ONE DIMENSION ANALYSIS OF RADIATING ELEMENT OF A TRANSDUCER

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ABSTRACT

A typical high power transducer consists of a number of individual radiators, each of which consists of several PZT elements, connected by adhesive layer, tail mass, head mass and quite often other materials for adjusting the bandwidth. An analysis is presented here in which a computer programme enables the computation of resonance frequency, bandwidth and projector response. The method consists in reducing all elements, other than the piezoelectric elements, to their ABCD matrix form from their equivalent circuits in order to combine various materials. The resultant ABCD matrix is again converted to a T-network and a mesh analysis is done on the resultant circuit along with equivalent circuits of the piezoelectric elements. Radiation impedence could be taken into account considering it as an element of a large plane or a cylindrical transducer. Such a programme is also useful in selecting suitable material for adjusting the bandwidth and projector response and also for measuring the sound velocity in different materials.

APPLICATION OF B. B. BAUER'S TRANSFORMER COUPLING METHOD TO THE EQUIVALENT CIRCUITS OF UNDERWATER TRANSDUCERS

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1. INTRODUCTION

The use of equivalent circuits in the modelling of sonar transducers is popular for two main reasons:-

i) There is usually a direct physical correspondence between a circuit element and a transducer component, which results in a better understanding of both the mode of operation of the transducer and of the part played in that operation by the transducer's various parts.

ii) The powerful methods of circuit analysis and synthesis can be used with the equivalent circuit to determine performance parameters, such as resonant frequency and Q-factor, and to determine how they depend upon the transducer components.

The main limitation of the equivalent circuit approach is that it is only readily applicable to transducers with one-dimensional modes of vibration. In practice this is less restrictive than at first it might appear because often the effect of lateral boundaries and lateral vibrations are explicitly accounted for in the sound speed along the direction of interest. So in many cases an accurate one-dimensional equivalent-circuit model can be developed.

In cases where a truly two- or three-dimensional model is required then the numerical methods, such as finite-elements, takeover from equivalent circuits. In the longer term it is probable that most transducers will be designed using these numerical methods, however in the shorter term the advantages of equivalent circuits discussed above will ensure an interest in them continues.

Whilst such an interest does exist it is worthwhile attempting to refine the method by which these circuits are created. The purpose of this present paper is to show how a method developed by Bauer [1], for the generation of impedance-analogy equivalent circuits for complicated mechanical structures, can be extended and applied to sonar transducers. The central feature of the method is the introduction of ideal transformer couplings between all elements to enable the circuit to be found. The attractiveness of the method is the ease by which circuits can be generated directly from the mechanical structure.

As well as being useful as a design tool the author has also found this technique valuable as an educational aid.

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2. EQUIVALENT CIRCUITS OF MECHANICAL AND PIEZOELECTRIC ELEMENTS

As is well known in an impedance-analogy treatment mass is represented by inductance and compliance by capacitance. Now a free mass which is moving at a constant velocity is a single terminal device, whereas its equivalent, an inductance, is a two terminal device; effectively the mass in this situation is behaving as a velocity junction. So, how can this inductance be made to properly represent this junction? The answer proposed by Bauer [1] is to represent the free mass by a short-circuited inductance. In this way the current, which is of course analogous to the velocity, is fixed everywhere in the circuit and so a current junction is created. The problem then is that any sources of force applied to the mass and any other mechanical elements which may be interacting with it need to be coupled in some way. Bauer proposed this coupling be done with ideal transformers. So, for example, the equivalent circuit of a mass, M, driven by a force, F, which produces a velocity u, would be as shown in Fig.1(a). Following Bauer's symbolism, which will be used throughout this paper, the ideal 1:1 transformer in Fg.1(a) is represented by the pair of adjacent parallel lines with a 'squashed' circle intersecting them.

Now a free compliance, such as a spring, has two ends and it is the difference in velocities between them that determines the spring dynamics. So, in a similar manner to the mass, the equivalent circuit for the spring, proposed by Bauer, is as shown in Fig.1(b), where u_1 & u_2 are the terminal velocities of the spring and C_m is the spring compliance. Mechanical resistance, R_m, is treated in a similar way and so its representation is as shown in Fig.1(c).

This representation may be extended to distributed mechanical components. For example, Fig.2(a), gives the usual T-network transmission-line equivalent for an acoustic wave in a medium of finite length, l, where Z_0 is the mechanical characteristic impedance and γ is the propagation constant. The low-frequency, lossless, lumped version of this becomes as shown in Fig.2(b), where M is the mass of the short length of medium and C_m is its compliance, eg. [2].

A piezoelectric element, which can be represented by a Mason type of equivalent circuit, may also include coupling transformers at its mechanical terminals, as indicated in Fig.3.

3. EQUIVALENT CIRCUITS FOR MECHANICAL STRUCTURES

The strategy whereby equivalent circuits are constructed from the mechanical structure can best be illustrated by reproducing examples from Bauer's paper. The first example is the mechanical system of Fig.4, which comprises four masses and four springs. Each component in the mechanical structure is replaced directly in the equivalent circuit by the appropriate model given in Section 2; this leads to Fig.5(a). This process is logical and easily understood.

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The next step is the simplification of Fig.5(a) by the removal of the coupling transformers. The rule for doing this is straightforward, as can be seen by reference to Fig.5(a). Points such as a and b can be connected to points c and d respectively and the transformer removed, provided in the process no cicuit element is short-circuited. So applying this rule successively the circuit of Fig.5(b) results. The removal of the final two transformers cannot be carried out because now the mass M_3 would be shorted, however the mass can be redrawn on the opposite branch of its loop without changing the behaviour of the circuit. Removal of the last two transformers can now be accomplished to give the final circuit of Fig.5(c).

In the generation of the equivalent circuit of even more complicated mechanical structures it may not be possible to remove all of the coupling transformers directly, without additional circuit analysis, such an example is given by Bauer.

The second example taken from Bauer is the simple mechanical oscillator of Fig.6(a), which has been slightly modified in this present treatment to include the effect of sound radiation from the mass, M, where u is its velocity. F is the applied force and F_r is the reaction force of the boundary medium associated with radiation. The spring, whose compliance is C_m , and the mechanical resistance, R_m , are both connected to a rigid boundary.

The circuit of Fig.6(b) directly follows. Removing the transformers according to the rules previously discussed finally results in the expected series resonant circuit of Fig.6(c), in which the reaction force has been accounted for by the radiation impedance Z_r.

This equivalent circuit has been obtained directly by the logical replacement of each mechanical component by its equivalent electrical analogue, without the need to resort to any describing differential equation.

In the next section the application of this method to some sonar transducer examples will be considered.

4. SONAR TRANSDUCER APPLICATIONS

Consider a simple sandwich transducer whose cross-section is shown diagrammatically in Fig.7. The structure comprises; a head mass, M_h , a tail mass M_t, a ceramic stack, a tensioning bolt of mass M_b and compliance C_b, a nut of mass M_n and a decoupling washer of compliance C_w.

Applying the methods outlined in the previous sections the equivalent circuit for this structure, obtained by inspection, is shown in Fig.8(a). For convenience the usual lumped version for the equivalent circuit of the piezo-ceramic stack with its bonds has been used, where C_s is the total spacer/bond compliance, which includes those between the stack and the head and tail masses. This equivalent circuit for the stack can be obtained using the coupling transformer method applied to each stack component, but the removal of the transformers presents no difficulties and so the

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development of the stack equivalent proceeds in a conventional manner. This development has not been included in this paper because of lack of space. The tail is assumed to be air-backed and so the usual short-circuit boundary condition is applicable. The head is assumed to be radiating into water and so the radiation impedance is included.

Now, the removal of the transformers proceeds in a straight forward manner until the stage shown in Fig.8(b) is reached. Now the bolt and nut masses must be considered transferred to the opposite branches of their respective loops so that the remaining transformers can be removed. The circuit of Fig.8(c) finally results. It is worth noting how easily the bolt, nut and decoupling washer are included in this methodical derivation of the equivalent circuit.

The next example is the double-headed sandwich transducer, which is shown schematically in Fig.9. Compliant elements, shown here as three thin rods, couple the two heads together. These compliant rods and outer head together form a mechanical filter which has the effect of widening the bandwidth of the original simple sandwich transducer.

Using the coupling-transformer method the equivalent circuit for this mechanical filter is easily found; it is shown in Fig.10(a) coupled to the head mass M_h of the original sandwich. M_1 is the mass of the outer head, M is the mass of each rod and C is the rod compliance.

Removal of some of the transformers results in the circuit of Fig.10(b). This circuit is now changed into the form of Fig.10(c), so that the final pair of transformers may be removed. This modification is achieved by transferring the masses associated with the bottom rod to the opposite branches of the appropriate loops and similarly transferring the masses associated with the middle rod to the top branch and then combining them with the masses of the top rod. The final transformers can now be removed giving the circuit shown in Fig.10(d). Finally the two masses in the bottom branches can be transferred to the top branches and combined with the other component rod masses to give the circuit of Fig.10(e). Of course in this case Fig.10(e) could be obtained more directly by replacing the three rods by a single rod of three times the area of one of them. Then the mass of the single rod becomes $3M$ and its compliance becomes $\frac{C}{3}$; Fig.10(e) then immediately follows.

In other sonar transducer examples, such as those which comprise multi-layered structures, the application of the transformer coupling technique is trivial in that the transformers can be removed directly leaving a number of circuits in tandem. An example of this is of course the equivalent circuit of the stack in the sandwich transducer.

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5. CONCLUSIONS

The main conclusion to be drawn is that the use of ideal 1:1 coupling transformers between the equivalent circuits of the mechanical components, as proposed by Bauer, provides a logical and easily understood method of generating the equivalent circuits of some types of sonar transducers. It is considered that the method provides a good physical insight into the form of the circuit in relation to the transducer being modelled.

6. ACKNOWLEDGEMENT

The author is indebted to J.R.Dunn for reading the manuscript and making some useful suggestions.

7. REFERENCES

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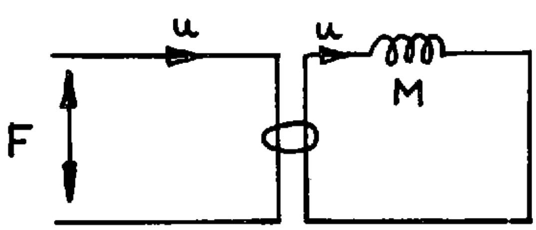


Fig.1(a)

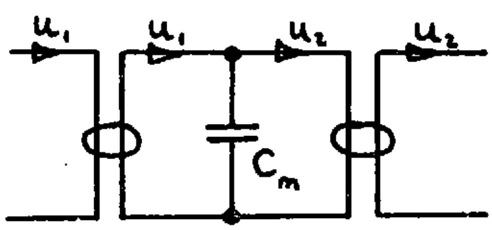


Fig.1(b)

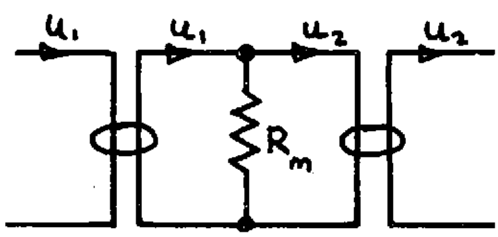


Fig.1(c)

EQUIVALENT CIRCUITS

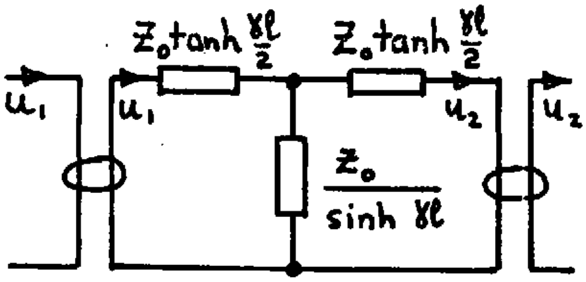


Fig.2(a)

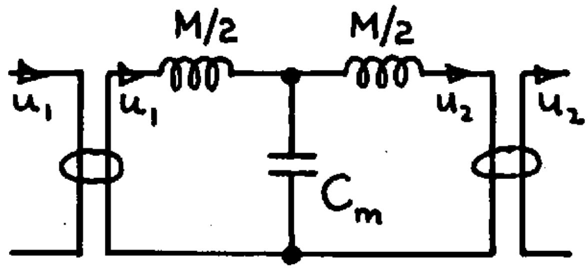


Fig.2(b)

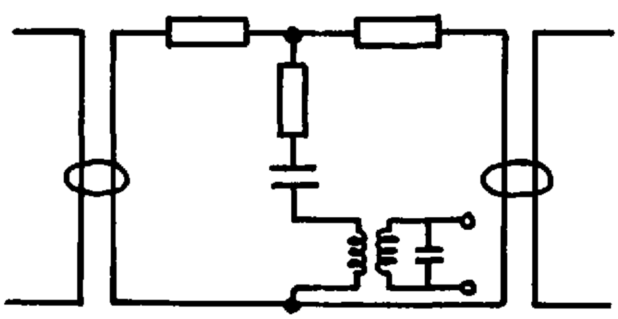


Fig.3

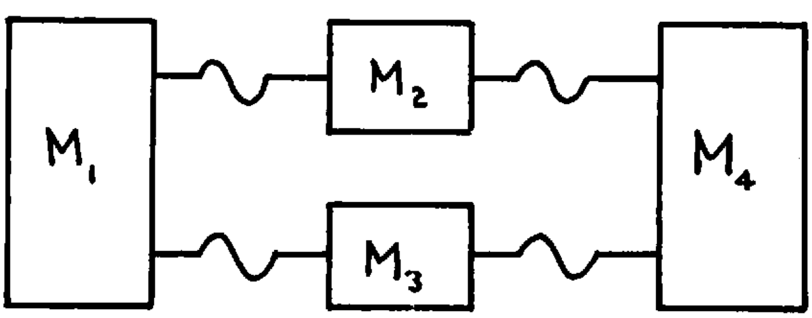


Fig.4

EQUIVALENT CIRCUITS

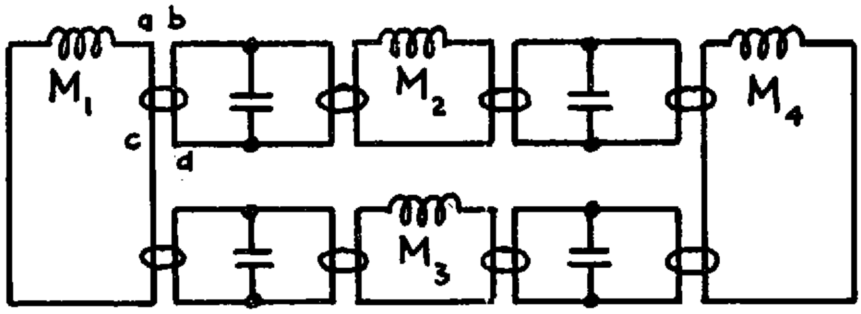
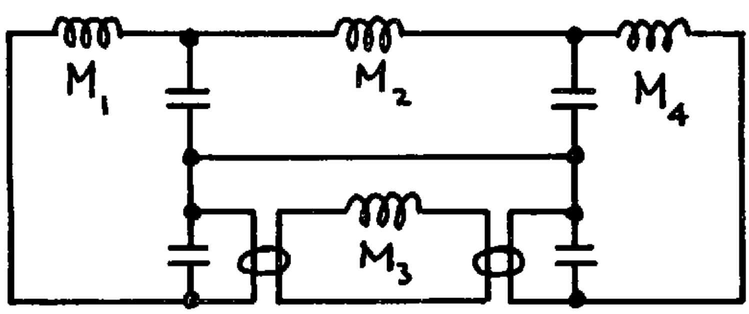


Fig.5(a)



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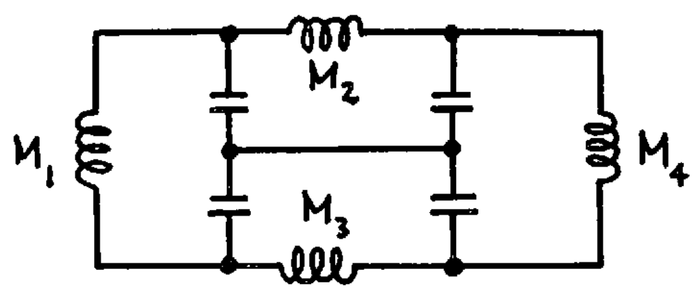

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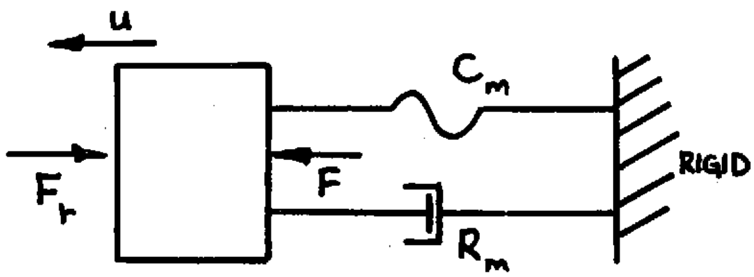

Fig.6(a)

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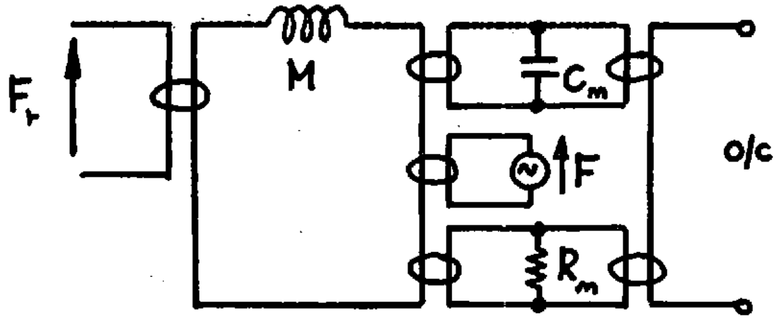


Fig.6(b)

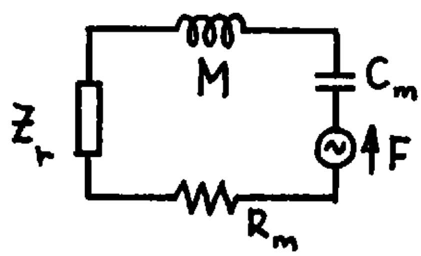


Fig.6(c)

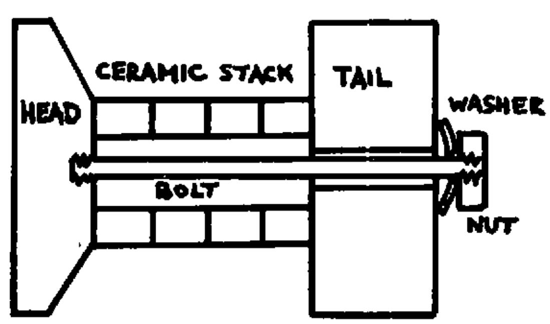
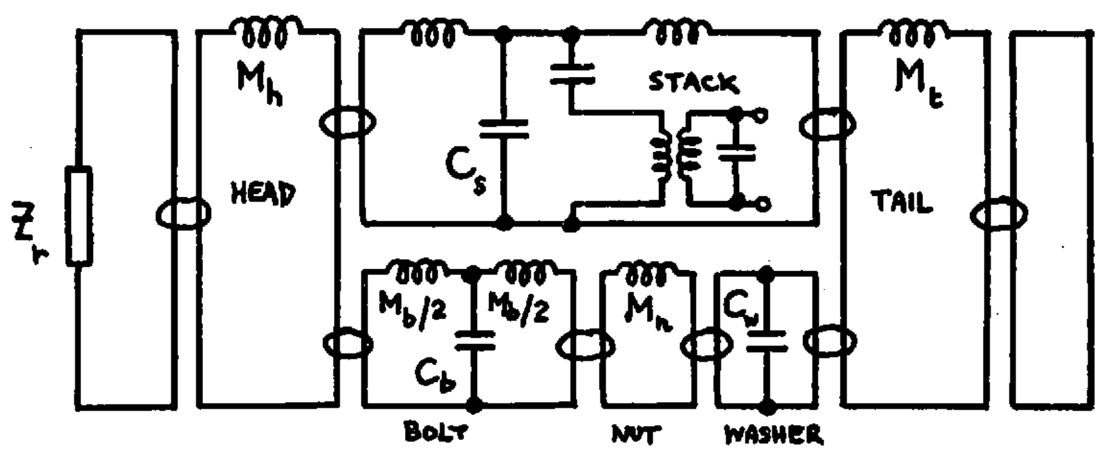


Fig.7

Fig.8(a)



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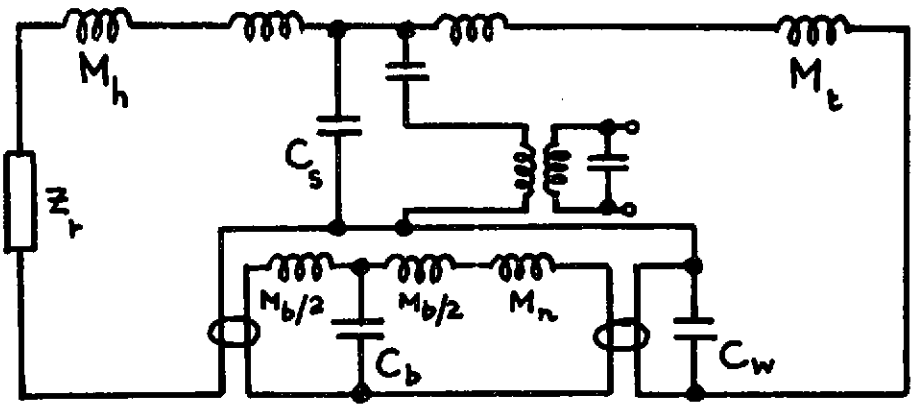

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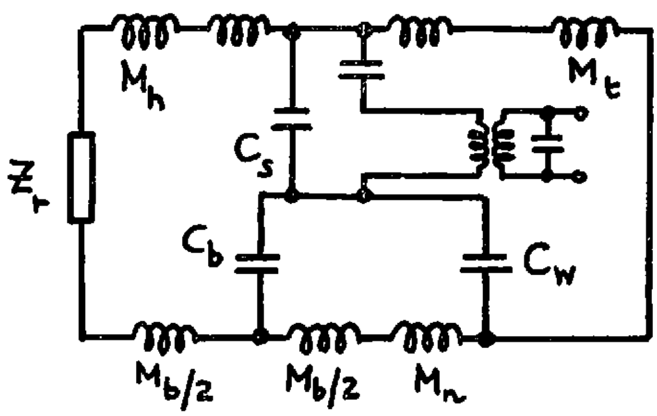

Fig.8(c)

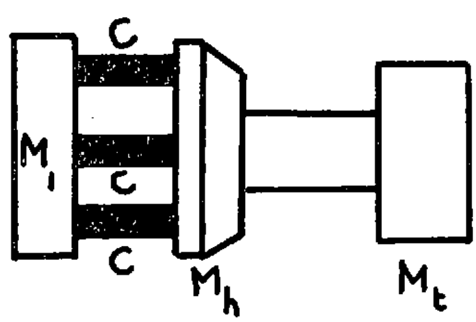

Fig.9

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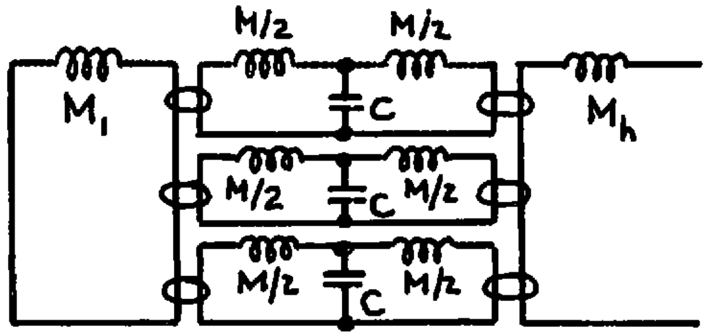
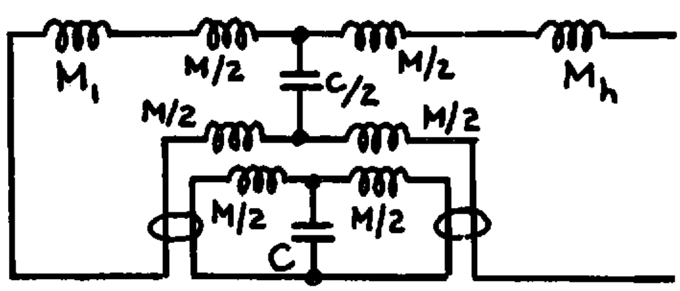


Fig.10(a)



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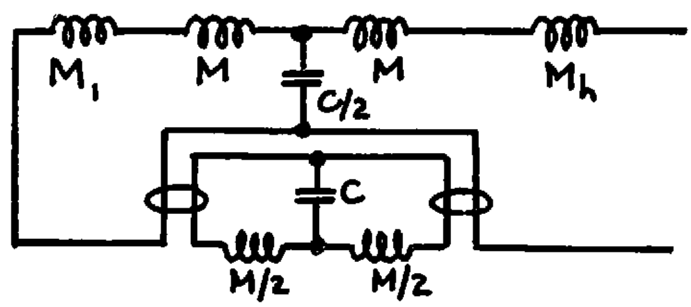
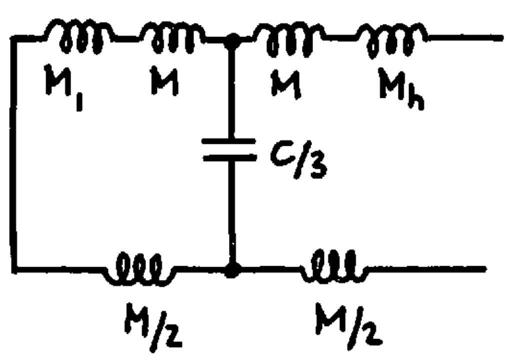


Fig.10(c)



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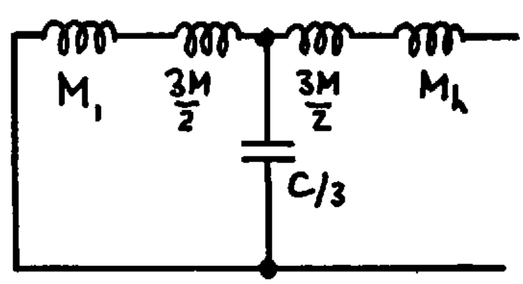

Fig.10(e)