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UNDERWATER ACOUSTIC TEST FACILITIES AND MEASUREMENTS.

"The Prediction of Farfield Radiation Patterns from Nearfield Measurements"

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INTRODUCTION

The pressure field of a transmitting sonar transducer is conventionally determined by measuring the pressure field directly at a chosen distance, normally within the farfield. In practice the distance required for direct farfield calibration can be prohibitively large for arrays whose dimensions are large compared with the wavelength in the medium. Under these conditions it is necessary to use alternative methods of calibration. This paper describes two methods of predicting the field of transmitting sources from measurements made in the nearfield thus obviating the need for large measuring distances. Both methods described rely on the use of the Helmholtz Integral formulation of the diffraction of scalar fields. The methods were tried for both plane and cylindrical arrays and compared with direct calibrations.

Theory

Consider a closed surface S, containing all the sources. The field at a point $\underline{X}_1$ a distance $\underset{\sim}{r}$ from the surface is given by the Helmholtz Integral;

$$1.\quad U(\underline{X}_1) = \frac{1}{4\pi}\iint_S \left\{ G(\underline{X}_o - \underline{X}_1)\frac{\delta U}{\delta n}(\underline{X}_o) \quad - \quad U(\underline{X}_o)\frac{\delta G}{\delta n}(\underline{X}_o - \underline{X}_1)\right\} ds$$

where $U(\underline{X}_1) = |U(\underline{X}_1)|\, e^{i\theta_1}$, is the pressure at point $\underset{\sim}{X}_1$ exterior to S, $U(\underline{X}_o) = |U(\underline{X}_o)| e^{i\theta_o}$, is the pressure on the surface S at the point $\underline{X}_o$ $\hat{n}$ is the outward normal to the surface S at $\underline{X}_o$, $i = \sqrt{-1}$ and $G(\underline{X}_o - \underline{X}_1)$ is the free space Green's function given as a solution to the time independent scalar wave equation with a delta function as a source term;

$$2.\quad (\nabla^2 + k_o^2)\, G(\underline{X}_o - \underline{X}_1) = \delta(\underline{X}_o - \underline{X}_1) \text{ where } \delta(\underline{X}_o - \underline{X}_1) = 1 \text{ if } \underline{X}_o = \underline{X}_1, \; = 0 \text{ if } \underline{X}_o \neq \underline{X}_1,$$

$k_o = \frac{2\pi}{\lambda_o}$ is the wave number, where λ_o is the free space wavelength, and the solution to eq. 2 is,

$$3.\quad G(\underline{X}_o - \underline{X}_1) = \exp(ik_o|(\underline{X}_o - \underline{X}_1)|)\Big/|(\underline{X}_o - \underline{X}_1)|$$

The normal pressure gradient $\frac{\delta U}{\delta n}(\underline{X}_o)$ at the surface S, needed in the evaluation of equation 1., is not particularly easy to determine experimentally. One method of reducing equation 1. to a function of pressure, $U(\underline{X}_o)$, only, thereby obviating the need to determine

$\frac{\delta U}{\delta n}(\underline{X}_o)$, is to find a Green's functions that satisfies both equation 2, and the boundary condition, $G(\underline{X}_o - \underline{X}_1) = 0 \quad \underline{X}_o$ on S;

A function that satisfies the above conditions is called an exact Green's function. The Helmholtz equation, eq. 1., can now be re-written,

$$4.\quad U(\underline{X}_1) = \frac{-1}{4\pi} \iint_S U(\underline{X}_o) \frac{\delta G_E}{\delta n}(\underline{X}_o - \underline{X}_1)\, ds$$

where $G_E(\underline{X}_o - \underline{X}_1)$ is an exact Green's function. For an infinite plane surface S, the normal derivative of the exact Green's function takes the form,

$$5.\quad \frac{\delta G_E}{\delta n}(\underline{X}_o - \underline{X}_1) = 2\cos(\hat{n}, \underline{r})\left(ik_o - \frac{1}{|\underline{r}|}\right) \exp(ik_o|\underline{r}|)/|\underline{r}|$$

where $|\underline{r}| = |\underline{X}_o - \underline{X}_1|$, and $\cos(\hat{n}, \underline{r})$ is the cosine of the angle between the normal to the surface at $\underline{X}_o$ and the vector defining the point $\underline{X}_1$ from $\underline{X}_o$.

For a cylindrical surface S,

$$6.\quad \frac{\delta G_E}{\delta n}(\underline{X}_o - \underline{X}_1) = \frac{1}{2\pi a} \sum_{\nu=-\infty}^{+\infty} \frac{H_\nu^{(1)}(k_o|\underline{r}|)}{H_\nu^{(1)}(k_o a)} \cos(\nu\theta)$$

where a is the radius of the cylindrical surface, θ is the cylindrical polar co-ordinate of $\underline{X}_o$ on S, and $H_\nu^{(1)}(k|\underline{r}|) = J_\nu(k|\underline{r}|) + iY_\nu(k|\underline{r}|)$ and is the Hankel Function of the first kind of order ν. Equation 4 may therefore be evaluated by using the appropriate Green's function for the surface under consideration and inserting the measured values of the pressure field on the closed surface S. In practice the integral need only be evaluated over that part of the surface where the contribution of the integrand is significant.

An alternative method of reducing the Helmholtz equation, eq. 1., to a function of pressure only is to find a suitable approximation for the pressure gradient, on the surface S, in terms of the pressure. This can be done simply for an infinite plane surface by assuming that the pressure gradient field at the surface, S, approximates to that due to a plane wave field.

Let $U(\underline{X}_o) = \exp(ik_o|\underline{R}|)/|\underline{R}|$ where $\underline{R}$ define a source within the surface S, then we have,

$$7.\quad \frac{\delta U}{\delta n}(\underline{X}_o) = \cos(\hat{n}, \underline{R})\left(ik_o - \frac{1}{|\underline{R}|}\right) \exp(ik_o|\underline{R}|)/|\underline{R}| \approx ik_o U(\underline{X}_o)$$

assuming that $\underline{R} \to \infty$ (ie, a plane wave field) and that the direction of propagation is nearly normal to the surface S. This approximation for the pressure gradient can also be used for gently curving cylindrical and spherical surfaces. It is now possible to reduce the Helmholtz equation to,

$$8.\quad U(\underline{X}_1) = \frac{1}{4\pi} \iint_S \left\{ ik_o G(\underline{X}_o - \underline{X}_1) - \frac{\delta G}{\delta n}(\underline{X}_o - \underline{X}_1) \right\} U(\underline{X}_o)\, ds$$

where $G(\underline{X}_o - \underline{X}_1) = \exp(ik_o|(\underline{X}_o - \underline{X}_1)|)/|(\underline{X}_o - \underline{X}_1)|$

As no boundary conditions are imposed on $G(\underline{X}_o - \underline{X}_1)$, the free space Green's function, it remains simple in form and independent of the measuring surface. Therefore by writing the approximate form of the Helmholtz equation in the required co-ordinates the pressure at $\underline{X}_1$, $U(\underline{X}_1)$, can easily be calculated from the pressure measurements on the surface S. The relative merits of equations 4 and 8 will be discussed in the conclusions.

The initial measurements were made on a $2\lambda_o$ by $2\lambda_o$ transmitting array of 16 transducers, suspended in a concrete tank, $12\lambda_o$ by $18\lambda_o$ by $8\lambda_o$ filled with fresh water. A small probe hydrophone was traversed in a plane parallel to and about $\lambda_o/8$ from the array. The hydrophone used was omnidirectional to within 1dB. The amplitude and phase of the signal measured by the hydrophone, when the array was transmitting, was recorded at points corresponding to a sampling distance of $\lambda_o/2$. These values of amplitude, $|U(\underline{X}_o)|$, and phase, θ_o, were used to evaluate the field at a distant point, $\underline{X}_1$, using equation 4, with the derivative of the exact Green's function, given by equation 5. A comparison was made by evaluating the field at a distant point, $\underline{X}_1$, using equation 8 with the same pressure data.

Measurements were also made on a cylindrical surface of $4.5\lambda_o$ radius surrounding a cylindrical transducer array of $4\lambda_o$ radius suspended from a calibration tender at sea. The field on this surface was measured at 5° intervals over an arc of 120° using a line hydrophone, the outputs of which were joined in parallel in the axial direction. The line hydrophone was omnidirectional to within 2dBs in the plane normal to the axis of symmetry of the surface. The measuring arc was limited to 120° by engineering difficulties. A 120° arc of the cylindrical array was available for transmission. The field at a distant point $\underline{X}_1$, was calculated using the measured values of the pressure over the 120° arc to evaluate the exact formulation, equation 4. A comparison was made by evaluating the field at a distant point, $\underline{X}_1$, using equation 8 in circular polars with the same data.

According to sampling theory it is only necessary to sample at intervals corresponding to a half wavelength at the highest spatial frequency present. As measurements were made in a region where the field contained both evanescent and propagating modes spatial frequencies higher than k_o were measured. This could lead to errors due to the overlapping of the side bands of the spatial frequency function for $\lambda_o/2$ sampling. However in this case the levels of the spectrum above k_o were well below those of the propagating field and so half wavelength sampling was sufficient.

Direct calibrations of the far field response of both the plane and cylindrical arrays were made so that the accuracy of the predicted fields could be assessed. All the patterns were normalised so that the field at 0° (the direction of propagation) was OdBs. The direct calibration was only accurate down to about -23dBs for the plane array and -12dBs for the cylindrical array. This restricted any comparisons to a range of about $\pm 70^\circ$ for both plane and cylindrical arrays.

RESULTS

The standard deviation of the errors between the predicted and calibrated farfield pattern for both the plane and cylindrical arrays was calculated to give an indication of the accuracy of prediction. The farfield of the plane array consisted of a central main lobe and two side lobes in the angular range considered.

The exact formulation, eq. 4 gave a standard deviation of the errors over the ranges $\pm 60^\circ$ and $\pm 70^\circ$ of 0.68 dBs and 1dB respectively. The approximate theory, eq. 8, gave values for the ranges $\pm 60^\circ$ and $\pm 70^\circ$ of 0.97 dBs and 1.1dBs respectively. The errors were calculated every two degrees and values of the calibrated pattern less than -23 dBs were excluded. Both methods predicted the angular position of the nulls to better than $\pm 1^\circ$. The main difference in the accuracy of the predictions is that the side lobes are much more accurately predicted using the exact formulation. The same data was used for both methods.

The farfield pattern of the cylindrical array was much more regular than that of the plane array and had no nulls in the angular range considered. The exact formulation, eq. 4, gave a standard deviation over the ranges $\pm 60^\circ$ and $\pm 70^\circ$ of 0.4dBs and 0.53dBs respectively. The approximate theory, eq. 8, gave values over the ranges $\pm 60^\circ$ and $\pm 70^\circ$ of 0.62dBs and 0.71dBs respectively. The errors were calculated very 5° and no values were excluded.

Samples of the nearfield were also made as frequently as every $\lambda_0/15$ but gave no improvement over $\lambda_0/2$ samples in the angular range considered.

The measurements on the plane array were made over an area corresponding to that of the array face. When a larger area was used the results deteriorated contrary to expectation. This was probably due to reflection from the tank.

CONCLUSIONS

The prediction of the farfield of transmitting arrays from nearfield measurements can be seen to be a reasonable method where direct calibration is not practicable due to large measurement distances. Both the methods described have their advantages. It appears from the limited results available that the exact formulation in this case proved more successful in predicting the farfield, however the simplicity of the computational form of the approximate method should be taken into account.

References:

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