Proceedings of the Institute of Acoustics

NON-R2 PSEUDO-NOISE SEQUENCES FOR TRANSFER FUNCTION MEASUREMENT.

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SUMMARY

The background and principle of measuring a system impulse response with pseudo-noise sequences is summarised. The properties of sequences suitable for such a measurement technique are reviewed together with their generation. It is proposed that it is possible to generate a pseudo-noise sequence with properties resembling a desired test stimulus. A sequence with appropriate properties is described and it is concluded that a measurement system employing such sequences could follow.

1.0 INTRODUCTION.

1.1 General Introduction.

The cross-correlation properties of pseudo-noise sequences enables estimation of the impulse response of practical systems [1] & [2] in [4]. The output signal from a system is related to the input signal by the convolution of the system response with the input sequence. Since the circular autocorrelation of certain pseudo-random sequences is an impulse it is the system impulse response which is measured by the application of such a sequence.

$$y(t) = h(t) * x(t) \qquad (1)$$

Where y(t) is the system output, h(t) is the system response and x(t) is the input to the system.

The main advantages associated with the technique are:

(i) Hardware simplicity. A measurement system can be constructed essentially with a single channel.

(ii) Transient noise immunity. The effects of a transient noise event during a measurement will be benign noise spread evenly over the measured periodic impulse response.

(iii) Computational efficiency. The computational complexity of the technique has been simplified by the development of efficient algorithms based on the fast Hadamard Transform for performing the desired cross-correlation [3].

1.2 Principle of Operation.

The origins of such techniques can be traced back to W.D.T. Davies 1966 [1] and M.R. Shroeder 1979 [2]. Efficient algorithms for the computation of the cross-correlation, such as that published by Borish and Angell [3], and the rapid evolution of modern microcomputers has enabled the production of a commercial analysis package based on this principle described by Rife and Vanderkooy [4].

Re-writing (1) for a discrete series, where ϕ_{ny} is the cross-correlation of the input and output and ϕ_{nn} is the auto correlation of the input signal, we have:

$$\phi_{ny}(k) = \phi_{nn}(k) * h(k) \qquad (2)$$

As described fully in [3], the impulse response may be recovered by cross-correlating the noise input n(k) with the output y(k). The desirable impulsive autocorrelation of pseudo noise sequences arises only under circular autocorrelation, so the indexing of sequences must be performed modulo n. Using the notation ((j))n for the residue of j modulo n, or simply ((j)), where the modulus can be inferred from the context, the cross-correlation operation is defined by:

$$\phi_{ny}(k) = 1/n \sum_{j=1}^{n-1} n(j)y((j+k)) \qquad (3)$$

By changing indices this expression is equivalent to:

$$\phi_{ny}(k) = 1/n \sum_{j=1}^{n-1} n((j-k))y(j) \qquad (4)$$

The circularity of the operation can also be achieved by performing linear cross-correlations with periodic versions of the original sequences defined by:

$$x_p(k) = x((k))_n \qquad (5)$$

In other words, each period of the periodic sequence is equal to the original sequence.

A computationaly efficient method for performing the desired cross correlation is based on the fast Hadamard transform.

1.3 Minimum Measurement Time.

The measurement time is important and must be sufficiently long to avoid temporal aliasing. As detailed in [4] if the true impulse response decays to a negligable value over M samples of the sequence, then M samples will have stabilised the system and a further L samples are required to measure the system response. Hence the minimum measurement time is T_m=(M+L) δt where δt is the interval between samples. In practice we ensure that L≥M to avoid time aliasing giving a minimum measurement time of $2L\delta t$.

2.0 PSEUDO-NOISE SEQUENCES.

2.1 Shift Register Theory.

Convenient methods for the generation of sequences with randomness properties which reproduce the characteristics of an ergodic random process are required in many engineering applications. A simple introduction to the generation of suitable sequences for vibration analysis is given in Newland [5], while a more detailed account of shift register theory, which is also used in encryption, is given by Golomb [6].

The sequences employed in [3] and [4] are known as maximum-length sequences. The theory for linear shift registers which may be used to generate maximum-length sequences is contained in [6] and summarised below.

A shift register is any mapping T of binary n-space into itself which satisfies the relation:

$$T(a_1,a_2,\ldots,a_n)= (a_2,a_3,\ldots,a_{n+1}) \qquad (6)$$

shift register with n stages may assume 2^n different states, therefore the longest periodic sequence which may be produced by the shift register is 2^n-1, since the all zero state is necessarily excluded (all zeros will lead to perpetual zeros).
Hence we have defined the maximum-length sequence as a pseudo-noise sequence of length 2^n-1.

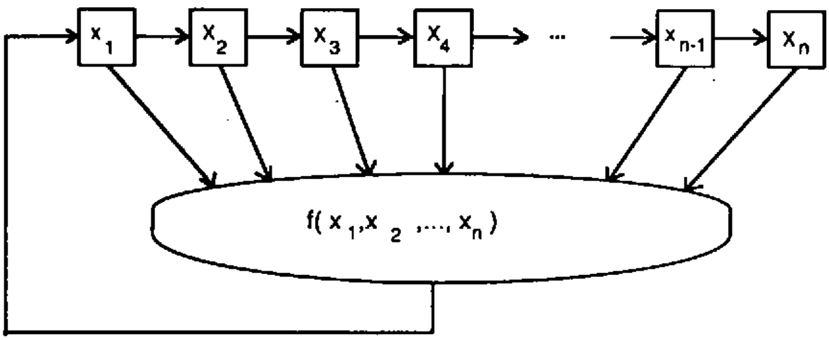

Figure 1 : General Diagram of a Feedback Shift Register.

The figure shows a general shift register with feedback. x_1 to x_n are binary storage elements and at periodic intervals (master clock) the contents of x_i is transferred into x_{i+1}. However to obtain a new value for location x_1 ,some function $f(x_1,x_2,\ldots,x_n)$ of all present terms in the shift register is computed and this value used in x_1.

A function with n binary inputs and one binary output is called a "Boolean function of n variables". There are 2^{2^n} different Boolean functions for a given number n of variables.

For a linear shift register,

$$f(x_1,x_2,\ldots,x_n) = c_1x_1 \oplus c_2x_2 \oplus \ldots \oplus c_nx_n \qquad (7)$$

Where the constants c_i are 0 or 1 and $\oplus$ denotes addition modulo 2. Since there are 2^n ways to pick the n binary constants c_i only 2^n of the 2^{2^n} shift registers with n stages are linear. If $a_n = c_1 a_{n-1} \oplus c_2 a_{n-2} \oplus \ldots \oplus c_r a_{n-r}$, where c_i to c_r are 0s or 1s, and do not depend on n, then any sequence a_n is a linearly recurring sequence. Taking the sequence $\{a_n\} = \{a_0, a_1, a_2, \ldots\}$ describing the history of the first stage for example, it is possible to associate the generating function.

$$G(x) = \sum_{n=0}^{\infty} a_n x^n \qquad (8)$$

The initial state may be thought of as $a_{-1}, a_{-2}, \ldots, a_{-r}$

$$G(x) = c_r / \; 1 - \sum_{i=1}^{r} c_i x^i \qquad (9)$$

Conveniently referred to as the r^{th} degree polynomial.

$$f(x) = 1 - \sum_{i=1}^{r} c_i x^i \qquad (10)$$

NB the characteristic polynomial of a shift register is obtained by taking $f(x) = 1 - \Sigma x^j$, where the j^{th} stages feedback.

The roots and hence the factorisation of these polynomials is the principle method for obtaining information about shift register sequences. The period of the sequence is the smallest positive integer p for which $1 - x^p$ is divisible by f(x). The primitive polynomial which defines the shift register generating function for a particular sequence may be conveniently represented as an octal.

2.2 Properties of Pseudo-Random Sequences.

Random sequences possess a special kind of autocorrelation function, peaked in the middle and and tapering off rapidly at the ends. If $\{a_n\} = \{a_0, a_1, \ldots\}$ is any sequence of real terms its autocorrelation function $C(\tau)$ is defined as:

$$C(\tau) = \lim_{N \to \infty} 1/N \sum_{n=1}^{N} a_n a_{n+\tau} \qquad (11)$$

In particular, if {an} is a periodic sequence of period p, this reduces to the finite sum:

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$$C(\tau) = 1/p \sum_{n=1}^{p} a_n a_{n+\tau} \quad (12)$$

Here τ can be thought of as a phase shift of the sequence $\{a_n\}$. $C(\tau)$ measures the amount of similarity between the sequence and its phase shift. This is always highest for $\tau=0$, and if $\{a_n\}$ is random, $C(\tau)$ is quite small for most other values of τ.

In general, the following properties are associated with a binary sequence with randomness:

R1. A balance of +1 and -1 terms:

$$\left| \sum_{n=1}^{p} a_n \right| \leq 1 \quad (13)$$

R2. Two runs of length n will exist for each run of length n+1. There are equally many runs of +1s and of -1s for each run length.

R3. A two level autocorrelation function:

$$pC(\tau) = \sum_{n=1}^{p} a_n a_{n-\tau} = \begin{cases} p \text{ if } \tau = 0 \\ K \text{ if } 0<\tau<p \end{cases} \quad (14)$$

Much of the literature deals with further evolution of the theory for sequences which obey these three randomness postulates and checks for compliance can be developed. The maximum-length sequences employed in [4] comply with these randomness postulates.

3.0 EVOLUTION OF THE MLS IMPULSE RESPONSE METHOD.

3.1. Usefulness of Spectrally Shaped Test Stimuli.

Many real systems contain strong and weak non-linearities. Examples of mechanisms are numerous and include transducer speech response, coding distortions, and voice switching, which represent sources of weak,medium and strong non-linear distortion. If the response of a system to a particular class of stimulus is of particular interest, it is possible to reduce the problem of universal characterisation of the system. In particular if the response of a communications system to speech is required the application of a speech-like test stimulus will yield the required result. This concept is not new and the use of composite speech-like signals has been documented by DBP Telekom [8] and in the CCITT [7].

3.2 Determination of the "Natural" System Response.

In order to measure a system response typical of that caused by a particular stimulus a pseudo-noise sequence with characteristics of the desired stimulus is required. In particular a more speech-like pseudo-noise sequence with appropriate properties could yield the "speech response" of the system under test while retaining the advantages of the

MLS impulse response measurement method. A prerequisite for development of the method is the existence of sequences which retain the properties required for fast Hadamard transformation but *do not* possess a "flat" frequency response.

4.0 NON-R2 PSEUDO NOISE SEQUENCES.

4.1 Properties for Hadamard Transformation.

It is the impulsive circular autocorrelation of certain pseudo-noise sequences which results in their usefulness for system response measurement. Further an overall balance between the numbers of -1s and 1s is desirable to provide a test stimulus with zero d.c. off-set. In general, sequences which comply with randomness postulates R1 and R3 are required for system response measurement. Therefore it follows that a non-R2 sequence which retained R1 and R3 properties can still be used for the analysis, and hence it is potentially possible to adjust the frequency spectrum of the applied sequence.

4.2 Sequence Generation and Properties.

Feedback conditions exist to produce sequences of length p, $1 \leq p \leq 2^n-1$ and when 2^n-1 is a prime it is known as a Mersenne prime. Maximum-length sequences satisfy all three randomness postulates. Generation of maximum-length sequences is readily accomplished in hardware or software. Software simulation of the shift register operation is simple and retains versatility during a development stage.
Maximum-length sequences defined by the octal generating functions 435 and 453 were generated in software and their frequency response characteristics verified.

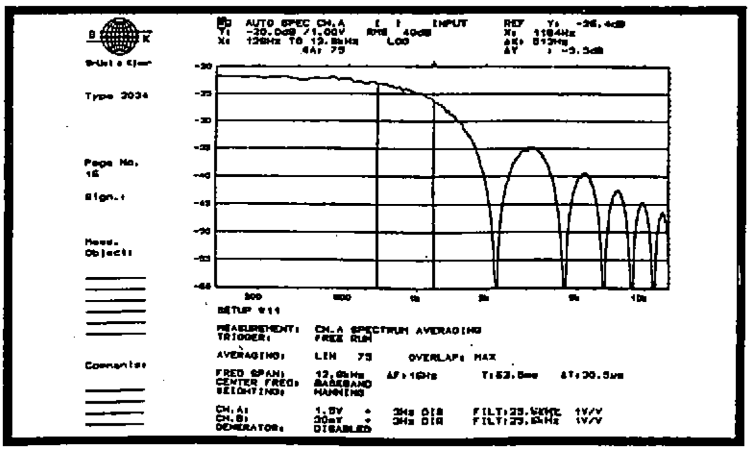

Figure 2 : Spectrum of 255 point MLS.

However, there is another family of sequences which satisfy R1 and R3 but not in general R2 called Legendre sequences. As described in [6], p is an odd prime and the Legendre symbol (n/p) is defined as:

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$$(n/p) = \begin{cases} 1 & \text{if there is an integer } x \text{ for which } x^2 \equiv n \pmod p \\ -1 & \text{otherwise} \end{cases}$$

Where $x \le p$. The sequence has the property R3 if p is of the form 4n-1.

For example, if p=7, the perfect squares mod7 are 0,1,4,2. Hence:

(0/7)=1, (1/7)=1, (2/7)=1, (3/7)=-1, (4/7)=1, (5/7)=-1, (6/7)=-1

Clearly then we may have sequences which satisfy R1 and R3 but not R2. It remains to develope generating functions which enable specific properties to be "built into" a sequence for use as a test stimulus.

Legendre sequences were generated in software using the programme kernel below.

```
100     REM INPUT D, DIM Sq(D)
110     FOR c=1 TO D
120     Sq(c) = -1
130     NEXT c
140     FOR c = 1 TO D
150     LET d = c^2
160     IF d<n GOTO 180
170     d=d-n
180     GOTO 150
190     Sq(d)=1
200     NEXT c
```

NB The Legendre sequence for the odd prime D is generated in the array Sq(*).

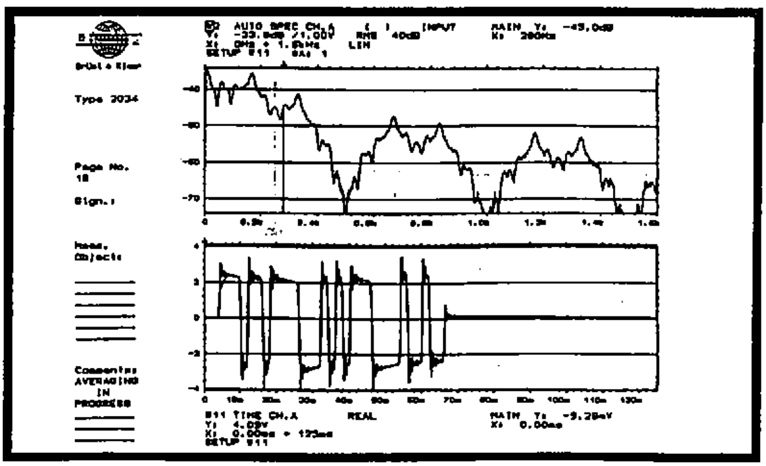

Figure 3 : Spectrum of 31 point Non-R2 Sequence.

5.0 CONCLUSION.

The existence of pseudo-noise sequences which exhibit R1 and R3 but not R2 properties indicates that it is possible to produce a spectrally shaped pseudo-noise test stimulus suitable for cross-correlation by the fast Hadamard transform method. Such a sequence could have a frequency spectrum consistent with that of male or female speech or even that of an ambient noise source required to be simulated.

There are many applications where a speech-like test stimulus is desirable including the testing of speech transducers, codecs and echo cancellers. The non-R2 sequences are more speech-like than conventional pseudo-random sequences due to their spectral content. Software generation of suitable sequences is feasible and a measurement system based on non-R2 sequences could follow.

6.0 REFERENCES.

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