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ACTIVE CANCELLATION OF NOISE USING MULTIPLE SOURCES

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1. INTRODUCTION

Active noise control (ANC) uses the intentional superposition of acoustic waves to create a destructive interference pattern and, thereby, reduce the level of noise. This is realised by artificially generating cancelling source(s) of noise through detecting and processing the noise by an electronic controller of suitable frequency-dependent characteristics. In this manner, the interference of the component waves leads to creating a pattern of zones of cancellation and reinforcement in the medium. The extent of zones of cancellation depends primarily upon the maximum noise frequency and separation between the sources: for given maximum frequency of the noise, a decrease in separation between the sources leads to an increase in the physical extent of cancellation. In practice, this is limited by the physical dimensions of the sources. However, the situation can significantly be improved with a proper design methodology incorporating a suitable geometrical arrangement of a finite set of cancelling sources.

This paper presents an investigation of the process of noise cancellation in three-dimensional propagation. A quantitative description of the degree of cancellation in terms of source-related and geometry-related parameters of the system is given on the basis of a multiple-source configuration. The design of a single-input multi-output ANC system on the basis of optimum (complete) cancellation of noise at a finite set of observation points in the medium is given. The physical extent of cancellation achieved with such a system is investigated through a quantitative measure of cancellation in terms of transfer characteristics of system components. A three-dimensional description of the interference pattern is given and merits of design relating the physical extent of cancellation with the number and geometrical arrangement of sources and transducers are identified.

2. THE FIELD CANCELLATION FACTOR

Consider a primary (unwanted) point source of noise emitting a wave $p(t)$, as function of time t, with spectral density $G_{pp}(\omega)$, where ω is the radian frequency, into a non-dispersive propagation medium. Moreover, consider a set of k secondary (cancelling) point sources respectively located at distances d_i, relative to the primary source and emitting waves $s_i(t)$ with spectral densities $G_{ssi}(\omega)$ ($i = 1, 2, \ldots, k$) into the medium (see Fig.1). In propagating through the medium the wave $p(t)$ results in a wave $p_o(t)$ with spectral density $G_{ppo}(\omega)$ at a point with distance r_g relative to the primary source and distances $\{ r_{hi} \}$ relative to the secondary sources. Similarly, the secondary wave $s_i(t)$ results in a wave $s_{io}(t)$ with spectral density $G_{ssio}(\omega)$ at the point. Let the resultant field due to the superposition of the primary and secondary waves at the point be given by $o(t)$ with spectral density $G_{cco}(\omega)$. Thus,

$$G_{ppo}(\omega) = |\, P_o(j\omega)\, |^2 = |\, \frac{A}{r_g}\, e^{-j\frac{\omega}{c} r_g} P(j\omega)\, |^2 = (\, \frac{A}{r_g}\,)^2\, G_{pp}(\omega)$$

$$G_{ssio}(\omega) = |\, S_{io}(j\omega)\, |^2 = |\, \frac{A}{r_{hi}}\, e^{-j\frac{\omega}{c} r_{hi}} S_i(j\omega)\, |^2 = (\, \frac{A}{r_{hi}}\,)^2\, G_{ssi}(\omega) \qquad (1)$$

$$G_{cco}(\omega) = |\, O(j\omega)\, |^2 = |\, \frac{A}{r_g}\, e^{-j\frac{\omega}{c} r_g} P(j\omega) + \sum_{i=1}^{k} \frac{A}{r_{hi}}\, e^{-j\,[\frac{\omega}{c} r_{hi} + \theta_i(\omega)\,]} S_i(j\omega)\, |^2$$

where $P(j\omega)$, $P_o(j\omega)$, $S_i(j\omega)$, $S_{io}(j\omega)$ and $O(j\omega)$ respectively are the frequency-domain representations of $p(t)$, $p_o(t)$, $s_i(t)$, $s_{io}(t)$ and $o(t)$. A is a constant, c is the speed of sound in the medium and $\theta_i(\omega)$ is the phase by which $p(t)$ leads $s_i(t)$.

For a quantitative description of cancellation, the field cancellation factor K is defined as the ratio of the cancelled spectrum $G_{ppo}(\omega) - G_{cco}(\omega)$ to the primary spectrum $G_{ppo}(\omega)$ that existed at the point prior to the superposition of the secondary waves [1];

$$K = 1 - \frac{G_{cco}(\omega)}{G_{ppo}(\omega)} \qquad (2)$$

Thus, the cancellation factor has an upper limit of unity; $K \leq 1$. It follows from Eq.(2) that for the primary wave to be cancelled K must be between zero and unity;

$$0 < K \leq 1 \qquad (3)$$

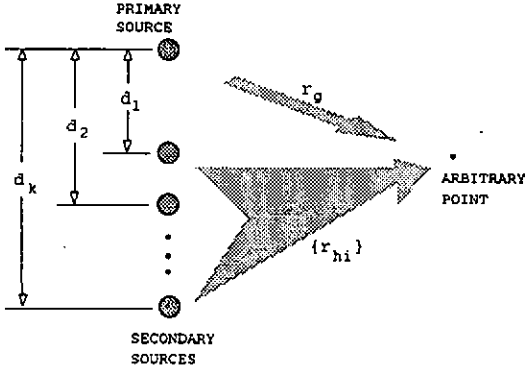


Fig.1: Process of interference of component waves.

where $K = 1$ means complete (optimum) cancellation, $K = 0$ means no cancellation and if $K < 0$ then the situation corresponds to a reinforcement of the primary wave.

Substituting for $G_{ppo}(\omega)$ and $G_{cco}(\omega)$ from Eqs.(1) into Eq.(2) and simplifying yields

$$K = -\sum_{i=1}^{k} (\alpha_i + 2\sqrt{\alpha_i}\, \beta_i) - 2 \sum_{i=1}^{k-1} \sum_{j=2}^{k} \sqrt{\alpha_i\, \alpha_j}\, \beta_{ij} \quad ; \quad i \neq j \tag{4}$$

where α_i is the power ratio and β_i is the cross-spectral density factor between the secondary signal $s_{io}(t)$ and the primary signal $p_o(t)$ and β_{ij} is the cross-spectral density factor between the secondary signals $s_{io}(t)$ and $s_{jo}(t)$:

$$\alpha_i = \frac{G_{ssio}(\omega)}{G_{ppo}(\omega)} = \left(\frac{r_g}{r_{hi}} \right)^2 \alpha_{si} \quad ; \quad \alpha_{si} = \frac{G_{ssi}(\omega)}{G_{pp}(\omega)}$$

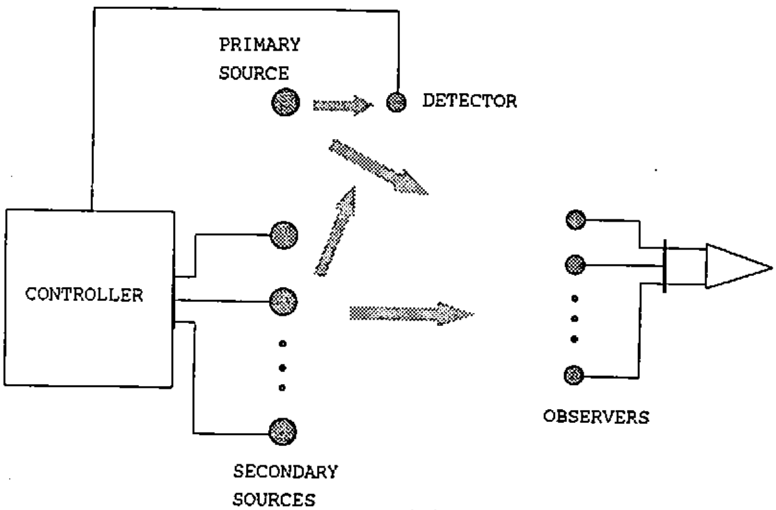
$$\beta_i = \cos\left[\frac{\omega}{c} (r_g - r_{hi}) - \theta_i(\omega) \right] \tag{5}$$

$$\beta_{ij} = \cos\left[\frac{\omega}{c} (r_{hi} - r_{hj}) - \theta_i(\omega) - \theta_j(\omega) \right] \quad ; \quad i \neq j$$

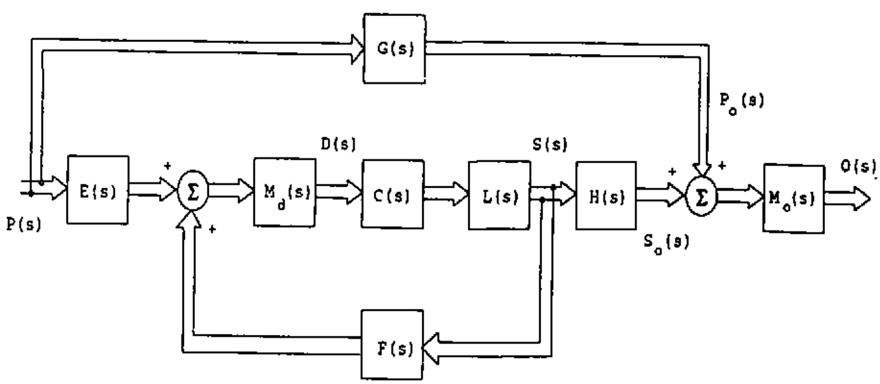
Equation (4) gives a quantitative measure of the degree of cancellation in terms of the power ratios, interpreted as the relative amplitudes, and the cross-spectral density factors, interpreted as the relative phases, of the component waves. Note that the first summation on the right hand side of this equation is due to the interference of the secondary fields with the primary field whereas the double summation term is due to the interference with one another of the secondary fields. For cancellation of the primary wave to be achieved at a point in the medium the amplitudes and phases of the secondary waves relative to the primary wave should be adjusted such that the condition in Eq.(3) is satisfied.

3. CONTROL STRUCTURE

A schematic diagram of the geometric arrangement of the ANC structure is shown in Fig.2a. The primary signal is detected by the detector, processed by the controller and fed to a set of k secondary sources. The secondary signals thus generated are superimposed onto the unwanted noise so that the noise level is reduced at a set of k observation points. An equivalent block diagram of this structure in the complex frequency, s, domain is shown in Fig.2b where $E(s)$, $F(s)$, $G(s)$ and $H(s)$ respectively represent transfer functions (in matrix form) of acoustic paths between primary source and detector, detector and secondary sources, primary source and observers and secondary sources and observers. Similarly, $M_d(s)$, $M_o(s)$, $C(s)$ and $L(s)$ respectively represent transfer functions of detector, observers, controller and secondary sources including amplifiers and other necessary electronics. $P(s)$ and $S(s)$ respectively represent primary and secondary signals before propagation



(a)



(b)

Fig.2: Active noise control structure;
(a) Schematic diagram,
(b) Block diagram.

whereas $P_o(s)$ and $S_o(s)$ respectively represent primary and secondary signals at the observation points. $D(s)$ is the detected signal and $O(s)$ represents an array of the observed signals.

The objective in Fig.2 is to reduce the level of noise to zero at the observation points. This requires that the observed primary and secondary signals at each observation point be equal in magnitude and opposite in phase. Thus, using Fig.2b the required controller transfer function is obtained as

$$C(s) = M_d^{-1} [G H^{-1} F - E]^{-1} G H^{-1} L^{-1} \quad (6)$$

This represents the required controller design relation for optimum cancellation of broadband noise at the observation points.

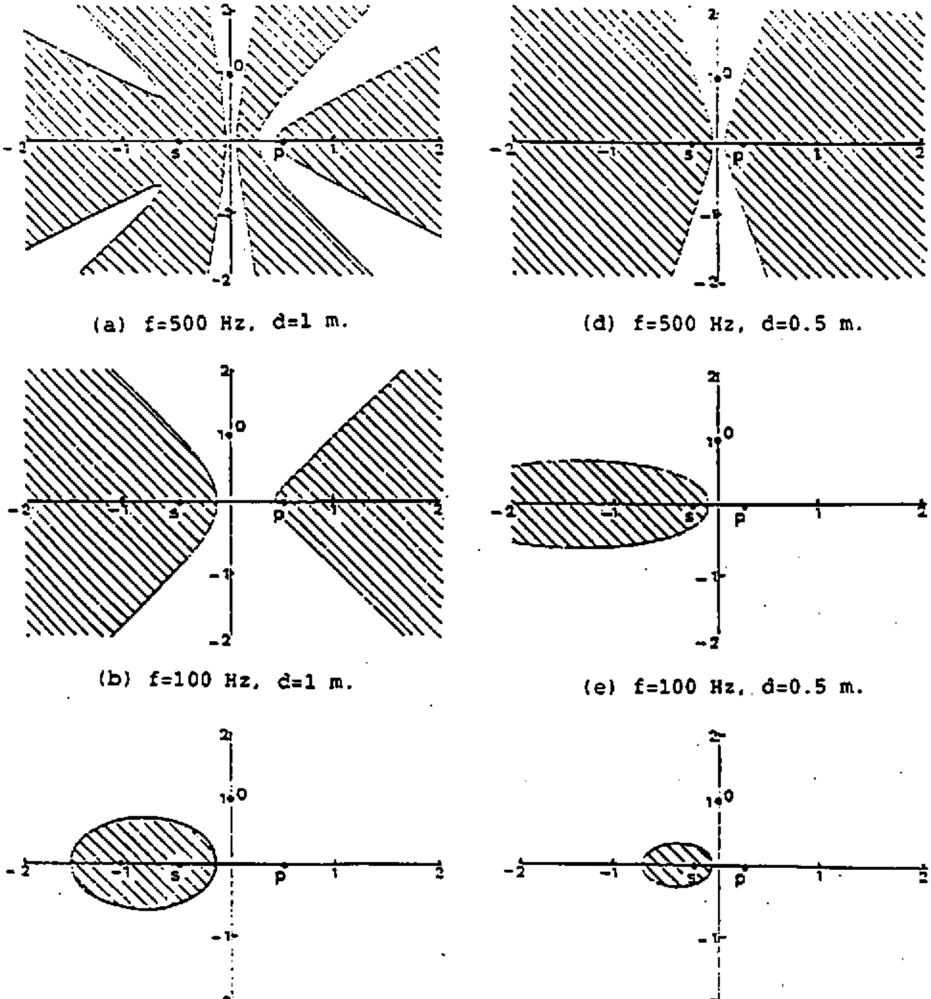
4. THREE-DIMENSIONAL DESCRIPTION OF CANCELLATION

To obtain a three-dimensional description of cancellation, consider Fig.1 with the secondary sources generated through an implementation of the controller transfer function in Eq.(6), for $s = j\omega$, within the ANC structure of Fig.2. Thus, obtaining $G_{ppo}(\omega)$ and $G_{cco}(\omega)$, substituting into Eq.(2) and simplifying yields the cancellation factor at the arbitrary point as

$$K = 1 - | 1 - g^{-1} GH^{-1} h |^2 \quad (7)$$

where g and $h = \{ h_i \}$ respectively are transfer functions of the acoustic paths through distances r_g and $\{ r_{hi} \}$. Equation (7) gives a quantitative measure of the degree of cancellation achieved with the ANC system, under stationary (steady-state) conditions, at points with distances r_g and $\{ r_{hi} \}$ relative to the primary and secondary sources respectively. Under such a situation, the amount of cancellation is dependent on transfer characteristics of the acoustic paths from the primary and secondary sources to the observation points and the point in question only. Therefore, for a given frequency of the noise the amount and physical extent of cancellation is described by the geometrical arrangement of the sources and observers. A three-dimensional description of the pattern of zones of cancellation and reinforcement can thus be obtained by calculating the cancellation factor K for given noise frequency and system geometry using Eq.(7).

The set of two-dimensional diagrams in Fig.3 show the interference patterns created by a primary source and a secondary source respectively located at points P and S with a distance d apart. In these diagrams the axes are graduated in metres, the plain regions represent zones of cancellation, the shaded regions represent zones of reinforcement and the controller is designed for complete cancellation of noise at point O. Similarly, the set of diagrams in Fig.4 show the interference patterns created by a primary source at point P and two secondary sources located at points S_1, with distance d_1 relative to P, and S_2, with distance d_2 relative to P, respectively. The controller is designed for complete cancellation of the noise at points O_1 and O_2. Revolution of each diagram in Figs.3 and 4 around the horizontal axis will result in the corresponding three-dimensional description.



(a) f=500 Hz, d=1 m.

(d) f=500 Hz, d=0.5 m.

(b) f=100 Hz, d=1 m.

(e) f=100 Hz, d=0.5 m.

(c) f=10 Hz, d=1 m.

(f) f=10 Hz, d=0.5 m.

Fig.3: Interference pattern with one secondary source.