f}_{S2} \end{pmatrix}$$
(2)

where, because E and S are assumed to be rigidly connected at contact points, we have

$$\mathbf{f}_{\mathbf{F}2} = -\mathbf{f}_{\mathbf{S}1} \tag{3}$$

$$\mathbf{u}_{\mathrm{E2}} = \mathbf{u}_{\mathrm{S1}} \tag{4}$$

f<sub>S2</sub> is the forcing on S in region y and is assumed to be zero.

From these definitions and boundary conditions a number of useful results follow. Firstly it can be shown that

$$u_{SZ} = S_{21} S_{11}^{-1} u_{S1} = H u_{S1} say$$
 (5)

i.e. that noise  $(u_{S2})$  at y can indeed be predicted from the excitations  $(u_{S1})$  on the paths; the matrix H is in effect a set of transfer functions from one to the other. Secondly,

$$\mathbf{u}_{S1} = \mathbf{S}_{11} \left[ \mathbf{S}_{11} + \mathbf{E}_{22} \right]^{-1} \mathbf{E}_{21} \mathbf{f}_{E1} \tag{6}$$

which gives us the transfer functions from our forcings of E to the resulting path excitations. Lastly,

$$\mathbf{u}_{S2} = \mathbf{S}_{21} \left[ \mathbf{S}_{11} + \mathbf{E}_{22} \right]^{-1} \mathbf{E}_{21} \mathbf{f}_{E1}$$
 (7)

which gives us the corresponding transfer functions from forcings of E to noise at y.

The process of identifying the noise paths can be represented as follows. We apply unit excitation forces to E at one position at a time; for excitation number i, the ith element of  $f_{E1}$  is unity, the remainder are zero. Suppose the resulting excitations at y are  $u_{S2}^i$ , and the path excitations are  $u_{S1}^i$ ; then using (7) the result of the complete set of excitations can be represented as

$$\mathbf{U}_{S2} = \mathbf{S}_{21} \mathbf{S}_{11}^{-1} \mathbf{U}_{S1} \tag{8}$$

(9)

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where

$$U_{S2} = \begin{bmatrix} u_{S2}^{1}, u_{S2}^{2}, ..., u_{S2}^{N} \end{bmatrix}$$

$$U_{S1} = \begin{bmatrix} u_{S1}^{1}, u_{S1}^{2}, ..., u_{S1}^{N} \end{bmatrix}$$
(9)

and

We then estimate the matrix of path strengths which we require by a matrix inversion:

$$\widehat{\mathbf{H}} = \mathbf{U}_{SZ} \mathbf{U}_{SI}^{-1} \tag{10}$$

For  $U_{S1}^{-1}$  to exist, N must be such that  $U_{S1}$  is square: we must apply as many excitations to E as there are sensors on paths.  $U_{S1}$  must also, of course, have rank N: we consider this point more fully in the next section.

Having estimated the path strengths, we run the noise source and record the resulting excitations; call them  $\mathbf{u}_{51}^0$  and  $\mathbf{u}_{52}^0$ . We use these to check our path strength estimates by forming an estimate of  $\mathbf{u}_{52}^0$  for comparison:

$$\widehat{\mathbf{u}}_{S2}^0 = \widehat{\mathbf{H}} \ \mathbf{u}_{S1}^0 \tag{11}$$

The phasor plot in Figure 1 illustrates the end result of this process. We consider the noise at one of the noise sensors in region y: this is one element of  $\mathbf{u}_{\Sigma \Sigma}^0$ . It is the dot product of one row of  $\widehat{\mathbf{H}}$  with  $\mathbf{u}_{\Sigma \Sigma}^0$ : i.e. the sum of products of estimated path strengths with path excitations. In Figure 1, each light phasor represents the amplitude and phase of one such product, i.e. the contribution due to one path; the heavy phasor represents the sum. This sum can then be compared with the amplitude and phase of the corresponding element of  $\mathbf{u}_{\Sigma \Sigma}^0$ , as a check on accuracy.

### THE METHOD IN PRACTICE

The first decision the experimenter must make is how to divide the system between noise source E and substructure S in Figure 2. The boundary is defined by path sensor positions. Three main factors determine the choice:

- The boundary must intersect all possible noise paths, and the intersections must be fully
  instrumented. It is therefore desirable to place the boundary so as to keep the number of
  degrees of freedom of connection across it, and hence the number of sensors required, to a
  reasonable figure.
- Sensors should be placed where noise reduction measures are likely to affect their signal levels in an obvious way. This makes results much more useful. For example, sensors could be placed close to mounting points where resilient elements could be installed.

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 Sensors should be placed where good signal-to-noise ratios can be obtained when the noise source is forced.

The next major decision is how many different forcings (N) of E are required. We saw from equation (10) that we might expect this to equal the number of path sensors, so that  $U_{S1}$  is square. However, from (6), we see that in this case,

$$\mathbf{U}_{S1} = \mathbf{S}_{11} \left[ \mathbf{S}_{11} + \mathbf{E}_{22} \right]^{-1} \mathbf{E}_{21} \mathbf{I}_{N} \tag{12}$$

This raises another possibility:  $E_{21}$  contains the dynamics of the noise source, and this may have fewer than N degrees of freedom. In the worst case, if E behaves as a rigid body,  $E_{21}$  will have a rank of 6 or less. Thus if N is greater than 6,  $U_{S1}$  may have a rank of less than N and will then be singular: in effect the noise source E obscures our view of the path strengths. H in (10) must then be estimated by using a pseudo-inverse of  $U_{S1}$ . Our estimated  $\widehat{H}$  will be consistent with any observations we can make on the system, but will not be unique: we can add any vector from the nullspace of  $U_{S1}^T$  to each row of  $\widehat{H}$  without affecting the fit.

In practice we deal with this problem by using singular value decomposition (SVD). This allows us to examine the rank of our excitations matrix after each forcing; we are able to judge the point at which further forcings are no longer affecting the singular values. If there are then fewer than N significant singular values, we obtain a pseudo-inverse by inverting the significant singular values and subtituting zero elsewhere. This procedure seems to give useful results even for quite large rank deficiencies: a situation where transmission is dominated by a few noise paths can be accurately diagnosed for a ratio of N:(rank) greater than three, in the presence of many dozens of noise paths.

In extreme cases, however, and where this is practicable, the noise source E may be removed and forcings applied directly to the paths. This may, however, result in significant errors if a large change in static loading, e.g. on resilient machinery mounts, changes the path dynamics.

#### HARDWARE SYSTEM

A system has been constructed to realise this technique. We have christened the technique as implemented on our system 'Multiple Path Signature Decomposition' (MPSD).

The system is based around a PC-compatible computer connected via a serial link to a purpose-built periodic data logging unit. The user drives the system through a menu system implemented on the PC, which in turn controls the data logging unit. A signal generator controlled from the data logger over an IEEE488 bus provides a synchronising signal for all data logging and excitation generation, while programmable signal conditioning allows the system to make maximum use of its dynamic range. The data logging system itself provides the excitation signals for the generation of the forcing f<sub>E1</sub>. Figure 3 is a block diagram of the system, which also shows how the sensors and actuators couple the MPSD system into the environment under investigation. The system was developed with assistance from the Admiralty Research Establishment, which also provided

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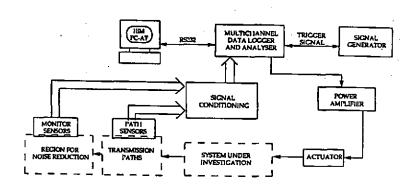


Figure 3 Topexpress path identification system facilities for full scale testing of the system in a marine environment.

The current MPSD system uses multiplexing to collect data over 128 sensor channels. The data logger transforms the sensor data to the frequency domain and transfers the results to the PC where it is stored. When the user believes that sufficient data has been taken, the transmission path matrix can be calculated, and phasor plots displayed on the PC screen. Sensors can be grouped together according to the physical paths they represent in order to simplify the display. The system has been extensively tested in laboratory, and has been used in the field on several marine and road vehicle applications, where it has proved to be both reliable and flexible.

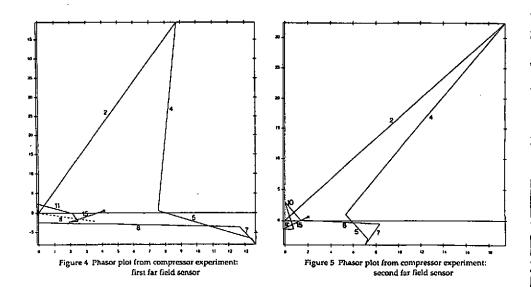
The main advantage of the technique is that it is largely non-invasive. It is unnecessary to block or sever paths of the system, although it is usually preferable to user sensors and actuators inside liquid- or gas-carrying pipes where possible. However, techniques of sensing and actuation have been developed to overcome this restriction in many cases. The non-invasive aspect of the technique makes it particularly easy to apply.

### AN EXAMPLE APPLICATION

This section describes a practical signature decomposition which was performed in the laboratory at Topexpress. The noise source was a small electrical air compressor which has been used by Topexpress for the testing of vibration measurement and control systems in the past.

The compressor was mounted on three rubber feet which, for the purposes of this test, were fixed to a wooden floor to enhance the vibration path. The aim of the experiment was to determine the route by which sound was radiated to the target point, which here was taken to be a point 5m from the compressor. The two noise target sensors deployed were thus vibration insensitive microphones spaced 1m apart at a distance of 5m from the compressor. The paths to the targets were assumed to be the direct airborne path and the vibration path through the mounts and the floor. These paths were instrumented by means of five microphones located around the compressor at a distance of 0.5m, and by a total of 18 accelerometers located six on top of each mount in such a way that all six degrees of freedom of the mount were measured. Thus a total of 23 path and 2 target channels were used.

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The fundamental compressor tone was 24.5Hz. The MPSD was performed at the second overtone of this fundamental, namely 73.5Hz. This was to ensure that the signals were well within the frequency range of all the sensors and actuators employed for the experiment. In addition, this frequency was low enough to ensure that the results would be reasonably invariant under small path changes. (In other applications, MPSD has been applied at up to 13 frequencies in one experiment.)

The signature of the compressor was measured first. It was then switched off and forcings were applied using a GWV46 electro-magnetic shaker and an 8-inch loudspeaker. In all, a total of 19 forcings were used, because by this stage it was found that further forcings were not significantly affecting the results.

Figure 4 shows the phase plot for the first target microphone. The broken line S is the measured signature, while the solid lines denote the components  $H_{1n}x_n$  (n = 1,...,23). The resultant of the vectors is the predicted signature  $\mathbf{u}_{52}$ ; its end is marked with a cross on the figure. Figure 5 is a similar plot for the second target microphone.

The vector loops are not closed, but the error between the predicted and measured target signatures is small on the scale of the largest vectors.

Also shown on Figure 4 and 5 are the sensor numbers of the channels producing the largest components of the predicted signature. It is interesting to note that the three largest components (2, 4 and 8) are all associated with lateral translations of the mounts at the motor end of the compressor. Moreover, although these components are large, there is a high degree of cancellation between them. One would therefore predict that any noise reduction technique which removed one

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or two of these components would substantially increase the overall target noise level. It may also be noticed that no channel numbers above 16 appear, which means that no microphone signals contribute to the target. This is consistent with the fact that the sound of the compressor in the near field appears to be strongly dominated by higher harmonics. It also implies that placing an acoustic enclosure around this machine would give very little noise reduction. It is costly mistakes of this type that the MPSD system is intended to help us avoid.

#### **CONCLUSIONS**

Multiple Path Signature Decomposition (MPSD) allows the contributions of different noise path excitations to transmitted noise to be estimated. The sum of these contributions may be compared with the observed transmission to provide a check of the accuracy of estimation.

Interpreted with care, these results allow the noise control engineer to decide which paths to modify to reduce noise transmission.

Topexpress has implemented MPSD on a versatile hardware/software system. In this form it has been proven on a laboratory rig and on practical marine and road vehicle noise problems.

#### ACKNOWLEDGEMENT

The MPSD system has been developed with the support of the Procurement Executive of the UK Ministry of Defence.

Software for the MPSD system was largely developed under Dr Chris Dorling.