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ON THE SENSITIVITY OF END-SHIELDED AND END-CAPPED THIN, HOLLOW CYLINDRICAL HYDROPHONES

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1. INTRODUCTION

The derivation of the receiving sensitivity of radially, longitudinally and tangentially polarized hollow cylindrical piezoelectric hydrophones was carried out by Langevin (1) for signal frequencies well below the lowest resonance frequency.

The main purpose of this paper is to develop expressions for the open circuit receiving sensitivities of air filled, thin, end-shielded and end-capped radially polarized cylindrical hydrophones near the mechanical resonance frequency which is of particular interest in determining the sensitivity of echo-ranging systems.

2. THE EQUIVALENT CIRCUIT FOR A RADIALLY POLARIZED, HOLLOW, CYLINDRICAL TRANSDUCER

It has recently been shown (2) that a radially polarized, hollow, thin cylindrical transducer, having a length L, a thickness t and inner and outer radii a and b, can be represented by the equivalent circuit shown in Figure (1a). The capacitance C due to the static compliance and the clamped capacitance C_o of a cylindrical transducer are given by:

$$C_o = \frac{2bL\, e_{33}^T}{t}\left[1 - \frac{2d_{31}{}^2 Y}{(1-\sigma)e_{33}^T}\right] \quad .... (1)$$

$$C = 4\pi bL\, d_{31}{}^2 Y/t(1-\sigma) \quad .... (2)$$

where e_{33}^T is the permittivity component at constant stress; d_{31} is the piezoelectric strain constant; Y is the Young's Modulus at constant electric field defined by $1/s_{11}^E$; σ is the Poisson's ratio at constant electric field defined by $-s_{12}^E/s_{11}^E$ and s_{12}^E, s_{11}^E are the elastic compliance constants at constant electric field.

The resistances $R_{r1}/4N_1{}^2$ is due to the acoustic radiation resistance R_{r1} from the surface of the cylindrical transducer.

The electromechanical transformation factor N_1 which is defined as the ratio of the output mechanical force to the input voltage under clamped conditions, can be shown (2) to be given by:

$$N_1 = \frac{2\pi LYd_{31}}{1-\sigma} \quad .... (3)$$

It is more convenient to transform the equivalent circuit shown in Figure (1a) to that shown in Figure (1b). It can be shown that the equivalent series resistance R_{e1} is given by:

$$R_{e1} = R_{r1}/(4N_1^2 Z_m) \qquad \ldots. (4)$$

where: $$Z_m = \left[1+(1-x^2)(1/_{kd}{}^2-1)\right]^2 + x^2\omega_{rr}{}^2C_o{}^2R_{L1}{}^2 \qquad \ldots. (5)$$

$$x = f/f_{rr}$$

and f_{rr} is the radial resonance frequency of the cylinder and kd is the dynamic electromechanical coupling coefficient.

Before making use of the equivalent circuit for the cylindrical transducer, its limitations will be discussed. Love (4) and more recently Haskins and Walsh (3) developed a general equation for the resonance frequency of a cylindrical transducer. For the fundamental resonance it is given by:

$$(f/f_r)^4(1-\sigma^2)-(f/f_r)^2(1+\pi\bar{a}^2/L^2)+\pi^2\bar{a}^2/L^2 = 0 \qquad \ldots. (6)$$

where f_r is the ring radial resonance frequency defined by $c/2\pi\bar{a}$, $\bar{a}$ is the mean radius of the cylinder, c is the bar velocity defined by $(Y/\rho)^{\frac{1}{2}}$ and ρ is the density of the cylinder material.

The dependence of the radial and the longitudinal resonance frequencies on the ratio $L/\bar{a}$ is shown in Figure (2) for a piezoelectric material with Poisson's ratio $\sigma = 0.35$. It can be seen from Figure (2) that as the ratio $L/\bar{a}$ moves away from the region defined by $3.5 > L/\bar{a} > 2.5$ there is decreasing coupling between the radial and the longitudinal modes. Outside this region the radial resonance frequency f_{rr} is given approximately by:

$$f_{rr} \simeq c/2\pi\bar{a} \qquad \text{for } L/\bar{a} \text{ less than 2.5 and} \quad \ldots. (7)$$

$$f_{rr} \simeq c/2\pi\bar{a}\,(1-\sigma^2)^{\frac{1}{2}} \qquad \text{for } L/\bar{a} \text{ greater than 3.5} \quad \ldots. (8)$$

Whereas the longitudinal resonance frequency f_L is given approximately by:

$$f_L \simeq c/2L \qquad \text{for } L/\bar{a} \text{ either less than 2.5 or greater than 3.5} \quad \ldots. (9)$$

Within the region $3.5 > L/\bar{a} > 2.5$ the coupling between the radial and the length modes is strong especially when $L/\bar{a}$ is approximately equal to 3 and hence the equivalent circuit shown in Figure (1) is no longer valid since the circuit parameters will be frequency dependent. On the other hand, the cylindrical transducer can be represented by the equivalent circuit outside the region $3.5 > L/\bar{a} > 2.5$.

3. RECEIVING SENSITIVITY OF A THIN-WALLED, SHIELDED, CYLINDRICAL TRANSDUCER

The receiving sensitivity of a transducer M is defined as the ratio of the open-circuit voltage at the output terminals of the transducer to the free field acoustic pressure at any point near the transducer. Assuming that the transducer obeys the reciprocity principle, the receiving sensitivity M can be shown (2) to be given by:

$$M = \lambda(DR_e/\pi\rho_o c_o)^{\frac{1}{2}} \qquad \ldots. (10)$$

where λ is the acoustic wavelength in water, ρ_o is the density of the water, c_o is the velocity of sound in water, R_e is the real part of the transducer input impedance and D is the directivity factor.

For a cylindrical transducer having a length L greater than λ, the radiation resistance of a pulsating infinite cylinder will

EQUIVALENT CIRCUITS OF A PIEZOELECTRIC CYLINDRICAL TRANSDUCER

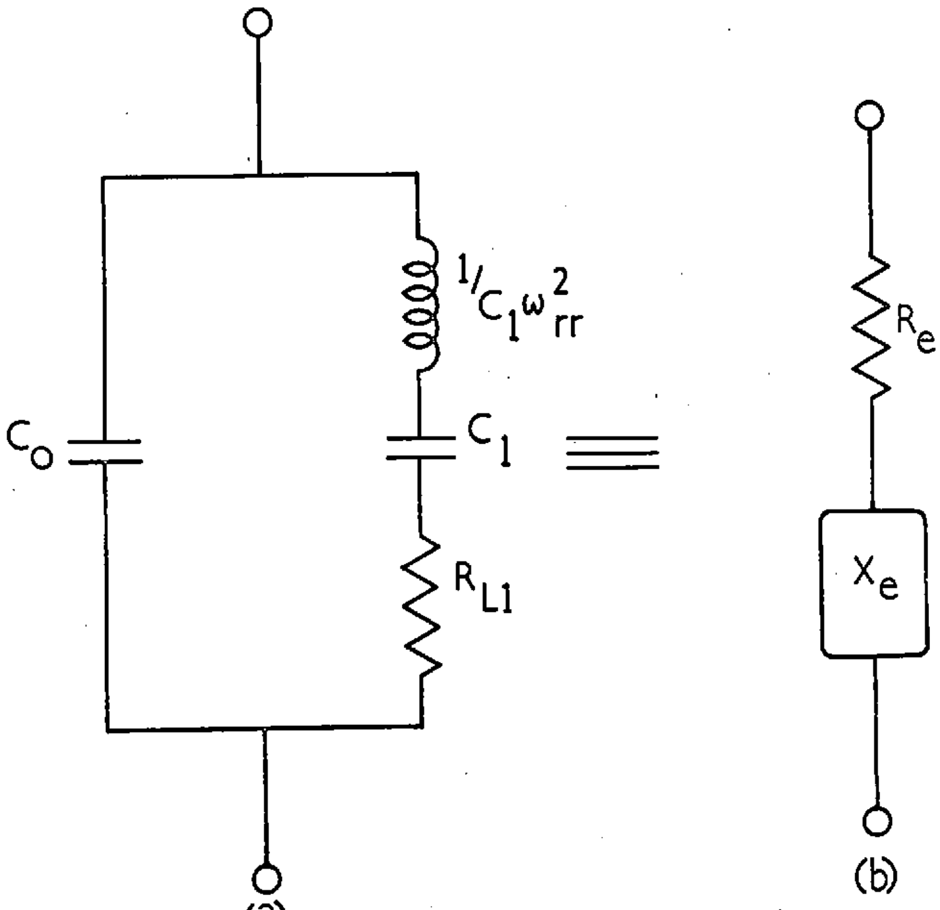

Fig. (1)

RESONANCE FREQUENCIES OF THE FUNDAMENTAL LENGTH
AND RADIAL MODES OF THIN-WALLED CYLINDER WITH $\sigma = 0.35$

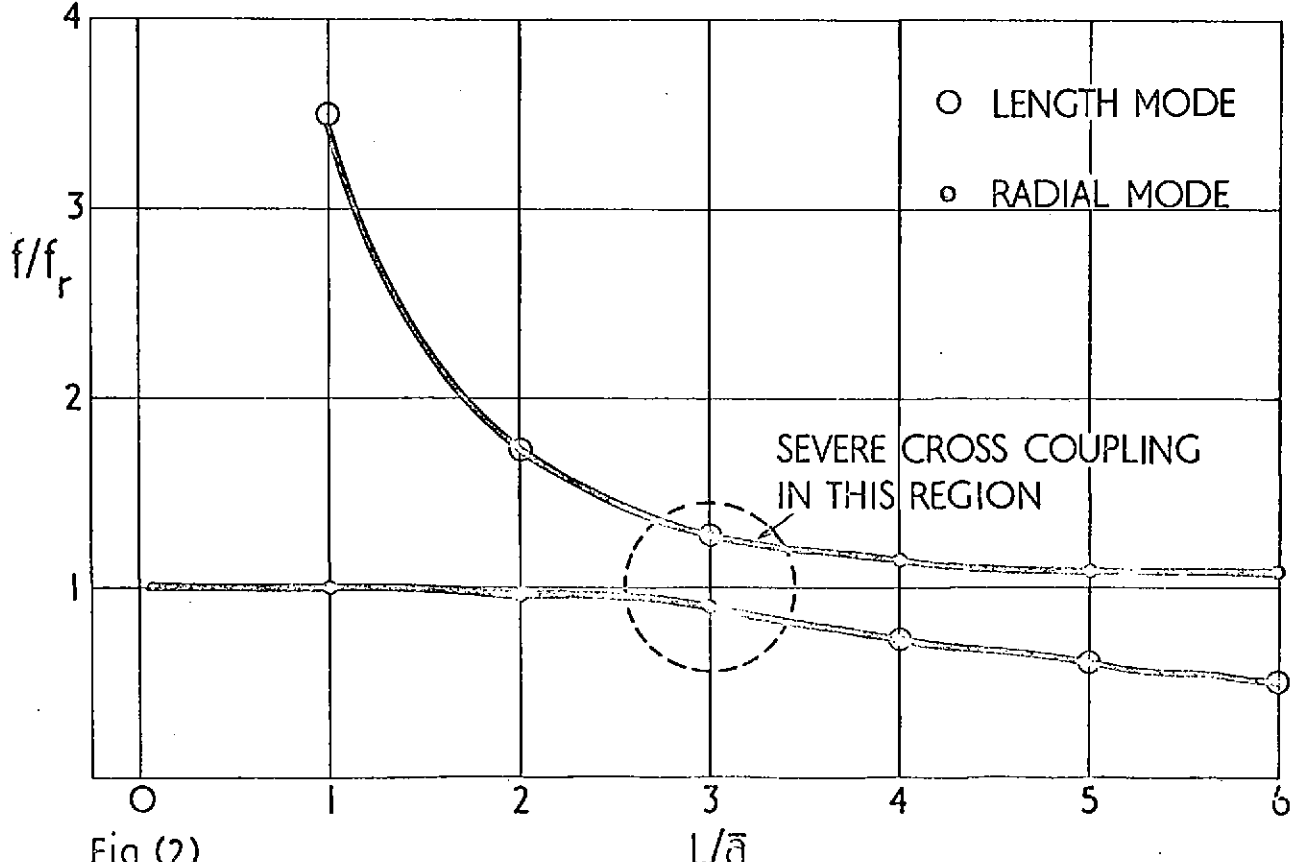

Fig. (2)

be used in determining R_{r1}.

The directivity factor D for a long cylindrical transducer is given (5) by:

$$D_1 = 2L/\lambda \qquad \ldots. \ (11)$$

The receiving sensitivity M_L of a thin long cylindrical transducer can be obtained from equation (10) by substituting for R_e and D_1 from equations (4) and (11), that is:

$$M_L = \frac{\lambda}{2N_1}\left[\frac{2LR_{r1}}{\pi\lambda\rho_o c_o Z_m}\right]^{\frac{1}{2}} \qquad \ldots. \ (12).$$

For a thin cylindrical transducer which is shorter than a wavelength, the directivity factor D_1 equals unity. Substituting for D_1 into equation (10), the receiving sensitivity M_s of such a transducer is given by:

$$M_s = \frac{\lambda}{2N_1}\left[\frac{R_{r1}}{\pi\rho_o c_o Z_m}\right]^{\frac{1}{2}} \qquad \ldots. \ (13)$$

An important difference between equation (12) and (13) is that the radiation resistance R_{r1} in equation (13) may be considered to be the same as that of a spherical source which has the same surface area as the short cylindrical transducer, whereas that in equation (12) is the radiation resistance of a long pulsating cylinder.

4. RECEIVING SENSITIVITY OF END-CAPPED CYLINDRICAL TRANSDUCER

So far we have developed an expression for the receiving sensitivity of a cylindrical transducer with its ends being shielded from the incident acoustic pressure, that is, the acoustic pressure on the ends of the transducer is zero. Two other cases are possible. In the first, the transducer ends are exposed to the incident acoustic pressure, whereas in the second case the transducer ends are closed by rigid caps so that the area $\pi(b^2-a^2)$ is subjected to pressure $b^2/(b^2-a^2)$ times the incident acoustic pressure. This last case is widely used since it gives a higher receiving sensitivity than the end-shielded case.

The receiving sensitivity M, for the end-capped cylindrical transducer is given by:

$$M = M_1+M_2 \qquad \ldots. \ (14)$$

where M_1 is the open circuit receiving sensitivity when the transducer ends are shielded, and M_2 is the open circuit receiving sensitivity due to the capped ends of the transducer. The open circuit receiving sensitivities M_1 and M_2 are given by equation (10) with the appropriate directivity factor and radiation resistance. Substituting for M_1 and M_2 from equation (10) into equation (14) gives:

$$M = \lambda(D_1 Re_1/\pi\rho_o c_o)^{\frac{1}{2}} + \lambda(D_2 Re_2/\pi\rho_o c_o)^{\frac{1}{2}} \qquad \ldots. \ (15)$$

where R_{e2} is the equivalent series resistance due to the acoustic radiation from the ends of the cylindrical transducer and D_2 is the corresponding directivity factor.

5. COMPARISON OF CALCULATED AND MEASURED SENSITIVITIES

Recently two end-capped, lead zirconate-titanate, cylindrical hydrophones H_1 and H_2 were designed for measuring ambient noise levels. A comparison is shown in Figures (3) and (4) between the measured open circuit free field sensitivities of the two hydrophones with those calculated. It should be noted that in calculating the theoretical sensitivities shown in Figures (3) and (4) losses have been neglected. It can be seen from Figure (3) that the measured free field receiving sensitivity of hydrophone H_2 is in good agreement with that calculated for the end-capped condition.

In the case of hydrophone H_1 it may be seen that the measurements are more in agreement with the end-shielded condition than with the capped condition. In fact inspection showed that the end-caps had become loose, though whether by sufficient amount to shield the end is uncertain.

Also for H_1, the ratio of its length to its mean radius is 2.7. It can be deduced from Figure (2) that for this ratio strong coupling between the radial and the length modes exists. It follows therefore that the parameters of the equivalent circuit representing hydrophone H_1 will be frequency dependent and hence the calculated receiving sensitivity based on the equivalent circuit will be less accurate.

6. CONCLUSION

One important conclusion which emerges from the analysis given above is that the use of the simple equivalent circuit shown in Figure (1) is quite adequate in obtaining the open circuit receiving sensitivity of the end-shielded and end-capped thin, hollow, cylindrical transducer provided that the ratio $L/\bar{a}$ of its length to the mean radius falls outside the region defined by $3.5 > L/\bar{a} > 2.5$.

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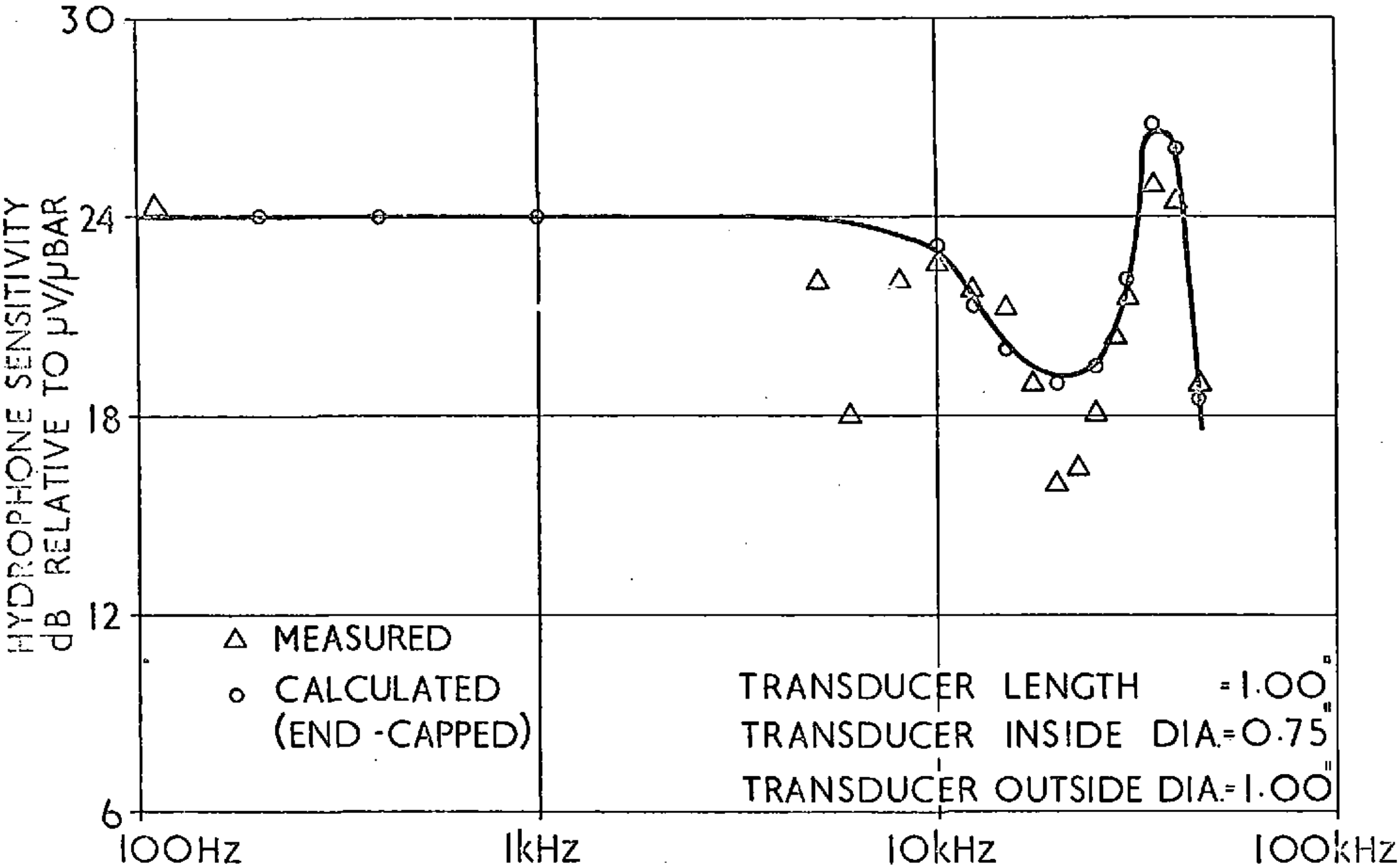
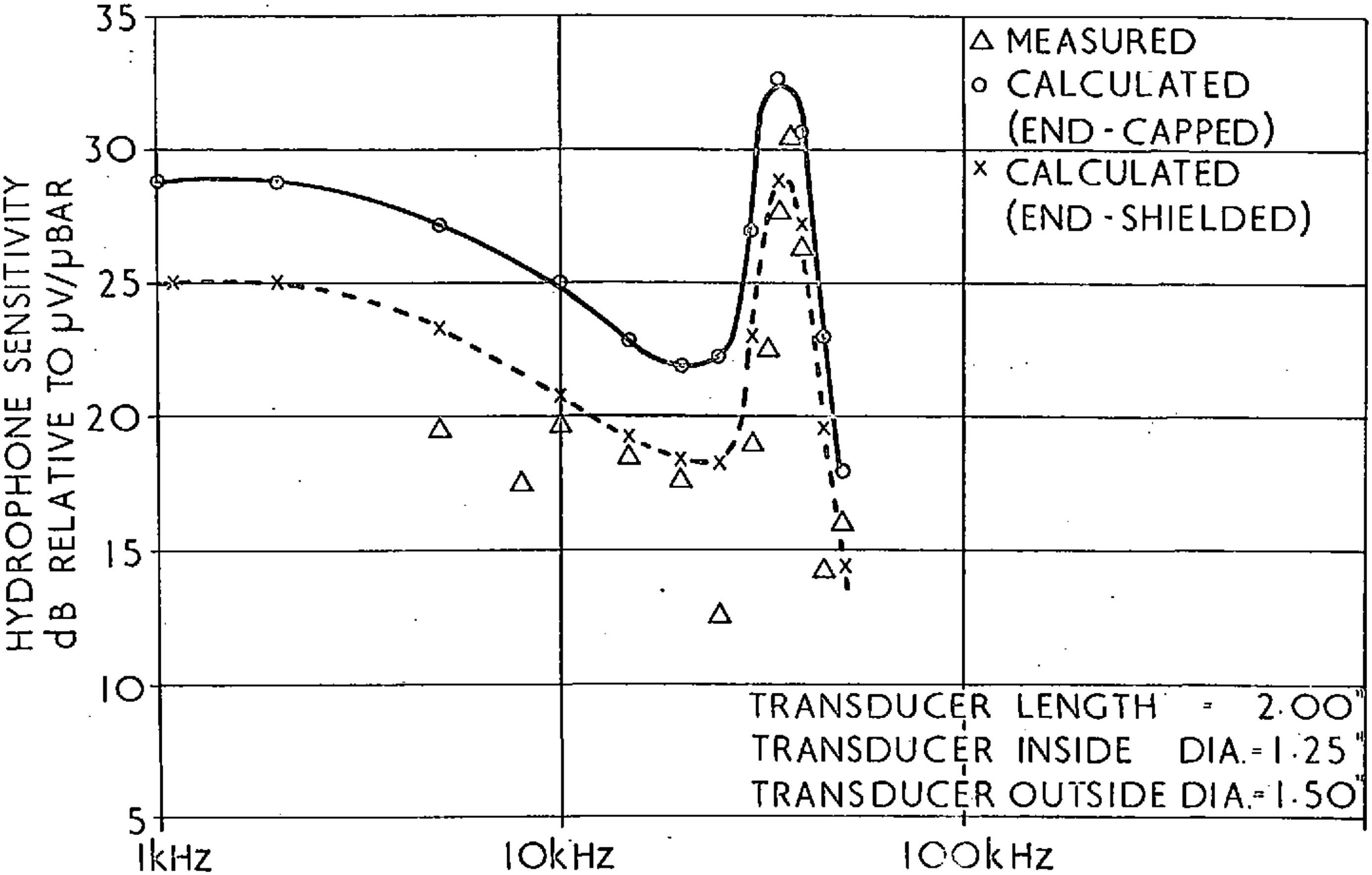

HYDROPHONE SENSITIVITY
dB RELATIVE TO μV/μBAR
30
24
18
12
6
100Hz
1kHz
10kHz
100kHz
FREQUENCY
△ MEASURED
○ CALCULATED
(END -CAPPED)
TRANSDUCER LENGTH =1.00"
TRANSDUCER INSIDE DIA.=0.75"
TRANSDUCER OUTSIDE DIA.=1.00"
Fig (3)



HYDROPHONE SENSITIVITY dB RELATIVE TO μV/μBAR
35
30
25
20
15
10
5
1kHz
10kHz
100kHz
FREQUENCY
△ MEASURED
○ CALCULATED (END - CAPPED)
× CALCULATED (END - SHIELDED)
TRANSDUCER LENGTH = 2.00"
TRANSDUCER INSIDE DIA. = 1.25"
TRANSDUCER OUTSIDE DIA. = 1.50"

Fig (4)