BRITISH ACOUSTICAL SOCIETY: Meeting on Wednesday, 22nd November 1972
at Loughborough University of Technology, Leics.

REAL TIME ANALYSIS.

Paper No.
72/ 82

DIGITAL ANALYSIS OF NONSTATIONARY DATA

MARTIN H. ACKROYD

LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

Summary

Virtually any physical random signal is nonstationary; that is, its statistical characteristics vary with time. In many applications the nonstationarity can be neglected but in other cases, such as in speech signal analysis, the statistics vary very rapidly and the signal cannot be treated as though it were stationary.

This paper is concerned with the short-time spectral analysis of signals to produce a time-dependent spectrum. Two problems arise in such a spectrum analysis. Firstly, what is the physical meaning of the result of such an analysis? The second problem is how to compute such a short-time spectrum from the physical data with reasonable economy in computer time.

The central concept in short-time spectrum analysis seems to be the time-frequency energy distribution of the signal, defined by $e(t,f) = s(t)S^*(f)\exp(-j2\pi ft)$ where $s(t)$ is the waveform of the signal and $S(f)$ is its Fourier transform. As an illustration of just one of the properties of this function, it is easily verified that its integral over the whole of the time-frequency plane gives the total energy of the signal. The ensemble average of this function gives $R(t,f)$, the time-dependent power spectrum of a random process which can be defined as the Fourier transform (with respect to τ) of the time-dependent autocorrelation function

$r(t,\tau) = \overline{s(t)\, s(t+\tau)}$.

Short-time spectra can be computed by two methods: by computing sections in time and by computing sections in frequency. The sections-in-time method uses a bank of bandpass filters, square-low rectifiers and lowpass filters, as shown in Fig.1. The output of each lowpass filter defines a section of the short-time spectrum in the time direction.

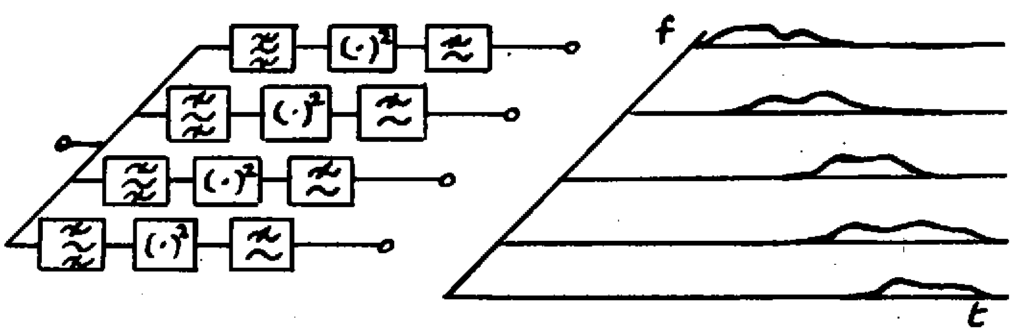

Fig.1. Short-time spectrum analyser producing sections in time.

With the sections-in-frequency method, the waveform to be analysed is multiplied by a window function centred at time t_1. The squared magnitude of the Fourier transform of this product waveform is computed and is then smoothed to some degree. This yields a section in the frequency direction centred at time t_1. The procedure is repeated with a succession of values of t_1 to yield a succession of sections in frequency.

It can be shown that a short-time spectrum produced by either the sections-in-time or the sections-in-frequency method can be regarded as being a version of $e(t,f)$ smoothed out in the time-frequency plane by a double convolution. The weighting function used in this convolution depends, in the sections-in-time case, on the bandpass and lowpass filter characteristics. The smoothed-out version of $e(t,f)$ can be regarded as providing an estimate of the time-dependent spectrum $R(t,f)$.

The sections-in-frequency method is implemented straightforwardly by the use of the fast Fourier transform (FFT). The sections-in-time method, which requires a set of bandpass filters, could also be implemented using the FFT, for it is well known that the FFT can be used to effect filtering. However, it seems that if the FFT is to be used, it is best applied to the sections-in-frequency method. The sections-in-time method can be implemented very economically by the use of the frequency sampling method of programming digital filters. In many applications this mode of implementation is even more economical than the use of the FFT to compute sections in frequency.