Array Gain of a Vertical Line Array in Shallow Water.

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ABSTRACT

The signal/noise performance of a vertical line array showing a horizontally directed main lobe in its angular response, and operating in a shallow channel of uniform depth, is discussed. The array gain is expressed as the product of the signal gain and the noise gain. Provided only a few modes contribute to the signal field the signal gain is on the order of N, where N is the number of hydrophones in the array. The noise gain depends on the anisotropy of the ambient noise field and in many cases can be expressed as a rational function of frequency with known coefficients.

SUMMARY OF THE DISCUSSION

A large proportion of U.K. coastal waters lying over the Continental Shelf are on the order of 100m deep. The question addressed in this paper is the following: in a uniform channel of such a depth, and assuming a constant sound velocity-depth profile in the water column, what is the array gain of a vertical line array showing a horizontally directed main lobe in its angular response pattern? In the discussion the signal and noise fields are treated independently, and the array gain is expressed as the product of the signal gain and the noise gain.

At the detector the signal field generated by a point source whose range is large compared with the water column depth, can be expressed as a sum of normal modes. Pekeris [1] was the first to obtain such a solution for iso-velocity water, and a full discussion of the problem in terms of the reflection coefficients of the channel boundaries may be found in reference [2]. For frequencies below 500 Hz, which is the range of interest here, the surface can be safely approximated as a plane, pressure-release boundary. In contrast, the bottom interface is an attenuating boundary whose effect is to introduce a factor

$$\exp -(m/\bar{m})^2$$

into the mode sum, where m is the mode number, $\bar{m} = 3\cdot9(fh/c)\sqrt{h/r}$, f is the frequency in Hertz, h is the water depth, c is the speed of sound in the channel and r is the range between the source and the detector. Evidently those modes for which $m < \bar{m}$ experience relatively little attenuation whereas when $m > \bar{m}$ the modes are severely attenuated. This is the phenomenon of mode stripping and it plays a significant part in determining the signal gain of the array.

The intensity of the acoustic field varies with depth in the water column, and it also depends on the depth of the source. In defining the signal gain it is therefore necessary to take the averaged response of an omnidirectional hydrophone with respect to both the source depth and the depth of the sensor; and the response of the vertical line array must also be averaged over the source depth. It can then be shown that the signal gain takes a value on the order M, where M is the number of hydrophones in the array, provided the aperture of the array is less than the distance between adjacent maxima in the $\bar{m}^{th}$-order mode. This criterion corresponds to the condition that the grazing angle of the $\bar{m}^{th}$-order equivalent ray should be less than half the angular width of the horizontal lobe at the -3dB points. Since $\bar{m}$ varies as $r^{-\frac{1}{2}}$, indicating that mode stripping increases with range, a high signal gain is more likely to be observed at long rather than short ranges.

The noise gain of the array has to be treated rather differently from the signal gain. Since the major sources of noise are randomly distributed over the surface it is appropriate to represent the noise field by a linear superposition of uncorrelated plane waves propagating in all directions [3]. As the vertical line array is axially symmetrical the azimuthal variation of the noise field is of no consequence; the noise gain is determined entirely by the integrated (over azimuth) vertical directionality of the noise. And this vertical directionality may be expressed in terms of a normalized directional density function.

The simplest directional density function to consider is independent of angle, corresponding to isotropic noise. For this case, when the frequency is such that the inter-element spacing in the array is a half wavelength, the complex correlation coefficient between noise fluctuations at different hydrophones in the array is zero, giving a noise gain of unity. The array gain is then equal to the signal gain. More generally, the noise gain can be expressed as a function of frequency in the form

$$G_n(\bar{\omega}) \simeq \bar{\omega}/\sigma\pi \qquad \text{for} \quad (\sigma-1)\pi \le \bar{\omega} < (\sigma+1) \quad \cdots (1)$$

where σ takes the values of the odd positive integers and

$$\bar{\omega} = \omega l / c \qquad \text{- - -} \qquad (2)$$

is the normalized angular frequency. In equation (2) l is the inter-element spacing and ω is the angular frequency. The approximate result in equation (1) is derived by taking the limit of the noise gain function as N goes to infinity (the high-N approximation) and replacing the resultant Fourier expansion by its equivalent polynomial form (which in this case is the first Bernoulli polynomial). Equation (1) is plotted in Fig.1 as a function of $\bar{\omega}$. The discontinuities in the figure appear as a result of the high-N approximation and may be interpreted as follows: the high-N approximation implies a notional array with an infinite aperture and a beam pattern showing very narrow main lobes. As the frequency increases, each time that $\bar{\omega}$ takes a value which is a multiple of 2π two new main lobes, one in the upward and one in the downward looking vertical directions, appear in the beam pattern of the array. When this occurs the noise seen at the output of the array increases abruptly, due to the narrowness of the beams, and a corresponding decrease occurs in the noise gain. A comparison of the approximate noise gain function in (1) with the exact expression for a five element array is shown in Fig.2. Notice that the approximation breaks down only in the vicinity of a discontinuity; elsewhere there is good agreement, indicating that in determining the noise gain over the frequency range of practical importance, the high-N approximation introduces virtually no error.

For the general case of a noise field with arbitrarily chosen anisotropy, the first step in determining the noise gain of the array is to expand the directional density function in a series of zonal harmonics. The coefficients in the series are related to the directional density function of the noise through an expression which is derived using the orthogonality of the Legendre polynomials. Again using the high-N approximation, and after some algebraic manipulation, the noise gain of the array is found to be

$$G_n(\bar{\omega}) \simeq \left[1 + \sum_{k=1}^{\infty} \frac{d_k(\bar{\omega})}{\bar{\omega}^k} B_k(\bar{\omega}/2\pi) \right]^{-1} \qquad \text{. . .} \quad (3)$$

where the coefficients $d_k(\bar{\omega})$ are related to the directional density function of the noise through the coefficients in the series of zonal harmonics, and $B_k(\bar{\omega}/2\pi)$ is the k^{th}-order Bernoulli polynomial. On substituting the actual polynomials for the terms $B_k(\bar{\omega}/2\pi)$ in (3), and provided the coefficients $d_k(\bar{\omega})$ are rational functions of frequency, the expression for $G_n(\bar{\omega})$ also reduces to a rational function of frequency with known coefficients. The expression for the noise gain in equation (3) is plotted in Fig.3 for a frequency-independent noise field showing a minimum in the horizontal direction. Note that at the practically important value of $\bar{\omega} = \pi$ (corresponding to a half-wavelength inter-element spacing) the noise gain is a factor of five higher than for the case of isotropic noise.

In practice the anisotropy of most noise fields shows some dependence on frequency. Such behaviour can be incorporated into the general expression for the noise gain given above by representing the actual directional density function of the noise by a frequency-dependent convex combination of frequency-independent noise fields. Thus for a large class of realistic noise fields the formulation in equation (3) provides a relatively straightforward means of deriving the noise gain of the array as a function of frequency.

REFERENCES

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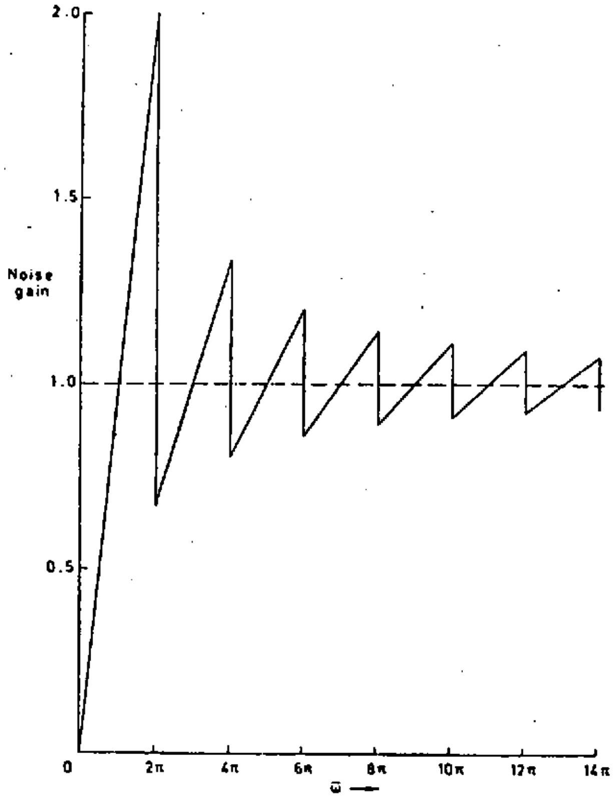

Fig. 1

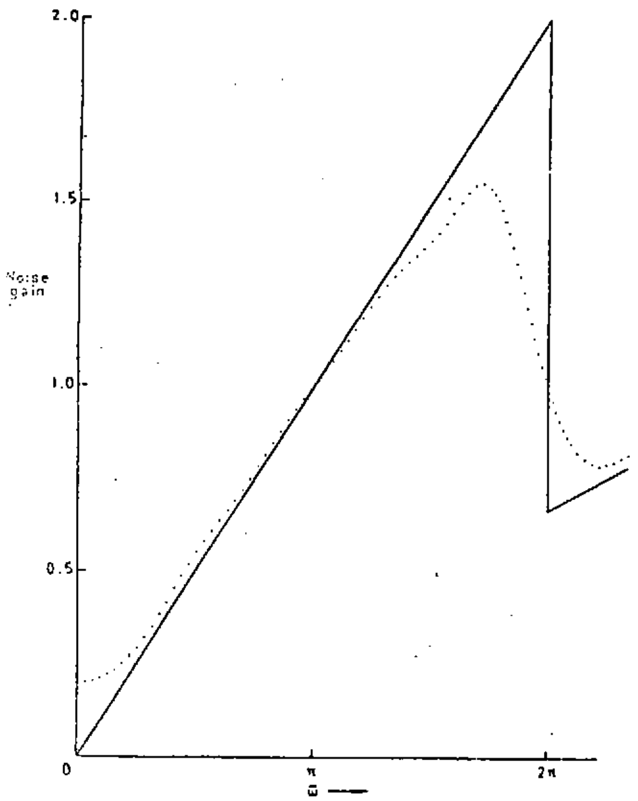

Fig. 2

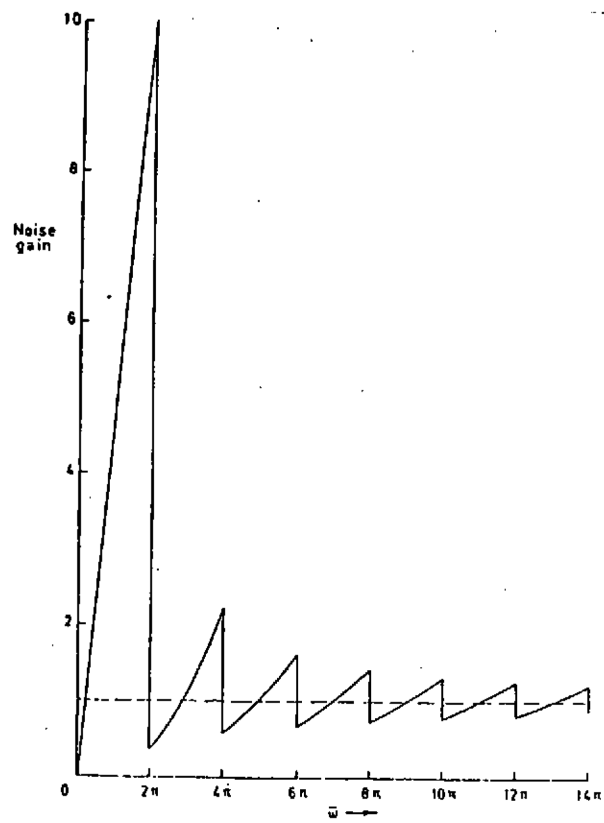

Fig. 3