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SOUND PROPAGATION IN TURBULENT PIPE FLOW

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Introduction

This paper outlines a theory of acoustic plane wave propagation in low Mach number turbulent pipe flow, and compares predicted attenuation rates with experimental results available in the literature. Further details are given in Reference (1).

Theory

Integration of the ensemble-averaged axial component of the momentum equation over the cross-section of the pipe yields for acoustic disturbances

$$\frac{Dv}{Dt} + \frac{1}{\rho_o}\frac{\partial p}{\partial x} = -\frac{a\nu}{A}\left(\frac{\partial v}{\partial y}\right)_o , \qquad (1)$$

where viscous stresses and acoustically induced perturbations in the Reynolds stress are ignored in the core of the pipe. In Equation (1) x is measured parallel to the mean flow, and $D/Dt = \partial/\partial t + U\partial/\partial x$, where U is the mean flow velocity, whose variation over the core region is neglected. Other quantities are defined as follows:

p = ensemble average acoustic pressure, assumed to be constant over a cross-section;
v = acoustic particle velocity;
ρ_o = mean density;
ν = kinematic viscosity;
a = perimeter of pipe;
A = cross-sectional area of pipe.

The derivative $(\partial v/\partial y)_o$ is evaluated at the pipe wall, y being a local coordinate normal to the wall ($y = 0$) directed into the flow, and the term on the right of Equation (1) represents a retarding force arising from coherent surface shear stress fluctuations. Similarly, the integrated continuity equation can be set in the form:

$$\frac{1}{c^2}\frac{Dp}{Dt} + \rho_o\frac{\partial v}{\partial x} = -\frac{a\rho_o\chi}{AT_o}\left(\frac{\partial T}{\partial y}\right)_o , \qquad (2)$$

where χ is the thermometric conductivity, T the acoustic perturbation temperature, T_o the mean temperature and c is the speed of sound.

The derivatives on the right hand sides of Equations (1), (2) depend on the structures of the acoustic momentum and thermal boundary layers, which in turn are governed by molecular and turbulent diffusion processes close to the wall. Howe (1) has shown that at low Mach number $M = U/c$, the momentum boundary layer profile for an acoustic wave proportional to $\exp\{i(kx-\omega t)\}$ is given by

$$v = \frac{kp}{\rho_o\omega}\left\{1 - \frac{H_o^{(1)}\left(\sqrt{\left(\frac{\kappa v_* y}{\nu} + 1\right)\frac{4i\omega\nu}{\kappa^2 v_*^2}}\right)}{H_o^{(1)}\left(\sqrt{\frac{4i\omega\nu}{\kappa^2 v_*^2}}\right)}\right\}, \tag{3}$$

in terms of the Hankel function $H_o^{(1)}(Z)$, where $\kappa \simeq 0.4$ is the von Karman constant, and v_* is the friction velocity. An analogous expression holds for the thermal boundary layer. The use of these results in Equations (1), (2) leads to the following approximate relation between the wavenumber k and frequency ω

$$k = \pm\frac{\left[\omega/c + i\alpha_p(\omega)\right]}{(1 \pm M)}, \tag{4}$$

the +/- sign being taken according as the propagation is in the +/- x-direction. The attenuation coefficient $\alpha_p(\omega)$ is real, and is given by

$$\alpha_p = \frac{\kappa v_* a}{4cA}\cdot \text{Real}\left(F\left(\sqrt{\frac{4i\omega\nu}{\kappa^2 v_*^2}}\right) + \frac{(\gamma-1)}{P}\cdot F\left(\sqrt{\frac{4i\omega\chi P^2}{\kappa^2 v_*^2}}\right)\right), \tag{5}$$

where P is the turbulence Prandtl number, γ the ratio of specific heats, and $F(Z) = ZH_1^{(1)}(Z)/H_o^{(1)}(Z)$. The wave amplitude decays as $\exp\{\pm\alpha_p x/(1 \pm M)\}$, and the dissipation occurs through the conversion of acoustic energy into turbulent fluctuations and, in the viscous sublayer, directly into heat.

Comparison with Experiment

Comparison is made in Figure 1 with the attenuation measurements in air of Ingard and Singhal (2) at a fixed acoustic frequency $\omega/2\pi$ = 1100 Hz and at various mean flow Mach numbers M. The pipe was of rectangular cross-section 1.905 x 2.223 cm^2, and in applying Equation (5) it is assumed that ν = 0.15 $cm^2.s^{-1}$; χ = 0.21 $cm^2.s^{-1}$; c = 34,000 $cm.s^{-1}$; v_* = 0.04 U. The dashed curve is the predicted attenuation when effects of heat transfer are ignored (γ = 1 in Equation (5)). Inclusion of heat transfer (γ = 1.67) greatly improves the agreement with experiment: the solid curve corresponds to a turbulent Prandtl number P = 0.8 - a value consistent with boundary layer measurements. The best fit to the experimental points at low Mach numbers is provided by the dotted curve, however, for which P = 2.5. The agreement between theory and experiment is satisfactory for M < 0.3.

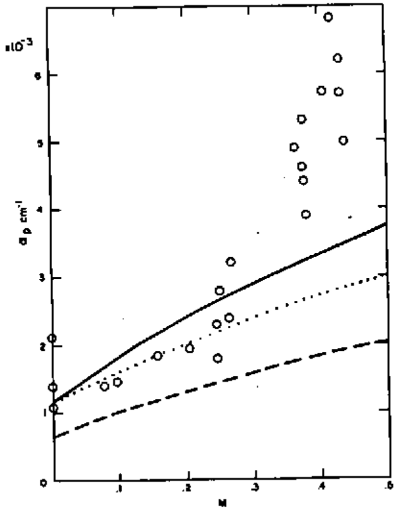

Figure 1

Further comparison, with the experiments of Ahrens and Ronneberger (3) using air in a circular pipe of diameter D = 7.5 cm, is presented in Figure 2 for M = 0 - 0.3 and a range of frequencies f = $\omega/2\pi$. There is a large spread in the data points, but reasonable agreement with theory is obtained for M ≤ 0.2. Significant differences are apparent for M = 0.3 and, as in the Ingard-Singhal experiment, this evidently sets an upper Mach number limit on the validity of the boundary layer solution given in Equation (3).

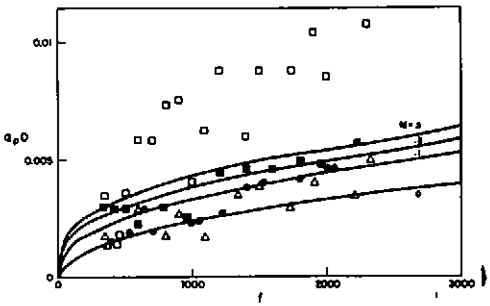

Figure 2: Experiment: ● M = 0; ▲ M = 0.1; ■ M = 0.2; □ M = 0.3; Theory ——. (P = 0.8).

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References

(1) M. S. HOWE 1978 (to appear in J. Fluid Mech.) The interaction of sound with low Mach number wall-turbulence, with application to sound propagation in turbulent pipe flow.

(2) U. INGARD and V. K. SINGHAL 1974 J. Acoust. Soc. Am. 55,535-538. Sound attenuation in turbulent pipe flow.

(3) C. AHRENS and D. RONNEBERGER 1971 Acustica 25, 150-157. Acoustic attenuation in rigid tubes with turbulent air flow.