"RANDOM ERRORS IN NEARFIELD MEASUREMENTS AND THEIR EFFECT ON THE PREDICTED FARFIELD"

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INTRODUCTION

The technique of calculating the farfield of a sonar array from nearfield pressure measurements made on a closed surface surrounding the array has been successfully demonstrated at AUWE for both plane and cylindrical measurement surfaces (ref 1, 2).

This paper describes a method of estimating the errors in the predicted farfield of a sonar array in terms of the random errors in the nearfield pressure amplitude and phase. The analysis is formulated in terms of the variances of an ensemble of similar nearfield amplitude and phase measurements and the variance of the resulting ensemble of predicted farfield pressure patterns. This enables the accuracy of a particular predicted farfield pattern to be estimated in probabalistic terms from knowledge of the variances of the nearfield pressure amplitude and phase measurements.

The related problem of estimating the errors in the farfield of radar antennae has been investigated by several authors (see for example ref 3). However, in this paper the problem has been formulated directly in terms of the variance of the farfield and nearfield pressures and the appropriate exact Green's functions for the particular nearfield measurement surface geometry. This enables the farfield errors to be calculated for nearfield measurements made on any surface where the appropriate exact Green's function can be evaluated. The theory developed applies to both transmitting and receiving arrays although for brevity only the former is described.

THEORY

The farfield of a sonar array can be calculated from knowledge of the nearfield pressure on a known surface enclosing the array by (see for example ref 1, 2),

$$T(\underline{X}_1) = \iint_S T(\underline{X}_o) \frac{\partial G(\underline{X}_1 - \underline{X}_o)}{\partial n} \, dS \qquad (1)$$

where $T(\underline{X})$ is the complex pressure (amplitude and phase) at a point $\underline{X}$, $\underline{X}_1$ is a point exterior to the surface of integration S, $\underline{X}_o$ is a point on the surface S and $\frac{\partial G(\underline{X}_1 - \underline{X}_o)}{\partial n}$ is the normal derivative of the exact Green's function for the particular geometry assuming Dirichlet boundary conditions on the surface S with an outward normal n. Thus by measuring the nearfield pressure on the surface S enclosing the array under test the farfield of that array may be calculated using relation (1). In practice there will be errors in the measurement of the nearfield pressure $T(\underline{X}_o)$ and it is the effect of these errors that is of interest here. It is assumed that the distribution of errors in the nearfield amplitude and phase are Gaussian with known variances and zero mean.

In practice the nearfield measurements are made at discrete points and so the integration is replaced by a summation which, using the mid point formula, gives the farfield pressure as,

$$T(\underline{X}_1) = \sum_m \sum_n A_{mn} \, T(\underline{X}_o^{mn}) \, \frac{\partial G(\underline{X}_1 - \underline{X}_o^{mn})}{\partial n} \qquad (2)$$

where A_{mn} is the area associated with each measurement.

For a function of several variables $F = F(x, y, z \ldots)$ we may write the variance of the function in terms of the variance of the arguments as (ref 4),

$$\sigma_F^2 = \left(\frac{\partial F}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial F}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial F}{\partial z}\right)^2 \sigma_z^2 + \ldots \qquad (3)$$

assuming that there is no correlation between the arguments or higher order terms ie,

$$\frac{\partial^2 F}{\partial x \partial y} = 0, \quad \frac{\partial^2 F}{\partial x^2} = 0 \text{ etc}$$

Hence this may now be applied to relation (2) to give,

$$\sigma^2_{|T(\underline{X}_1)|} = \sum_m \sum_n A_{mn}^2 \left| \frac{\partial G(\underline{X}_1 - \underline{X}_o^{mn})}{\partial n} \right|^2 \left\{ \sigma^2_{|T(\underline{X}_o^{mn})|} + |T(\underline{X}_o^{mn})|^2 \, \sigma^2_{\theta(\underline{X}_o^{mn})} \right\} \qquad (4)$$

This is a quite general result for the variance of the ensemble of farfield pressure amplitude, or root mean square power patterns in terms of the variances of the nearfield pressure amplitude and phase and the exact Green's function for the particular surface of integration.

The particular statistics of the resulting farfield amplitude will depend on the relative magnitude of its deterministic and random components. In cases where the deterministic component is large compared with the random component ie

$$|T(\underline{X}_1)|^2 \gg \sigma^2_{|T(\underline{X}_1)|}$$

the farfield pressure amplitude will obey Gaussian statistics. Where the random component dominates ie,

$$\sigma^2_{|T(\underline{X}_1)|} \gg |T(\underline{X}_1)|^2$$

the farfield pressure obeys Rayleigh statistics. This enables the probability of a given error in the farfield prediction to be calculated (see Ref 5).

SOURCE OF RANDOM ERRORS

In a nearfield measurement system the measurement transducers are used to measure the nearfield pressure amplitude and phase generated by the sonar array under test, at set positions $\underline{X}_o^{mn}$ around the array. These measured values $T(\underline{X}_o^{mn})$ are then used in the evaluation of the diffraction integral relation (1) for the particular measurement surface geometry, to give the farfield pressure, $T(\underline{X}_1)$.

The two main sources of random errors in the system are those due to the voltmeter and phasemeter used to measure the output of the measurement transducers and random errors in the positioning of the nearfield measurement transducers. The meter errors can be deduced from standard technical literature. The positional errors will depend on the particular relation between the nearfield and the measurement surface geometry.

As an example of estimating the effect of the positional errors on the nearfield measurements, consider a sonar array generating a plane wave field in a given direction (ϕ = o) with measurements made around the array on a cylindrical surface. The nearfield phase is given approximately by,

$$\theta(\phi) = \frac{2\pi a}{\lambda} \cos\phi \quad \text{where a is the radius}$$

of the measuring surface and $\theta(\phi)$ is the phase at the point (a, ϕ) in polar co-ordinates. Now this may be rewritten in terms of the variance of the circumferential and radial positional errors, σ_x^2 and σ_a^2 respectively, using relation (3), as

$$\sigma_\theta^2 = \left(\frac{2\pi \sin\phi}{\lambda}\right)^2 \sigma_x^2 + \left(\frac{2\pi \cos\phi}{\lambda}\right)^2 \sigma_a^2$$

The amplitude errors will depend on the particular nearfield distribution for which it is difficult to give a general form, but normally the amplitude errors will be less significant than the phase errors.

DISCUSSION

It can be seen from relation (4) that the magnitude of the farfield error due to the nearfield amplitude errors is independent of the deterministic nearfield amplitude or phase. Thus to minimise the errors, nearfield measurements should only be made in regions where the deterministic amplitude makes a significant contribution to the farfield. The nearfield phase errors, however, contribute to the farfield amplitude in proportion to the corresponding deterministic nearfield amplitude. Thus to minimise the errors the nearfield phase accuracy should be greatest where the deterministic amplitude is greatest. It should be noted that the farfield errors due to the nearfield phase errors do not in general have the same angular distribution as those due to nearfield amplitude errors.

The relative importance of the nearfield amplitude and phase errors can be clearly seen by rewriting relation (4) as,

$$\sigma^2_{|T(\underline{X}_1)|} = \sum_m \sum_n A^2_{mn} \left| \frac{\partial G(\underline{X}_1 - \underline{X}_o^{mn})}{\partial n} \right|^2 \left| T(\underline{X}_o^{mn}) \right|^2 \left\{ \frac{\sigma^2_{|T(\underline{X}_o^{mn})|}}{|T(\underline{X}_o^{mn})|^2} + \sigma^2_{\theta(\underline{X}_o^{mn})} \right\} \qquad (5)$$

Thus the variance of the nearfield amplitude normalised by the square of the deterministic value will contribute with the same magnitude as the variance of the nearfield phase (radians).

In addition the accuracy of the predicted farfield amplitude will improve in proportion to the square root of the number of independent measurements. Consider the ratio of the farfield variance to the square of the deterministic farfield pressure amplitude from relations (2) and (5). This has the form of the ratio of the "sum of the squares" to the "square of the sum". For simplicity consider a plane nearfield measurement surface with $\underline{X}_1$, in the farfield, then the Green's function is independent of $\underline{X}_o^{mn}$ and we may write,

$$\frac{\sigma^2_{|T(\underline{X}_1)|}}{|T(\underline{X}_1)|^2} = \frac{\sum_m \sum_n \left\{ \sigma^2_{|T(\underline{X}_o^{mn})|} + |T(\underline{X}_o^{mn})|^2 \sigma^2_{\theta(\underline{X}_o^{mn})} \right\}}{\left| \sum_m \sum_n T(\underline{X}_o^{mn}) \right|^2}$$

It can be seen clearly in this case that the farfield amplitude accuracy will improve as the square root of the number of independent measurements, assuming that the amplitude and phase variances are independent of m and n.

EXAMPLES

As an illustration of the foregoing theory consider the case of a cylindrical array enclosed by a cylindrical measuring surface radius $a = 4.85\lambda$ whose beam pattern had a maximum sensitivity of 0dB and first side lobe about -14dB re the maximum. Suppose also that 64 nearfield measurements are made circumferentially of the nearfield of this array and the farfield predicted from these values.

Case I:- random errors in the nearfield amplitude only with a standard deviation of 32% of the maximum deterministic nearfield amplitude. This would give rise to a farfield variance at its highest of about -15dB re the maximum of the deterministic pattern calculated from relation (4). This is much larger than would be expected in a real system and would completely mask the true farfield patterns except for the main lobe. The side lobes almost certainly obeying Rayleigh statistics. See Fig 1.

Case II:- Random errors in the nearfield amplitude only with a standard deviation of 8% of the maximum deterministic value. This would give rise to a farfield variance of about -27dB re the maximum of the deterministic pattern calculated from relation (4). This is about typical for a real system with the first -20dB of the beam pattern being quite faithfully reproduced, and obeying Gaussian statistics. See Fig 2.

Case III:- Random errors in the nearfield phase only with a standard deviation of 0.16 rads ($\sim 9^{o}$). This would give rise to a farfield variance of similar level to Case II although with a different angular distribution, calculated from relation (4). If these phase errors were caused by radial positional errors only then this would imply a standard deviation in the radial position of about 0.025λ. See Fig 3.

It can be seen from this last example that the positional errors can contribute significantly to the overall farfield errors.

CONCLUSIONS

The main points for consideration in designing a nearfield measurement system, so as to reduce the effect of random errors in the nearfield measurements, are:-

- that in general the positional errors are the major factor contributing to the overall errors,
- that in general the larger the number of significant nearfield measurements the more accurate the resulting farfield prediction will be,
- that nearfield measurements should only be made over regions where the nearfield deterministic amplitude is significant,
- that the greatest nearfield phase accuracy should be in those regions where the nearfield deterministic amplitude is greatest.

REFERENCES

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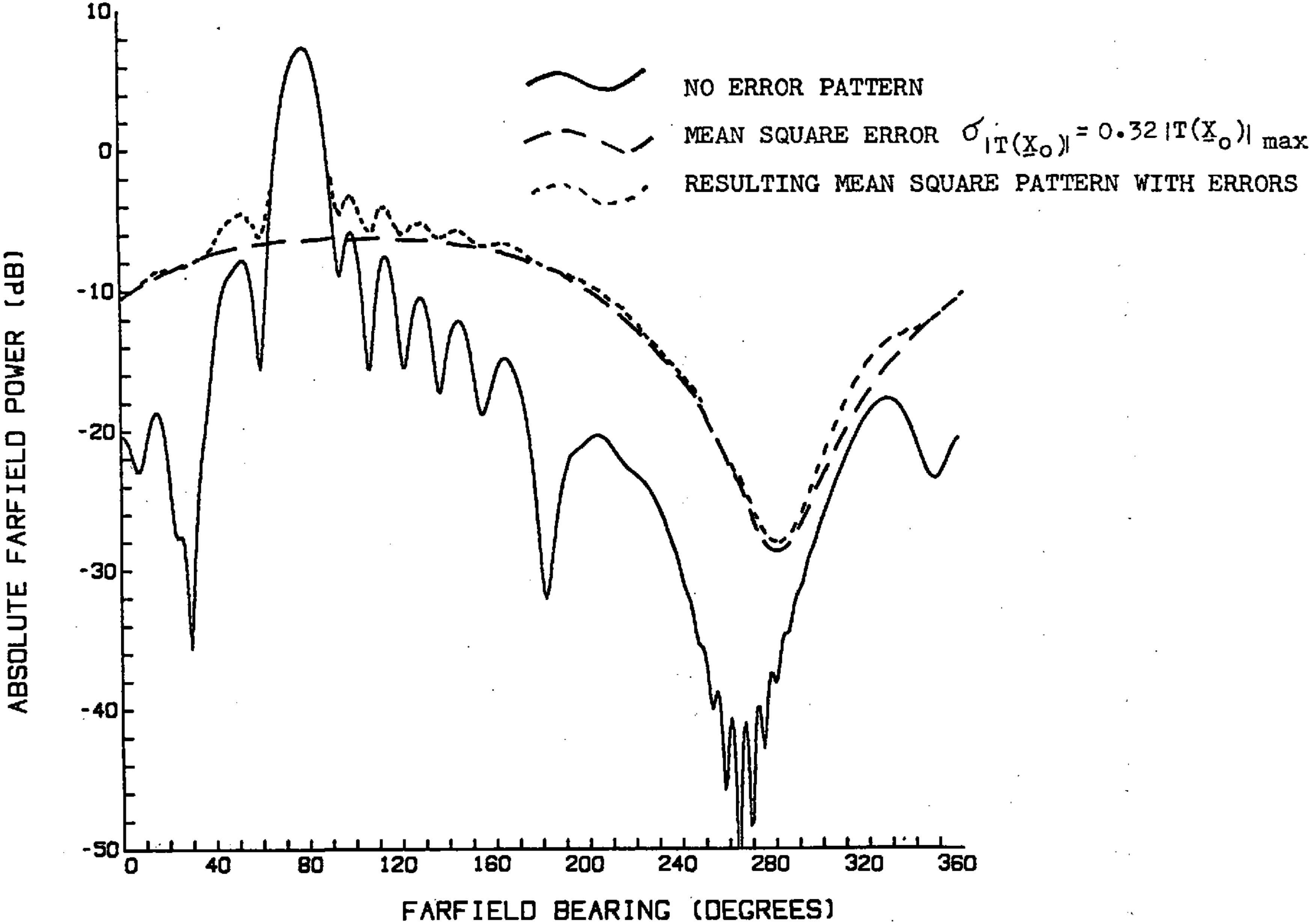

FIG. 1 . ABSOLUTE FARFIELD POWER

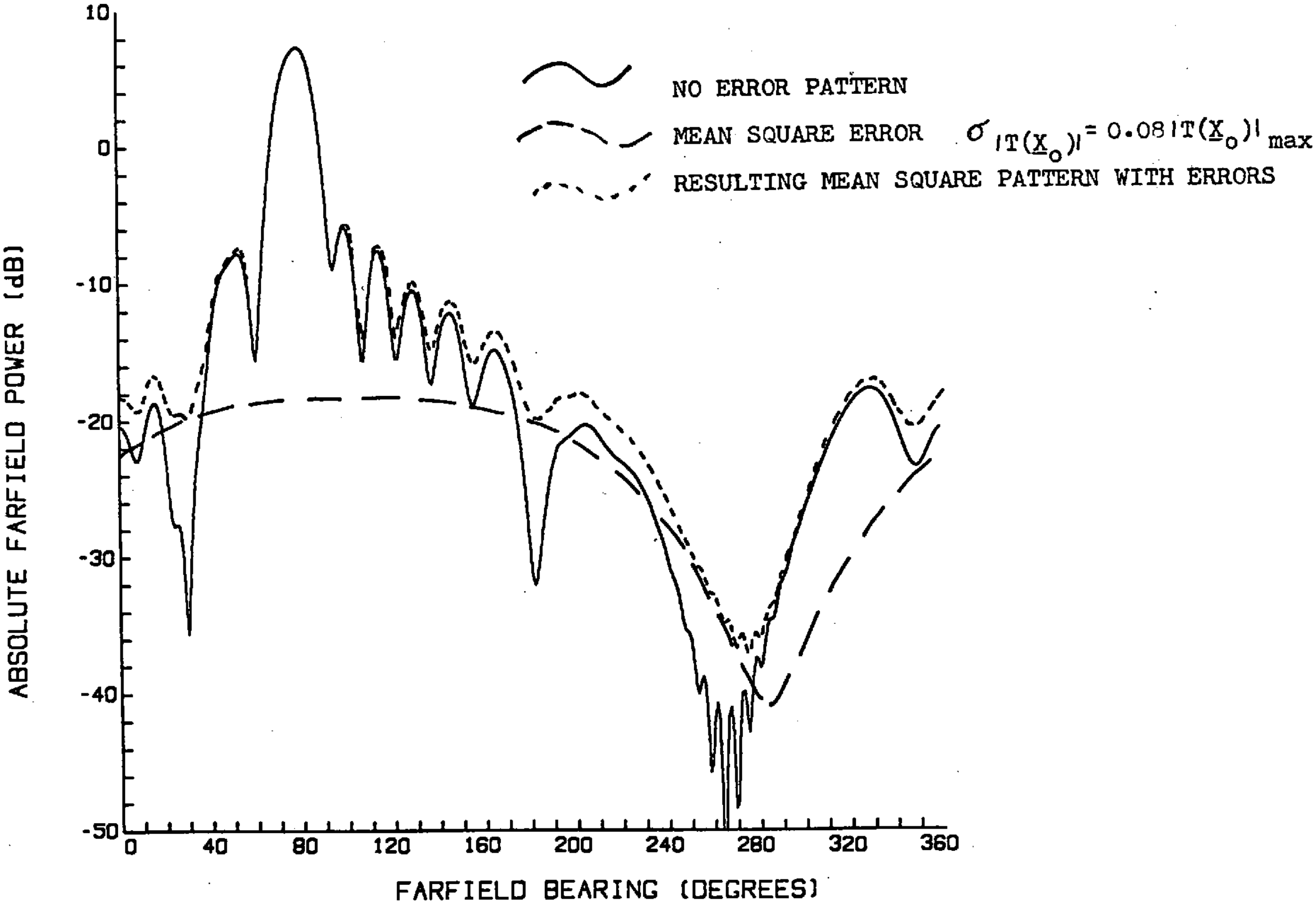

FIG. 2. ABSOLUTE FARFIELD POWER

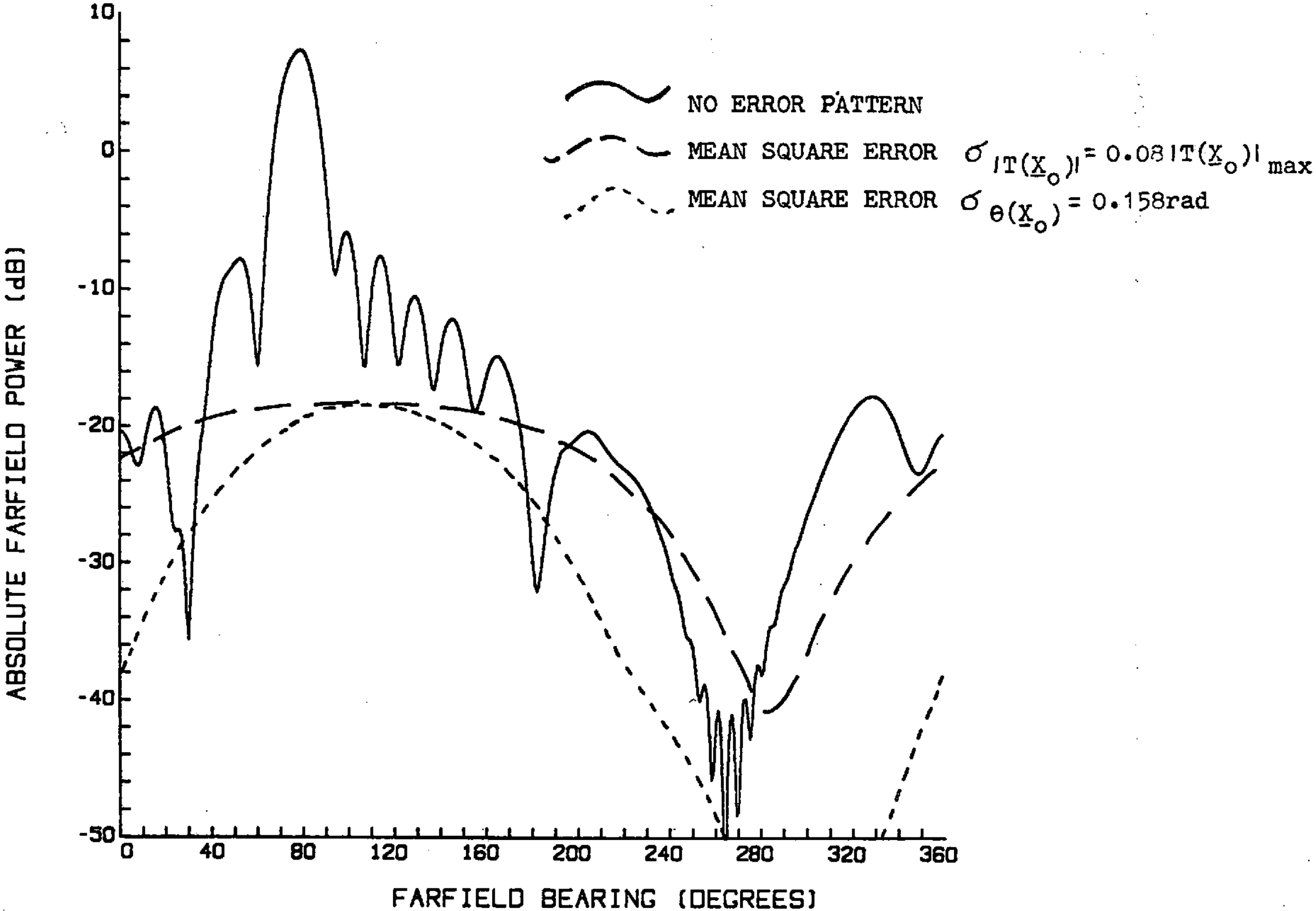

FIG. 3. ABSOLUTE FARFIELD POWER