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Asymmetric all-pass crossover alignments

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0 Introduction

A set of low-frequency crossover alignments is described that exhibit composite all-pass characteristics but have individual high-pass and low-pass transfer functions of different order. Of specific interest is the alignment using a first-order, high-pass filter although more general, higher-order asymmetric alignments are also discussed. This work is presented for two reasons, first the existence of asymmetric filters generalises the already well documented crossovers belonging to the all-pass family and secondly, the application of this class of filter to low-frequency, crossover design can represent a significant simplification in network topology. However, the main application regime is restricted to low frequency primarily because of asymmetric phase response and the association of lobing errors [Lipshitz *et al* 1983] when there are significant time delays between drive unit acoustic centres.

As an example, a basic scheme is described in Section 3 that combines a broad bandwidth loudspeaker with a sub-woofer. The system uses a first-order network in the satellite loudspeaker channel yet accommodates a second-order network of lower cut-off frequency in the subwoofer feed to enhance suppression of high frequency signal components.

1 Review of crossover systems

Crossover systems can be sub-divided into three broad categories, namely: constant voltage, delayed derived and all-pass.

1.1 Constant voltage

For a two-way crossover with high- and low-pass transfer functions $A_H(s)$, $A_L(s)$ a constant voltage filter [Small 1971] is defined as:

$$A_H(s) + A_L(s) = 1 \qquad \ldots 1.1$$

The most common alignment of this class is the first-order system where,

$$A_H(s) = \frac{as}{1 + as}$$

$$A_L(s) = \frac{1}{1 + as}$$

for which the solution is exact and both filters exhibit equal rates of attenuation. However, asymmetric, higher-order, constant-voltage alignments also exist, where for example,

$$A_L(s) = \frac{1}{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}$$

$$A_H(s) = \frac{a_1 s \left[1 + \frac{a_2 s}{a_1} + \ldots + \frac{a_n s^{n-1}}{a_1} \right]}{1 + a_1 s + \ldots + a_n s^n}$$

ie at low frequency, $A_H(s) \rightarrow a_1 s$ and corresponds to a pseudo, first-order response.

1.2 Delay derived

Delay-derived filters [Lipshitz *et al* 1983] are not well suited to analogue applications as they effectively combine s-domain and z-domain techniques. Their main application is in digital crossover filter design [Bews 1987] where a prototype low-pass filter is specified and evaluated using for example a Parks McClellan optimisation [McClellan 1973] or Kaiser Window procedure and a complementary, high-pass filter derived. Specifying filters in the z-domain gives,

$$A_H(z) + A_L(z) = z^{N/2} \qquad \ldots 1.2$$

where z^N is the filter duration. Such filters are not well matched to the synthesis of low-frequency analogue filters due to the excessive number of coefficients N required to accurately approximate the impulse response.

1.3 Symmetric, all-pass crossover function

The class of symmetric all-pass crossover has found relatively wide application particularly in active loudspeaker systems where filter realisation is well suited to operational amplifier circuits [Bohn 1983, Hawksford 1988].

This group of filters is described by,

$$kA_H(s) + A_L(s) = \frac{P_n(-s)}{P_n(s)} \qquad \ldots 1.3$$

where $P_n(s)$ is a general polynomial of order n, and k is either 1 or -1 depending upon system order and alignment.

Four common examples occur from order 1 to 4, though the existence of higher-order alignments is recognised [Bohn 1988].

(i) First order

$$A_H(s) = \frac{as}{1 + as}, \qquad A_L(s) = \frac{1}{1 + as}$$

whereby

$$A_L(s) - A_H(s) = \frac{1 - as}{1 + as}$$

However, by inverting the hf channel, the first-order, constant-voltage crossover emerges, hence the all-pass alignment has minimal advantage, other than demonstrating a tolerance to wiring errors in loudspeaker assembly.

(ii) Second-order. If,

$$A_H(s) = \frac{a^2s^2}{(1 + as)^2}, \qquad A_L(s) = \frac{1}{(1 + as)^2}$$

then

$$A_L(s) - A_H(s) = \frac{1 - as}{1 + as}$$

(iii) Third-order. If,

$$A_H(s) = \frac{a^3s^3}{(1 + as + a^2s^2)(1 + as)}$$

$$A_L(s) = \frac{1}{(1 + as + a^2s^2)(1 + as)}$$

then,

$$A_L(s) + A_H(s) = \frac{1 - as + a^2s^2}{1 + as + a^2s^2}$$

and

$$A_L(s) - A_H(s) = \frac{1 - as}{1 + as}$$

again the odd-order alignment exhibits a tolerance to hf channel inversion, although the first-order, all-pass offers the best composite alignment due to reduced group-delay distortion.

(iv) Fourth-order. The best known fourth-order alignment is the Linkwitz-Riley LR-4 [Linkwitz 1976], where

$$A_H(s) = \frac{a^4s^4}{\left(1 + \sqrt{2}\,as + a^2s^2\right)^2}, \quad A_L = \frac{1}{\left(1 + \sqrt{2}\,as + a^2s^2\right)^2}$$

and,

$$A_L(s) + A_H(s) = \frac{1 - \sqrt{2}\,as + a^2s^2}{1 + \sqrt{2}\,as + a^2s^2}$$

2. Asymmetric, all-pass alignments

The general expression for an all-pass alignment was given by equation 1.3. To develop a method for identifying asymmetric crossovers we express the transfer functions $A_L(s)$, $A_H(s)$ as

$$A_L(s) = \frac{LN(s)}{LS(s)}, \qquad A_H(s) = \frac{HN(s)}{HD(s)}$$

where from equation 1.3

$$k\frac{HN(s)}{HD(s)} + \frac{LN(s)}{LD(s)} = \frac{P_n(-s)}{P_n(s)}$$

Rearranging in terms of the LPF numerator polynomial LN(s),

$$LN(s) = \frac{LD(s)}{P_n(s)HD(s)}\left[P_n(-s)HD(s) - kP_n(s)HN(s)\right]$$

Since LN(s) is a polynomial of finite order then,

$$LD(s) = P_n(s)HD(s) \qquad \ldots 2.1$$

and,

$$LN(s) = P_n(-s)\,HD(s) - kP_n(s)HN(s) \qquad \ldots 2.2$$

Hence by specifying both the high-pass filter denominator polynomial and the polynomial $P_n(s)$, the transfer function of the corresponding the low-pass filter can be derived. The following sub-sections describe a range of crossover examples where the orders of LD(s) and HD(s) differ:

2.1 First-order, high-pass filter with polynomial $P_n(s)$ of order N.

In this first class of asymmetric filter, the high-pass filter is defined as first order where,

$$A_H(s) = \frac{b_1 s}{1 + b_1 s} \qquad \ldots 2.3$$

that is

$$HN(s) = b_1 s$$

$$HD(s) = 1 + b_1 s$$

The polynomial $P_n(s)$ of order n is expressed as,

$$P_n(s) = \sum_{r=0}^{n} a_r s^r \qquad \ldots 2.4$$

whereon the denominator of the low-pass filter LD(s) follows from equation 2.1,

$$LD(s) = \left(1 + b_1 s\right) \sum_{r=0}^{n} a_r s^r \qquad \ldots 2.5.$$

In evaluating the numerator LN(s) from equation 2.2, there are two conditions: for n odd, k = -1 while for n even, k = 1. These conditions ensure that terms in s^{n+1} cancel, they also represent the relative inverted and non-inverted connections between high-pass and low-pass channels.

Hence for odd n

$$LN(s)\Big|_{n\ odd} = \sum_{r=0}^{\frac{n-1}{2}} s^{2r}\left[a_{2r} + \left[2b_1 a_{2r} - a_{2r+1}\right] s\right] \qquad \ldots 2.6$$

and for even n

$$LN(s)\Big|_{n\ even} = a_0 - \sum_{r=1}^{\frac{n}{2}} [a_{2r-1}\, s^{2r-1} + \left(2b_1 a_{2r-1} - a_{2r}\right)s^{2r} \qquad \ldots 2.7$$

Using equations 2.5, 2.6, 2.7 the low-pass filter transfer function $A_L(s)$ can be evaluated for any order of polynomial $P_n(s)$ when matched to the first-order high-pass function of equation 2.3, where the all-pass function follows directly from equation 1.3.

In Section 2.1.1 asymmetric examples using the first-order, high-pass filter together with low-pass filters of orders n=2 to 4 are considered, although by following the procedure, higher orders can also be determined.

2.1.1 First-order, high-pass filter, with polynomial $P_n(s)$ order n = 4

A n = 4 example demonstrates the evaluation procedure and allows simple modification to n = 2 by setting appropriate coefficients to zero. From equation 2.5, 2.7 the transfer function for $A_L(s)$ is given as,

$$A_L(s) = \frac{LN(s)}{LD(s)} = \frac{a_0 - a_1 s - a_3 s^3 - \left(2b_1 a_1 - a_2\right)s^2 - \left(2b_1 a_3 - a_4\right)s^4}{\left(1 + b_1 s\right)\left(a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4\right)} \qquad \ldots 2.8$$

and the all-pass function as

$$\frac{P_4(-s)}{P_4(s)} = \frac{a_0 - a_1 s + a_2 s^2 - a_3 s^3 + a_4 s^4}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4} \qquad \ldots 2.9$$

The order of the polynomial LN(s) can be minimised (thus maximising the high frequency, rate of attenuation of $A_L(s)$), by setting

$$a_4 = 2b_1 a_3$$

$$a_2 = 2b_1 a_1$$

and normalising the dc gain to 1 i.e $a_0 = 1$,
whereby,

$$A_L(s) = \frac{1 - a_1 s - a_3 s^3}{(1 + bs)\left(1 + a_1 s + 2b_1 a_1 s^2 + a_3 s^3 + 2b_1 a_3 s^4\right)} \qquad \ldots 2.10$$

and,

$$\frac{P_4(-s)}{P_4(s)} = \frac{1 - a_1 s + 2b_1 a_1 s^2 - a_3 s^3 + 2b_1 a_3 s^4}{1 + a_1 s + 2b_1 a_1 s^2 + a_3 s^3 + 2b_1 a_3 s^4} \qquad \ldots 2.11$$

2.1.2 First-order, high-pass filter, with polynomial $P_n(s)$ order n = 2

Equation 2.10, 2.11 are modified to order n = 2 by setting coefficient $a_3 = 0$, whereby

$$A_L(s) = \left(\frac{1 - a_1 s}{1 + b_1 s}\right) \cdot \frac{1}{\left(1 + a_1 s + 2b_1 a_1 s^2\right)} \qquad \ldots 2.12$$

and,

$$\frac{P_2(-s)}{P_2(s)} = \frac{1 - a_1 s + 2b_1 a_1 s^2}{1 + a_1 s + 2b_1 a_1 s^2} \qquad \ldots 2.13$$

In equation 2.12, we note the special case where if $a_1 = b_1$, $A_L(s)$ becomes second-order in cascade with a first-order all-pass transfer function.

However, a characteristic revealed by equation 2.10 is that as the order of n is increased the hf, rate of attenuation approaches only 12 dB/octave. Hence there is little advantage in seeking larger values of n particularly as the phase distortion described in the all-pass transfer function (equation 2.9, 2.11) becomes more severe.

2.1.3 First-order, high-pass filter, with polynomial $P_n(s)$ order n = 3

A similar procedure to section 2.1 is followed but this time $A_L(s)$ is derived from equation 2.5, 2.6 whereby for n = 3

$$A_L(s) = \frac{LN(s)}{LD(s)} = \frac{a_0 + a_2s^2 + [2b_1a_0 - a_1]s + [2b_1a_2 - a_3]s^3}{(1 + b_1s)(1 + a_1s + a_2s^2 + a_3s^3)} \quad \ldots 2.14$$

$$\frac{P_3(-s)}{P_3(s)} = \frac{1 - a_1s + a_2s^2 - a_3s^3}{1 + a_1s + a_2s^2 + a_3s^3} \quad \ldots 2.15$$

Again LN(s) can be reduced in order by setting

$$a_3 = 2b_1a_2$$

$$a_1 = 2b_1a_0$$

$$a_0 = 1$$

whereby,

$$A_L(s) = \frac{1 + a_2s^2}{(1 + b_1s)(1 + 2b_1s + a_2s^2 + 2b_1a_2s^3)} \quad \ldots 2.16$$

and

$$\frac{P_3(-s)}{P_3(s)} = \frac{1 - 2b_1s + a_2s^2 - 2b_1a_2s^3}{1 + 2b_1s + a_2s^2 + 2b_1a_2s^3} \quad \ldots 2.17$$

This example is characterised by a third-order, all-pass composite transfer function and a real transmission zero in LN(s).

.1.4 First-order, high-pass filter, with polynomial $P_n(s)$ order n = 1

By setting $a_2 = 0$ in equation 2.16, 2.17 the first-order all-pass response results,

$$A_L(s) = \frac{1}{(1 + b_1s)(1 + 2b_1s)} \quad \ldots 2.18$$

$$\frac{P_1(-s)}{P_2(s)} = \frac{1 - 2b_1s}{1 + 2b_1s} \quad \ldots 2.19$$

This alignment is possibly the most useful as $A_L(s)$ remains second order and is all pole, while the all-pass function is first order thus offering reduced phase distortion, also equation 2.18 shows that the second pole is located at one half the frequency of the high-pass filter pole, consequently $A_L(s)$ has a greater attenuation at and above the high-pass filter break frequency, yet retains only gradual curvature in the amplitude response. In fact at the high-pass filter 3 dB break frequency, $|A_L(s)|$ exhibits 10 dB attenuation and the frequency of maximum group-delay distortion is one half of the 3 dB break frequency of $A_H(s)$.

2.2.1 Second-order, high-pass filter with polynomial $P_n(s)$ of order n = 2.

The second-order transfer function $A_H(s)$ is defined as,

$$A_H(s) = \frac{b_2 s^2}{1 + b_1 s + b_2 s^2} \qquad \ldots 2.20$$

that is

$$HN(s) = b_2 s_2$$

$$HD(s) = 1 + b_1 s + b_2 s^2$$

For n = 2, let $P_2(s) = a_0 + a_1 s + a_2 s^2$

Where from equations 2.1, 2.2, $A_L(s)$ follows as,

$$A_L(s) = \frac{LN(s)}{LD(s)} = \frac{a_0 - (a_1 - a_0 b_1)s + (a_2 - a_1 b_1)s^2 - (2a_1 b_2 - a_2 b_1)s^3}{\left(a_0 + a_1 s + a_2 s^2\right)\left(1 + b_1 s + b_2 s^2\right)} \qquad \ldots 2.21$$

and

$$\frac{P_2(-s)}{P_2(s)} = \frac{a_0 - a_1 s + a_2 s^2}{a_0 + a_1 s + a_2 s^2} \qquad \ldots 2.22$$

To reduce the order of LN(s), let $a_0 = 1$ and,

$$a_1 = b_1, \quad a_2 = a_1 b_1, \quad 2a_1 b_2 = a_2 b_1$$

i.e.

$$A_L(s) = \frac{1}{\left(1 + \sqrt{2b_2}\, s + 2b_2 s^2\right)\left(1 + \sqrt{2b_2}\, s + b_2 s^2\right)} \qquad \ldots 2.23$$

$$\frac{P_2(-s)}{P_2(s)}=\frac{1-\sqrt{2b_2}\,s+2b_2s^2}{1+\sqrt{2b_2}\,s+2b_2s^2} \qquad \ldots 2.24$$

$$A_H(s)=\frac{b_2s^2}{1+\sqrt{2b_2}\,s+b_2s^2} \qquad \ldots 2.25$$

2.2.2 Second-order, high-pass, filter with polynomial $P_n(s)$ of order n = 3

For n = 3 let $P_3(s) = a_0 + a_1s + a_2s^2 + a_3s^3$

where from equation 2.1, 2.2, $A_L(s)$ given by,

$$A_L(s)=\frac{LN(s)}{LD(s)}=\frac{a_0-(a_1-a_0b_1)s+(a_2+2a_0b_2-a_1b_1)s^2-(a_3-a_2b_1)s^3+(2a_2b_2-a_3b_1)s^4}{\left(a_0+a_1s+a_2s^2+a_3s^3\right)\left(1+b_1s+b_2s^2\right)} \qquad \ldots 2.26$$

Setting $a_0 = 1$ and reducing the order of LN(s)

$$A_L(s)=\frac{1-\left(a_1-\sqrt{2b_2}\right)s}{\left(1+a_1s+\left(a_1\sqrt{2b_2}-2b_2\right)s^2+\left(2a_1b_2-(2b_2)^{3/2}\right)s^3\right)\left(1+\sqrt{2b_2}\,s+b_2s^2\right)} \qquad \ldots 2.27$$

where

$$\frac{P_3(-s)}{P_3(s)}=\frac{1-a_1s+\left(a_1\sqrt{2b_2}-2b_2\right)s^2-\left(2a_1b_2-(2b_2)^{3/2}\right)s^3}{1+a_1s+\left(a_1\sqrt{2b_2}-2b_2\right)s^2+\left(2a_1b_2-(2b_2)^{3/2}\right)s^3} \qquad \ldots 2.28$$

and $A_H(s)$ is again given by equation 2.25.

Examination of equation 2.27 shows that by increasing n from 2 to 3, the hf rate of attenuation compared with equation 2.23 remains unaltered because of a zero in LN(s). Hence with a second-order, high-pass filter there is little advantage in seeking a polynomial $P_n(s)$ order > 2, a result that mirrors the first-order case of Section 2.1. Hence we may generalise by saying that for a high-pass filter of order r, the order of polynomial $P_n(s)$ should not exceed r if zeros in $A_L(s)$ are to be avoided, where it follows that the maximum order of $A_L(s)$ is 2r.

2.2.3 Second-order, high-pass filter with polynomial $P_n(s)$ of order n = 1

The order of the all-pass function in equation 2.23 reduces to 1 when

$$a_1 = \sqrt{2b_2},$$

where from equation 2.27, 2.28

$$A_L(s) = \frac{1}{\left(1 + \sqrt{2b_2}\, s\right)\left(1 + \sqrt{2b_2}\, s + b_2 s^2\right)} \qquad \ldots 2.29$$

$$A_H(s) = \frac{b_2^2 s^2}{1 + \sqrt{2b_2}\, s + b_2 s^2} \qquad \ldots 2.30$$

and

$$\frac{P_1(-s)}{P_1(s)} = \frac{1 - \sqrt{2b_2}\, s}{1 + \sqrt{2b_2}\, s} \qquad \ldots 2.31$$

This result is interesting to compare with the symmetrical, third-order, all-pass alignment discussed in Section 1.3.

2.3.1 Third-order, high-pass filter with polynomial $P_n(s)$ of order n = 3

The third-order, high-pass filter is defined as,

$$A_H(s) = \frac{b_3 s^3}{1 + b_1 s + b_2 s^2 + b_3 s^3} \qquad \ldots 2.32$$

where,

$$HN(s) = b_3 s^3$$

$$HD(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3$$

As discussed in Section 2.2.2, the highest order polynomial $P_n(s)$ to yield an all-pole, low-pass filter $A_L(s)$ is 3 where,

$$P_3(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3$$

Following equation 2.1, 2.2 the transfer function $A_L(s)$ is given as (for $a_0 = 1$),

$$A_L(s) = \frac{LN(s)}{LD(s)} = \frac{1 - (a_1 - b_1)s + (a_2 + b_2 - a_1b_1)s^2 - (a_3 + a_1b_2 - a_2b_1 - 2b_3)s^3 + (a_2b_2 - a_3b_1)s^4 - (a_3b_2 - 2a_2b_3)s^5}{\left(1 + a_1s + a_2s^2 + a_3s^3\right)\left(1 + b_1s + b_2s^2 + b_3s^3\right)}$$

Equating LN(s) = 1 yields two solutions:

$$A_L(s) = \frac{1}{\left(1 + \sqrt{2b_2}\, s + b_2s^2 + \frac{b_2^{3/2}}{\sqrt{2}} s^3\right)\left(1 + \sqrt{2b_2}\, s + b_2s^2 + \frac{b_2^{3/2}}{2\sqrt{2}} s^3\right)}$$

. . . 2.33

$$A_H(s) = \frac{\frac{b_2^{3/2}}{2\sqrt{2}} s^3}{1 + \sqrt{2b_2}\, s + b_2s^2 + \frac{b_2^{3/2}}{2\sqrt{2}} s^3}$$

. . . 2.34

and

$$\frac{P_3(-s)}{P_3(s)} = \frac{1 - \sqrt{2b_2}\, s + b_2s^2 - \frac{b_2^{3/2} s^3}{\sqrt{2}}}{1 + \sqrt{2b_2}\, s + b_2s^2 + \frac{b_2^{3/2}}{\sqrt{2}} s^3}$$

. . . 2.35

The second solution corresponds to the first-order polynomial when $a_2 = a_3 = 0$ and is described in Section 2.3.2.

2.3.2 Third-order, high-pass filter with polynomial $P_n(s)$ of order n = 1

Following the second solution in Section 2.3.1,

$$A_L(s) = \frac{1}{\left(1 + \sqrt{b_2}\, s\right)\left(1 + \sqrt{b_2}\, s + b_2s^2 + \frac{b_2^{3/2}}{2} s^3\right)}$$

. . . 2.36

$$A_H(s) = \frac{\frac{b_2^{3/2}}{2} s^3}{1 + \sqrt{b_2}\, s + b_2 s^2 + \frac{b_2^{3/2}}{2} s^3}$$

... 2.37

and

$$\frac{P_1(-s)}{P_1(s)} = \frac{1 - \sqrt{b_2}\, s}{1 + \sqrt{b_2}\, s}$$

... 2.38

This result yields a first-order composite all-pass, with fourth-order low-pass and third-order high-pass transfer functions.

2.3.3 Third-order, high-pass filter with polynomial $P_n(s)$ of order n = 2

To complete the third-order, high-pass set of asymmetric crossover the even-order polynomial $P_n(s)$ for n = 2 is considered where

$$P_n(s) = a_0 + a_1 s + a_2 s^2$$

Using the high-pass transfer function defined in equation 2.3.2 and applying equation 2.1, 2.2,

$$A_L(s) = \frac{LN(s)}{LD(s)} = \frac{1 - (a_1 - b_1)s + (a_2 + b_2 - a_1 b_1)s^2 - (a_1 b_2 - a_2 b_1)s^3 + (a_2 b_2 - 2a_1 b_3)s^4}{\left(1 + a_1 s + a_2 s^2\right)\left(1 + b_1 s + b_2 s^2 + b_3 s^3\right)}$$

Reducing LN(s) = 1 gives,

$$A_L(s) = \frac{1}{\left(1 + \sqrt{2b_2}\, s + b_2 s^2\right)\left(1 + \sqrt{2b_2}\, s + b_2 s^2 + \frac{b_2^{3/2}}{2\sqrt{2}} s^3\right)}$$

... 2.39

$$A_H(s) = \frac{\frac{b_2^{3/2}}{2\sqrt{2}} s^3}{1 + \sqrt{2b_2}\, s + b_2 s^2 + \frac{b_2^{3/2}}{2\sqrt{2}} s^3}$$

... 2.40

$$\frac{P_2(-s)}{P_2(s)} = \frac{1 - \sqrt{2b_2}\, s + b_2 s^2}{1 + \sqrt{2b_2}\, s + b_2 s^2}$$

... 2.41

3. Loudspeaker system using an asymmetric crossover with a first-order, high-pass filter in the satellite channel

Consider a satellite loudspeaker with a transfer function $A_s(s)$ which exhibits a second-order, high-pass response with a 3 dB break frequency circa 70-80 Hz. This system is to be interfaced with a subwoofer system which has a low-frequency transfer function $A_w(s)$ where in the frequency range 100 to 500 Hz both satellite and subwoofer show well behaved responses. The crossover alignment selected for this example is given in Section 2.1.4 where $A_H(s)$, $A_L(s)$ and $P_1(-s)/P_1(s)$ are described by equation 2.3, 2.18 and 2.19 respectively.

In Fig.1 the two-way system is shown where the asymmetric high-pass and low-pass filters are implemented using passive R-C circuits. The crossover frequency (-3 dB) is set at 200 Hz and the filters designed using the equations presented in Fig.1. The subwoofer in the example has an extended response to 20 Hz and a Q = 0.5 while the satellite has an undamped natural resonance of 70 Hz and a Q = 0.7. Computed results are then presented in Fig. 2a,b that show individual satellite and subwoofer amplitude and phase responses both with and without the associated crossover filters while in Fig. 2c the overall response is described.

The advantage of the asymmetric alignment is evident where a well controlled, composite response is displayed. By setting the crossover to 200 Hz, the response of the satellite is adequately curtailed at low frequency thus reducing distortion through excessive cone excursion. However, the subwoofer commences its attenuation region at 100 Hz rather than 200 Hz and, being second order, achieves a respectable attenuation at mid and high frequencies.

3 Conclusion

This paper has described a set of asymmetric all-pass crossovers up to the combination of third-order high-pass, sixth-order low-pass alignments. However, the methods presented are sufficiently general that the results can be extended to any high-pass filter of order r where the maximum useful order for the low-pass filter is 2r. However, low-pass filters in the range r + 1 to 2r can also be accommodated within an asymmetric alignment with a corresponding reduction in order of the all-pass polynomial. If low-pass filters of order > 2r are sought the associated zeros in the numerator of $A_L(s)$ must be accepted which then restrict the ultimate attenuation, thus at hf, the rate of attenuation with frequency of a low-pass filter of order 2r is the same as a filter of order 2r + p. However, if the numerator of $A_L(s)$ contains for example, an s^2 term, a real transmission zero is introduced which may be used to increase attenuation in the region of the crossover although non-monotonicity in the amplitude response results.

The first-order, high-pass example described in Section 2 is particularly useful because of the minimal extra circuitry required in the satellite channel which bodes well for minimising signal impairment and the overlapping nature of the responses also increase their effectiveness.

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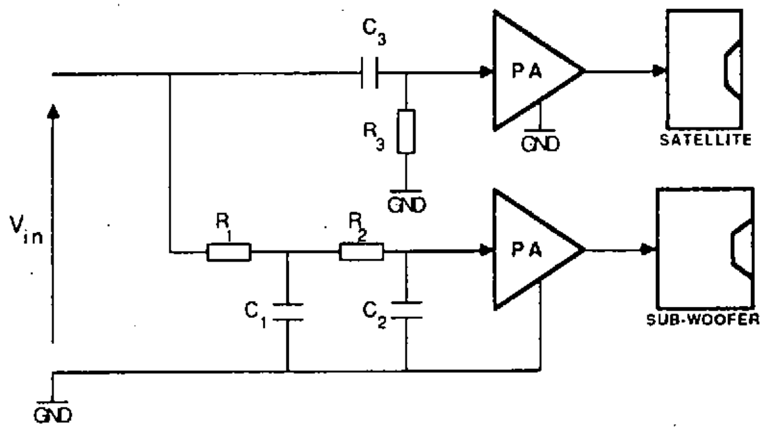

Design: Let satellite 3 dB break frequency = f_0 Hz

Set capacitors C_2, C_3, then $R_1 = 1/(12\pi f_0 C_2)$

$R_2 = 9R_1$

$C_1 = 8C_2$

$R_3 = 1/(2\pi f_0 C_3)$

Figure 1 Two-way active loudspeaker using asymmetric crossover with passive, low-level circuitry..

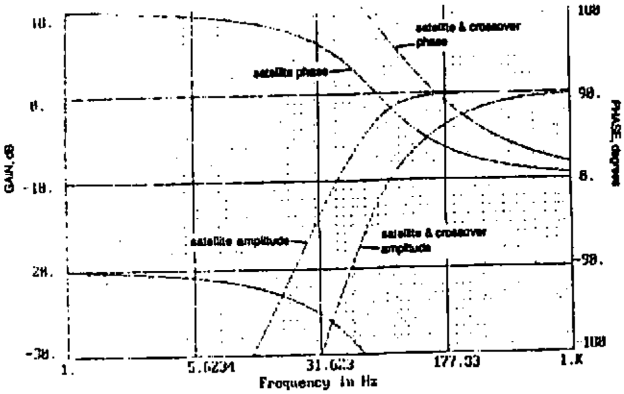

Fig.2a Satellite amplitude and phase responses with and without crossover filter

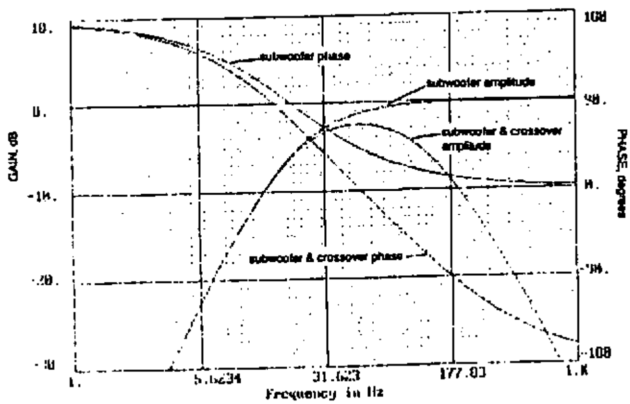

Fig.2b Subwoofer amplitude and phase responses with and without crossover filter

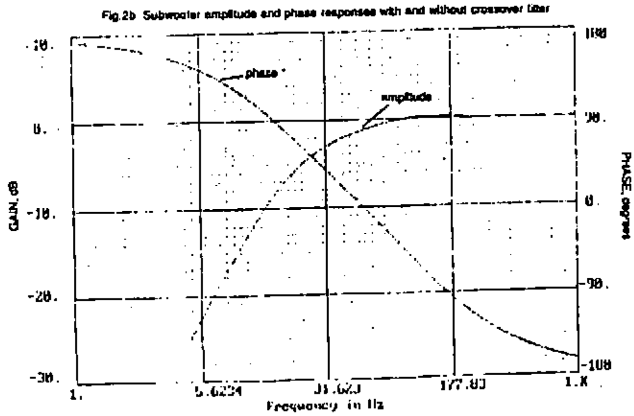

Fig.2c Composite amplitude response of satellite and subwoofer with asymmetric crossover filters