"Stick close by your desks and never go to sea ..." W S Gilbert

THE DEGRADATION OF SONAR ARRAY PERFORMANCE DUE TO FINITE SOURCE SIZE (U)

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I. INTRODUCTION

Normally when discussing the performance of passive sonar arrays and array processors, consideration is given to the case of a point source at infinity and attention is paid to the effects of various noise fields on sonar performance. However, a paper by Cox (ref 1) discussed the reduction in array gain of large arrays due to lack of signal coherence across the array. In his paper the signal coherence was considered to decay exponentially with sensor separation. A later paper by Green (ref 2) extended the work of Cox to include a linear decay of signal coherence. In both these papers the cause of the decay in signal coherence was assumed to be due to complexities in the propagation process.

In this paper the effect of the decay of signal coherence with increasing sensor separation is also considered. However the decay is here not assumed to be due to the propagation process, which certainly will be important, but due to a quite fundamental limit; that of the finite size of real sources. It will be shown in this paper that under many conditions of practical interest large reductions in the array gain of large arrays can be caused due solely to the finite source size.

The first part of this paper describes simple mathematical models of both "sum-square" and "cross-correlation" array processors. Then follows the calculations of the spatial correlation function of the radiated field of a finite source. The resulting spatial correlation function is used with the above models to evaluate the reduction in array performance in each case.

II. SONAR MODELS

Cross-Correlation Sonar

The system modelled consists of two half arrays of equi-spaced omni-directional hydrophones each with the same constant frequency response. The hydrophone outputs are first multiplied with the shading weights and then delayed by appropriate steering delays and finally summed to give the half array outputs. These two half array outputs are then cross-correlated and the normalised correlation coefficient for zero time delay found as a function of steering angle.

The output of the half array 'A' may be written as,

$$f_A(t) = \sum_{n=1}^{N} a_n \, f_n(t - (n - 1)\delta) \qquad (1)$$

where $f_n(t)$ is the hydrophone output, $(n - 1)\delta$ the steering delay and a_n the shading weight of the n^{th} hydrophone. The output of half array 'B' may similarly be given as,

$$f_B(t) = \sum_{m=1}^{M} b_m \, f_m(t - (m - 1)\delta - \delta^1) \qquad (2)$$

where $f_m(t)$ is the hydrophone output $(m - 1)\delta + \delta^1$ the steering delay and b_m the shading weight of the m^{th} hydrophone. It should be noted that the subscripts n, p, q and the variable a_n will be associated with half array 'A' only, and the subscripts m, r, s and the variable b_m with half array 'B'.

Now the output of the cross correlator is by definition

$$R_{AB}(\tau) = \int_{-\infty}^{+\infty} f_A(t) \, f_B(t + \tau) \, dt \qquad (3)$$

where the value for $\tau = o$ is required. Substituting for the half array outputs from relations 1 and 2 into relation 3 gives,

$$R_{AB}(o) = \sum_{n=1}^{N} \sum_{m=1}^{M} a_n \, b_m \, R_{nm} \{(n - m)\delta - \delta^1\} \qquad (4)$$

where $R_{nm} \{(n - m)\delta - \delta^1\}$ is the cross-correlation function between the acoustic field at the n^{th} hydrophone in the half array 'A' and the m^{th} hydrophone in the half array 'B'. Thus the output of the sonar, the normalised cross-correlation coefficient, may be written,

$$\rho_{AB}(o) = \frac{R_{AB}(o)}{\sqrt{R_{AA}(o) \, R_{BB}(o)}} \qquad (5)$$

where $R_{AA}(o)$ and $R_{BB}(o)$ are the mean square outputs of the half arrays 'A' and 'B' respectively.

Relation 5 is the expression that is used below to analyse the effect of finite sources on the performance of a cross-correlation sonar. It is important to note that in the case of a cross-correlation sonar, if there is no correlation of the acoustic field between half arrays the correlation coefficient will be zero even though the acoustic field may be correlated between hydrophones within a half array.

"Sum-Square" Sonar

In the case of a sum-square sonar the output is the mean square power which may be written as,

$$R_{AA}(o) = \sum_{n=1}^{N} \sum_{m=1}^{N} a_n \, a_m \, R_{nm} \{(n - m)\delta\} \qquad (6)$$

where R_{nm} $\{(n - m)\delta\}$ in this case is the cross correlation function between the acoustic field at the n^{th} and m^{th} hydrophones in the same array. In this case there will be a finite output even if the acoustic field is uncorrelated between individual hydrophones due to the auto correlation terms.

The performance of such a sonar is conveniently discussed in terms of the Array Gain given as

$$AG = \frac{\sigma_N^2 R_{AA}^S(o)}{\sigma_S^2 R_{AA}^N(o)} = \frac{\sum_{n=1}^{N}\sum_{m=1}^{N} a_n a_m R_{nm}^S\{(n - m)\delta\}/\sigma_S^2}{\sum_{n=1}^{N}\sum_{m=1}^{N} a_n a_m R_{nm}^N\{(n - m)\delta\}/\sigma_N^2} \qquad (7)$$

where the superscripts S and N denote the appropriate factors for signal only or noise only respectively and σ^2 is the variance.

System performance with perfectly coherent signals

As the purpose of this paper is to discuss the effects of lack of signal coherence rather than the effects of the noise structure, attention is given to the array gain in the presence of noise which is spatially uncorrelated and also is uncorrelated with the signal.

In the case of the sum-square sonar the signal component will give rise to values of the signal cross-correlation function, in the steering direction, of $\bar{S}$, the signal mean square power, ie

$$R_{nm}^S\{(n - m)\delta\} = \bar{S} \text{ for all } n,m.$$

The noise, being uncorrelated, will only have non zero values for n = m, ie

$$R_{nm}^N\{(n - m)\delta\} = \bar{N} \text{ only for } n = m$$

$$= 0 \text{ else,}$$

where $\bar{N}$ is the mean square noise power. The array gain is then from relation 7 and assuming unity for all the shading weights,

$$AG = \frac{\bar{N}\sum_{n=1}^{N}\sum_{m=1}^{N}\bar{S}}{\bar{S}\sum_{n=1}^{N}\bar{N}} = N \qquad (8)$$

Thus the signal to noise ratio has improved by a factor N, by using an array of N elements.

In the case of the cross correlation sonar consideration is given to the normalised cross correlation coefficient for zero time delay given by relation 5. Using the same ideal signal and uncorrelated noise as above gives, for half arrays of equal numbers of elements (N), the normalised correlation coefficient as,

$$\rho_{AB}(o) = \frac{\bar{S}}{\{\bar{S} + \bar{N}/N\}} \tag{9}$$

III. SPATIAL CORRELATION FUNCTION OF FINITE SOURCES

In the preceding sections the signal field has been assumed to be spatially correlated, independent of the hydrophone spacing; that is a point source at infinity. In practice most sources of interest in the sonar context are neither small nor at infinity. In this section the spatial correlation function of the radiated field of such finite sized sources is derived, and evaluated for the special case of a linear uncorrelated source.

The fundamental relationship between the spatial cross-correlation function of the radiated field of a finite partially correlated source and the surface cross power spectrum may be written as, (references 3, 4)

$$R(\underline{X}_p, \underline{X}_q, \tau) =$$

$$\frac{1}{2\pi} \int_{\omega} \iint_{S} \iint_{S} S(\underline{x}_r, \underline{x}_s, \omega) \frac{\partial G}{\partial n} (\underline{X}_p, \underline{x}_r, \omega) \frac{\partial G^*}{\partial n} (\underline{X}_q, \underline{x}_s, \omega)\, d\underline{x}_r\, d\underline{x}_s\, e^{i\omega\tau}\, d\omega \tag{10}$$

where $S(\underline{x}_r, \underline{x}_s, \omega)$ is the cross power spectrum of the surface pressure field on the surface S enclosing the source, $\underline{x}_r$ and $\underline{x}_s$ are surface points, $R(\underline{X}_p, \underline{X}_q, \tau)$ is the cross correlation function of the radiated field, $\underline{X}_p$ and $\underline{X}_q$ are exterior points and $\frac{\partial G(\;)}{\partial n}$ is the normal derivative of the Green's function for Dirichlet boundary conditions on the surface S.

In the case of a source that gives rise to a spatially uncorrelated surface pressure field relation 10 becomes on integrating with respect to $\underline{x}$,

$$R(\underline{X}_p, \underline{X}_q, \tau) =$$

$$\frac{1}{2\pi} \int_{\omega} \iint_{S} S(\underline{x}, \underline{x}, \omega) \frac{\partial G}{\partial n} (\underline{X}_p, \underline{x}, \omega) \frac{\partial G^*}{\partial n} (\underline{X}_q, \underline{x}, \omega)\, d\underline{x}\, e^{i\omega\tau}\, d\omega \tag{11}$$

In cases of practical interest the farfield approximations for the Green's functions can be used and assuming a uniform auto power spectrum given by

$$\begin{aligned} S(\underline{x}, \underline{x}, \omega) &= S(\omega) \text{ for } -L < x < L \\ &= 0 \text{ else} \end{aligned} \tag{12}$$

and a uniform frequency spectrum given by,

$$S(\omega) = \bar{S} \text{ for } \omega_1 < |\omega| < \omega_2 \qquad (13)$$

$$= 0 \text{ else}$$

gives the spatial correlation function of the farfield of an uncorrelated broadband line source as,

$$R(X_p, X_q, 0) = \frac{\bar{S}L}{4\pi^3 z^2 c^2} \left[\frac{\omega^2}{a} \{ j_1[\omega(a+\beta)] + j_1[\omega(a-\beta]\} \right]_{\omega_1}^{\omega_2} \qquad (14)$$

where $a = (X_p - X_q)L \cos\theta/zc$, $\beta = (X_p^2 - X_q^2)/2zc$. X_p and X_q lie on the measurement plane which makes an angle θ with the source plane and z is the perpendicular distance of the origin of the source plane coordinate from the measurement plane origin. In the narrow band approximation, relation 13 for the spatial correlation function may be written,

$$R(X_p, X_q, 0) = \frac{L\, \bar{S}\, \bar{\omega}^2 \cos \bar{\omega}\, \beta \sin \bar{\omega}\, a}{2\pi^4 z^2 c^2 \bar{\omega} a} \qquad (15)$$

It is this narrow band approximation for the spatial correlation function of the radiated farfield of a spatially uncorrelated line source of uniform intensity that is used below together with the sonar models described in Section II to determine the effect of such signal fields on sonar performance.

The cosine term in relation 15 is due to the basic curvature of the wavefront and is independent of the size of the source and for hydrophones symmetrically placed about the normal from the array to the source is unity. The sine term is dependent on the size and range of the source and it is clear from relation 15 that for small sources at large distances the field will be well correlated even for large hydrophone separations.

It will prove convenient in the following to use the correlation length Δ, defined as the separation corresponding to the first zero of the spatial correlation function, as a measure of the extent of the region of high correlation. This may be written for small β in the narrowband approximation as,

$$\Delta = \frac{z\, c\, \pi}{\bar{\omega}\, L \cos\theta} . \qquad (16)$$

IV. EFFECT OF FINITE SOURCES

In this section the spatial correlation function of the radiated field of a finite uncorrelated source is used to evaluate the degradation of sonar performance. The actual spatial correlation function used is for the narrowband approximation. In practice this will suffice for quite wide bandwidths and even for very large bandwidths the general trend of results given below still hold although if required the exact broadband correlation function given by relation 14 may be used. In general the wider the bandwidth the shorter the correlation length.

Sum-Square Sonar

The effect of the finite source on the array gain of a sum square sonar array processor can be seen from relation 8 using the above correlation function. It is then clear that for hydrophone separations small compared to the correlation length the ideal array gain will be maintained. However for separations greater than the correlation length the correlation function will have only small values. Approximating the correlation function by,

$$R^S_{nm}\{(n-m)\delta\} = 0 \text{ for } (n-m)\,d > \Delta/2$$
$$= \bar{S} \text{ for } (n-m)\,d \leqslant \Delta/2$$

allows the Array Gain using relation 7 to be written for uncorrelated noise,

$$AG = \frac{[N(2k+1) - k(k+1)]}{N} \quad \text{where } k = \frac{\Delta}{2d} \text{ and } N \geqslant \frac{\Delta}{2d}.$$

Thus an estimate of the reduction in array gain for large arrays may be written as,

$$\text{reduction} \sim \frac{\Delta}{Nd}, \quad \text{for } Nd \gg \Delta/2 \text{ and } \frac{\Delta}{2d} \gg 1. \tag{17}$$

Thus increasing the length of a sum square array beyond the length of high correlation ($\Delta/2$) gives no improvement in array gain.

Cross Correlation Sonar

In the case of the cross correlation sonar consideration is given to the effect on the normalised cross correlation coefficient $\rho_{AB}(o)$ given by relation 5. The dominating factor in this expression is the numerator, namely,

$$R_{AB}(o) = \sum_{n=1}^{N} \sum_{m=1}^{M} a_n b_m R_{nm}\{(n-m)\,\delta - \delta^1\}. \tag{5}$$

Thus for contiguous half arrays the reduction in the correlation coefficient $R_{AB}(o)$ due to the finite source size will be worse than the sum square array processor as there will be fewer small hydrophone separations contributing.

For large contiguous cross correlation arrays, where the half array length is greater than the half correlation length, the reduction in the cross correlation $R_{AB}(o)$ due to a finite source compared with the ideal value can be written in a similar manner to relation 17 as,

$$\text{reduction} \sim \frac{\Delta^2}{8(Nd)^2}, \quad \text{for } Nd \gg \Delta/2 \text{ and } \frac{\Delta}{2d} \gg 1 \tag{18}$$

where in this case Nd is the half array length. This then allows the normalised cross correlation coefficient to be written, from relations 5, 17 and 18 for uncorrelated noise and half arrays of N elements, as,

$$\rho_{AB}(o) = \frac{k(k+1)\,\bar{S}}{2\{[N(2k+1) - k(k+1)]\,\bar{S} + N\bar{N}\}} \quad \text{where } K = \frac{\Delta}{2d} \text{ and } Nd \geqslant \frac{\Delta}{2}.$$

V. CONCLUSIONS

In this paper the effect of the finite size of sources on the performance of sum square and cross correlation passive sonars has been quantified and discussed. It is clear that the effects of limited signal coherence due to finite source size are in reality as important to consider for large arrays as the effects of real noise fields, and that serious over estimates of sonar performance may be made by only considering ideal signal fields.

VI. REFERENCES

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