**Proceedings of the Institute of Acoustics**

# PREDICTION OF NOISE PROPAGATION FROM WIND TURBINE GENERATORS USING RAY THEORY

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## 1. INTRODUCTION

One of the major factors preventing wide scale installation of wind turbine generators in Britain is difficulty in estimating the environmental noise that they produce. This difficulty is compounded by uncertainties in predicting both their radiated sound power and the influence of atmospheric and topographical conditions on the received noise level at a distant observer.

There are various ways of modelling noise propagation including wave methods, finite element modelling and ray theory methods. The first two, whilst potentially providing accurate answers (particularly in the near field) become overwhelmingly intensive of computer time when used to study propagation over large distances. Ray theory, by making simplifying assumptions about the sound field and the length scale of changes in the atmosphere, provides a much more manageable form of solution.

## 2. RAY THEORY METHODS

A sound ray is a line through a sound field which is perpendicular to the wavefronts at all points along its path. It is possible to formulate so-called dispersion relationships which govern the ray paths, and using these one can find the paths of individual rays emanating from a noise source without determining the complete shape of the wavefronts. The principal assumption of ray theory is that the speed of sound varies on a length scale which is large compared with the wavelength of the sound.

The strength of the sound at any point in the field is evidently related to the rate at which rays spread apart with increasing distance from the source. There are two approaches to the problem of finding this rate of spreading. The mathematically and computationally simplest way is to trace large numbers of rays from the source, keeping track of those rays which pass close to the points of interest in the far field, and to calculate the sound intensity either from the spread of neighbouring rays or from the local density of rays. Although simple, this method potentially requires a considerable amount of computer storage when solving three dimensional problems over large distances. An advantage is that it is relatively easy to introduce complexities such as undulating terrain.

The alternative method of finding the local strength of the sound field is to use 'ray tubes', and this is the approach adopted here. The theory is described clearly by Candel [1] (whose notation we follow) and is illustrated in Figure 1. A ray path is characterised by initial azimuthal and polar angles, $\theta$ and $\alpha$. Distance along the path is parameterised by S, which can be thought of as a phase parameter. At any point on the ray the tangent to the ray $\frac{d\underline{x}}{dS}$ is given by:

$$\frac{d\underline{x}}{dS} = N^{-1} \left( \underline{v} + \underline{M} \right)$$

where N is effectively a speed of sound, $\underline{v}$ is the unit wave vector which is normal to the wavefront, and $\underline{M}$ is the local Mach number of the flow.

The changes in direction of $\underline{v}$ along the ray path are in turn governed by the differential equation:

$$\frac{d\underline{p}}{dS} = N^{-1} \left( \nabla N - (\nabla \underline{M}) \cdot \underline{p} \right) \qquad \text{where } \underline{p} = \left( \frac{N}{(1 + \underline{M} \cdot \underline{v})} \right) \underline{v}$$

These simultaneous equations are sufficient to trace a ray by path integration.

The spreading of the ray tube can be found by taking variations of the initial angles, $\theta$ and $\alpha$, so that the local sound intensity, I, is inversely proportional to the area, as shown in Figure 1.

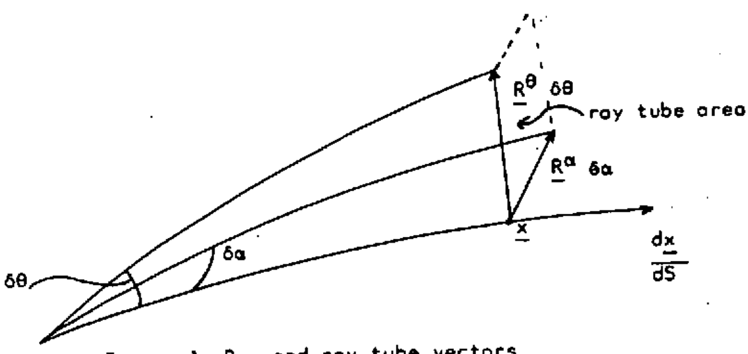


Figure 1. Ray and ray tube vectors

Mathematically at a point $\underline{x}$ on the ray path:

$$I \propto \frac{1}{|\underline{R}^{\theta} \times \underline{R}^{\alpha}|} \qquad \text{where } \underline{R}^{\theta} = \frac{d\underline{x}}{d\theta} \qquad \underline{R}^{\alpha} = \frac{d\underline{x}}{d\alpha}$$

Candel shows how $\underline{R}^{\theta}$ and $\underline{R}^{\alpha}$ can also be determined at any point along the ray path. He includes a description of the vector jump that occurs at ground reflections.

The ray tube method was chosen for the present problem because of the potential for using $\underline{R}^{\theta}$ and $\underline{R}^{\alpha}$ in an iterative scheme to locate the eigenrays which pass directly from the source to a particular observer point, as described below.

## 3. EVALUATING THE SOUND FIELD AT AN OBSERVER

In the absence of wind and temperature gradients, ray paths are straight and, given a ground plane, there are only two paths between any given source and observer. Otherwise, depending on distance, there may be fewer than two rays (as occurs in the upwind noise shadow zone) or more than two rays, as illustrated in Figure 2. It was found that, for this particular application, a convenient way of characterising a ray was by the number of times it passed through the observer height plane (direct ray = 1, first indirect ray = 2, etc).

One advantage of the ray theory method is that it is possible to separate the frequency independent process of finding the ray paths from the frequency dependent calculation of ground and atmospheric attenuation. Thus to evaluate the sound field at a particular point, all the eigenray paths are found and then the energy contribution of each ray is calculated.

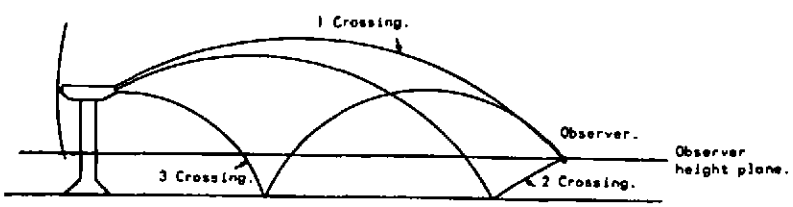


FIGURE 2.

### 3.1 Finding Eigenrays

The method chosen to find eigenrays was an iterative one of altering the initial ray angles, $\theta$ and $\alpha$. For the direct ray, the initial values of $\theta$ and $\alpha$ were taken as being the no wind direct ray angles. This ray was traced (with wind) until it had crossed the observer height plane at a point $x_1$ as shown in Figure 3. The requirement is to find the change in $\theta$, $\alpha$ and phase distance S so that a new ray can be traced which will arrive closer to the required observer position. By definition of $\underline{R}^{\theta}$, $\underline{R}^{\alpha}$ and $\frac{d\underline{x}}{dS}$ we can write to first order):

$$\Delta \underline{x} = \underline{R}^{\theta}\,\delta\theta + \underline{R}^{\alpha}\,\delta\alpha + \frac{d\underline{x}}{dS}\,\delta S$$

where $\delta\alpha$, $\delta\theta$ and $\delta S$ are the requisite changes to $\alpha$, $\theta$ and $S$. It is possible to solve this vector equation for the three unknowns.

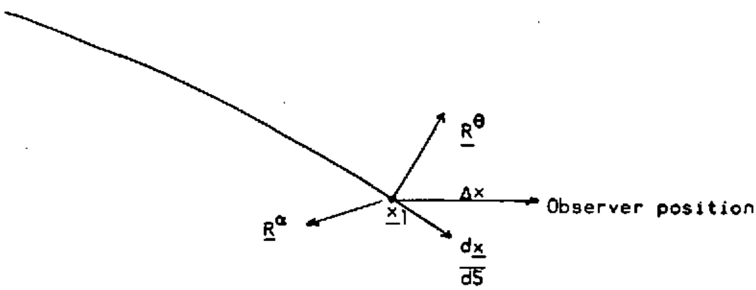


Figure 3. Iteration towards the observer position

Once a required ray has been found, the ray with the next number of height crossings is found by continuing to trace the current ray until another height crossing occurs (perhaps with a reflection along the path), from which point the iterative procedure continues.

A useful aid to visualising some of the phenomena that have to be coped with by the iteration scheme is Figure 4. This shows the result of tracing downwind rays with initial azimuthal angles between -8° and +8° to the horizontal until they reach a horizontal distance of 1600 metres from the source. The graph shows arrival height versus initial angle.

For an observer at a height of 5 metres, eigenrays occur with the following initial angles, number of reflections and number of height crossings:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Initial angle (degrees) | +7.4 | +7.3 | +4.2 | +4.0 | 0 | -1.5 | -6.3 | -6.5 |
| Number of reflections | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 1 |
| Number of height crossings | 1 | 2 | 3 | 4 | 5 | 4 | 3 | 2 |

Eigenrays for a 5 metre Observer, from Figure 4

RAY THEORY

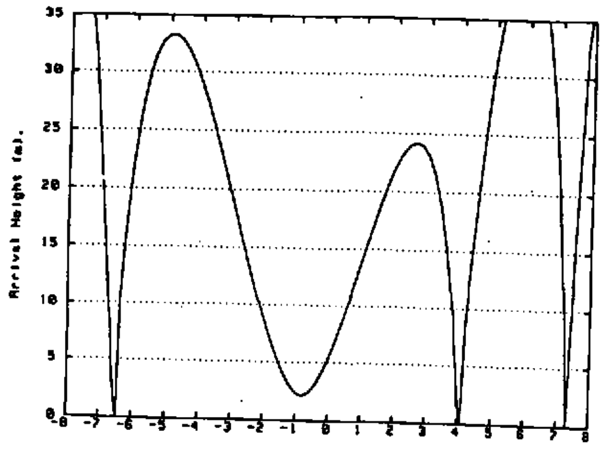


FIGURE 4 Arrival Height of Rays vs Launch Angle 1600m downwind. (Wind profile 10m/s @ 10m b = 0.2).

The gradient of this graph (by definition the rate of change of position X with initial angle θ, subject to a constraint on remaining at a fixed horizontal distance), is obviously closely related to one component of $R^{\theta}$, and the intensity associated with each ray will be inversely proportional to this gradient. It can be seen that the rays with more height crossings have a higher intensity. In particular the sound field is focused to a caustic at this distance along the ray with initial angle -0.9 degrees.

3.2 Summing Rays

When all the rays have been traced, the sound intensity associated with each path is found from a) the initial source strength and directivity, b) the reflection coefficient at any ground reflections and c) the atmospheric absorption for the geometrical length of the ray path. All these factors are likely to be frequency dependent.

By using the phase distance from the ray calculation and retaining phase information in the ground reflection coefficient, it is possible to add rays either coherently or incoherently. The well known ground interference dip is due to coherent addition of direct and indirect rays.

3.3 Atmospheric and Ground Models

In order to control the complexities of the problem the computer implementation of the model was restricted to propagation over flat ground, and the effect of temperature variations was neglected as being of secondary importance to the effect of wind profile.

RAY THEORY

The wind speed, W, was taken to be a horizontal sheared flow with a profile governed by the power law:

$$W(Z) = a\ Z^b$$

where the power, b, typically takes a value of about 0.14 (1/7th power law) and the constant, a, is determined from the wind speed at a reference height. Z is height above ground level.

The ground impedance model was due to Delaney and Bazely (described in [2]) and allows for the effective flow resistivity and porosity of the ground. This model was used in conjunction with the plane wave reflection coefficient as a function of ground impedance.

Atmospheric absorption was calculated according to the model described by Bazely [3].

## 4. PREDICTIONS FROM THE MODEL

The model was used to perform a parametric study, the aim of which was to provide general design rules on noise attenuation with distance. The parameter plotted is the decibel level relative to the sound pressure level at 1m from a notional source.

Figures 5a) and 5b) show the variation of this relative level with distance downwind and upwind, with a wind speed of 10 metres/sec at a height of 10 metres. The source height was 20 metres and results are shown for three observer heights.

Simply by drawing the propagation paths of rays (as in Figure 2), it is evident that sound is channelled downwind. Figure 5a) shows that we predict a stratification of the sound field with sound level increasing with height. The additional rays occur at a caustic of the sound field and, as shown above, low level observers receive these additional rays further downwind. These predictions are for 100 Hz where both ground and atmospheric attenuation will be small, so that multiply-reflected rays will not have been strongly attenuated. Figure 5b) demonstrates that the distance to the upwind noise shadow increases with observer height and also that a caustic occurs at the shadow boundary.

Figures 6a) and 6b) show the effect of varying source height for an observer height of 1.2 metres. Downwind, the onset of additional rays occurs further away the higher the source so that for a large distributed source like a WTG the sources of sound near the ground may carry more strongly. Close to the source in the two ray zone, the highest source produces the highest relative level downwind and the lowest relative level upwind. The distance to the upwind noise shadow increases with source height.

RAY THEORY

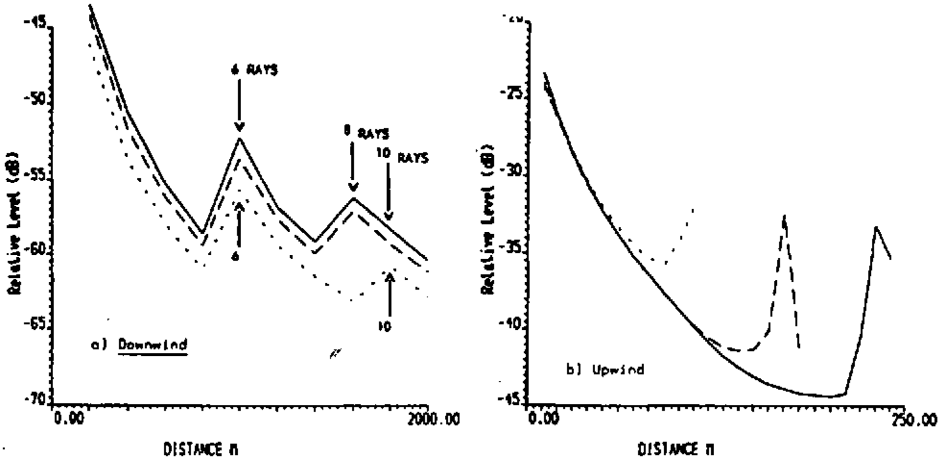


Figure 5 Variation with observer height (100Hz).
7m ————
4m — — —
1.2m - - - - - -

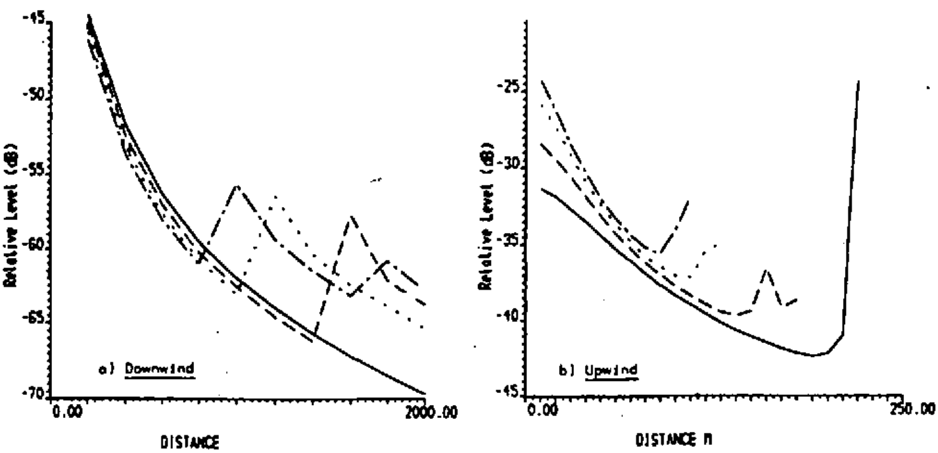


Figure 6. Variation with source height (100Hz).
50m ———— 25m - - - - - -
35m — — — 20m — · — ·

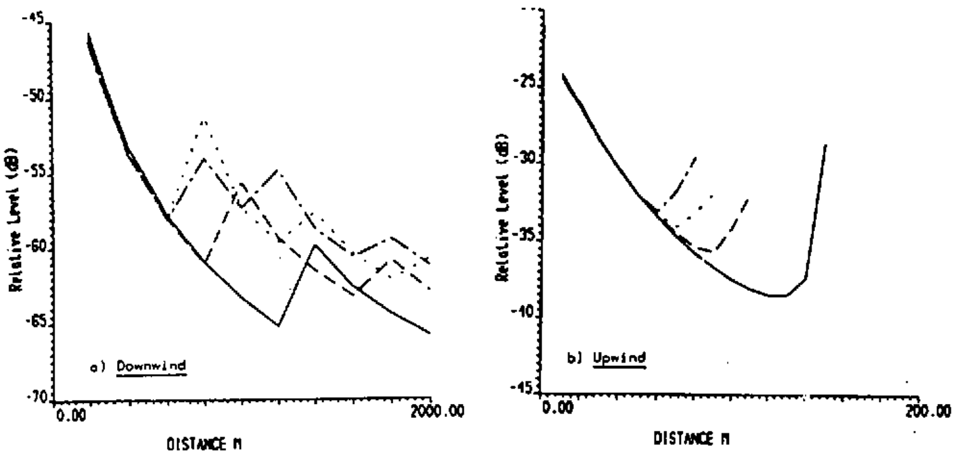


Figure 7 Variation with wind speed (100Hz).

5m/s ———— 15m/s - - - - - - -
10m/s — — — 20m/s — - — -

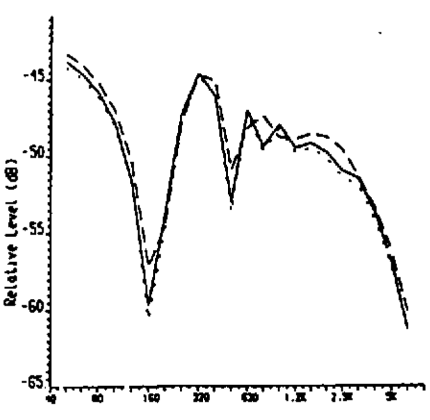


FIGURE 8a - Relative level at 200m downwind.

Windspeed : 5m/s — — —
10m/s ————
20m/s - - - - - -

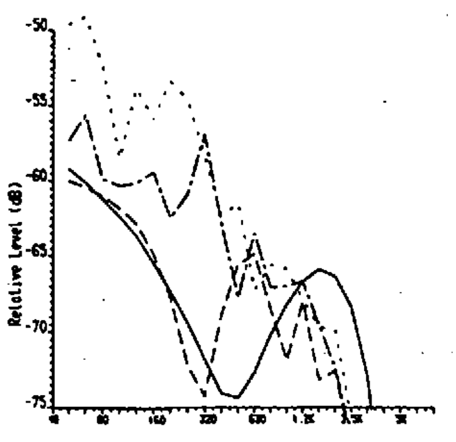


FIGURE 8b - Relative level at 1000m downwind.

No wind ———— 5m/s . — —
10m/s - - - - - - 20m/s — - — -

Figures 7a) and 7b) showing variation with wind speed, indicate as expected that the stronger the wind, the closer to the source the wind effects become apparent.

Figures 8a) and 8b) show the relative level spectrum in 1/3 octave bandwidths at two distances downwind. Coherent addition of rays has been used and thus the ground effect dip is apparent with zero wind speed. The level has been calculated as an average over nine discrete frequencies in each 1/3 octave band, and this has the effect of smoothing out the predicted high frequency coherent effects that are never seen in practice. Within the two ray zone (Figure 8a)), the wind speed has little effect but Figure 8b) demonstrates the dramatic variations predicted in the multiple ray region as wind speed varies. These effects arise principally because the arrival of extra rays disrupts the ground interference effect, rather than because of the effects of caustics, though these will obviously add further variability to the predictions.

## 5. CONCLUSION

A ray theory scheme for predicting sound propagation under windy conditions has been developed. The method uses a novel way of iterating to find the eigenrays between the source and observer, and this has the advantage that it utilises computing power quite efficiently. A disadvantage is that it may be more difficult to extend the method to complex topography.

The predictions show that at large distances (beyond the two ray zone) the noise shows marked dependency on source height, observer height and wind conditions, and that small changes in wind speed can produce large variations in noise level.

Within the two ray zone variations are much less marked, and the no wind predictions with ground and atmospheric attenuation effects provide a reasonable approximation. For a 20 metre source and wind speed of 10 m/s this zone extends out to 800 metres.

## 6. ACKNOWLEDGEMENT

ISVR Consultancy Services was supported in this work by the Department of Energy, Great Britain, through the Energy Technology Support Unit, Harwell Laboratory.

## 7. REFERENCES

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