IDENTIFICATION OF DAMPED FOUNDATIONS IN ROTATING MACHINERY

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In earlier work, a procedure for identifying the foundation of a rotor bearing foundation system (RBFS) via modal parameters using rotor and foundation motion as input data was outlined. The procedure was evaluated numerically using a RBFS comprising an unbalanced flexible rotor running in fluid film bearings fixed to a damped flexibly supported rigid foundation block. While reasonable identification was achieved when the input data was truncated to 2 digit accuracy to simulate measurement error in practice, the iterative approach proved problematic and not conducive to investigating the effects of measurement speeds (input data) on identification accuracy. In this paper, a simpler approach is outlined. Comparison of the identification obtained via this simpler approach using the same input data showed no significant improvement, predicating the need for further investigations to minimise the effect of round off errors.
Keywords: system identification, rotor foundation, damping

1. Introduction

Modelling the foundations of rotating machinery is an invaluable asset for efficient operation and balancing [1-3]. There are two common procedures for such modelling. The first uses appropriate experimental vibration measurements to identify an equivalent foundation (a foundation which reproduces the system unbalance response over the speed range of interest); the other models the foundation by finite elements, this latter approach being limited by difficulty of modelling [2]. This paper follows the former approach and is concerned with developing a procedure which is applicable to existing turbomachinery installations without requiring rotor removal.

The problem is to identify appropriate mass, damping and stiffness matrices or appropriate modal parameters for this equivalent foundation. Such an identification procedure invariably requires as input data the forces transmitted to the foundation via the bearing pedestals as well as the motion of the foundation at appropriate locations. Provided the dynamic properties of the rotor are known (not regarded to be a significant problem), such force data can be obtained from existing performance monitoring instrumentation, viz. displacement transducers measuring the relative motion between the rotor journals and the bearing housings, and accelerometers measuring the absolute motion of the foundation at the housings. Such a rotor-model-based force determination approach, which relies on knowledge of the rotor unbalance and on the dynamic properties of the rotor, has been experimentally proven to give satisfactory identification of a simple flexible pedestal bearing support in a laboratory test rig [4]. Thus, it is assumed for the foundation identification procedures to be developed below, that all externally applied dynamic forces to the foundation at the housing supports (due to rotor unbalance) are available as input data, together with all required foundation motion measurements. This is so even if the actual rotor unbalance is unknown; in which case all measurements need to be repeated with an added known unbalance [4].

Assuming that the foundation damping can be approximated by a diagonalisable damping matrix, earlier work has successfully identified, via numerical experiments, an equivalent foundation for a RBFS comprising an unbalanced flexible rotor running in hydrodynamic bearings which are fixed to a damped flexibly supported rigid block (a six degrees of freedom (DOF) foundation) [5]. This system is shown schematically in Figure 1. Though reasonable identification was achieved when the input data was truncated to 2 digit accuracy (to simulate measurement error in practice), the iterative approach proved problematic and not conducive to investigating the effects of measurement speeds (input data) on identification accuracy. In this paper a simpler approach, which avoids iteration, is outlined and compared to the earlier approach. The identification problem for the more general case, when the foundation damping matrix need only be assumed to be symmetric, is significantly more difficult [6] and is left for future work.

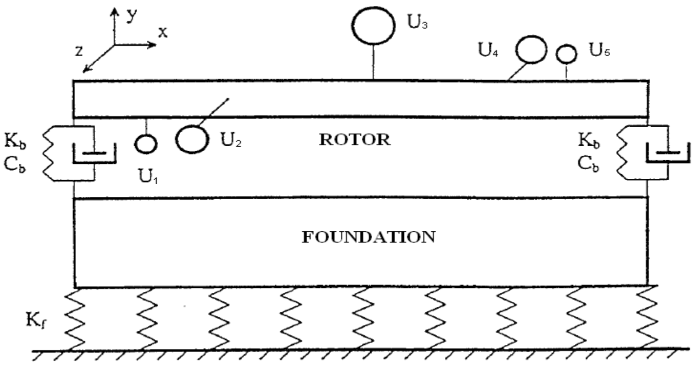

Figure 1: Unbalanced rotor mounted via two hydrodynamic bearings on damped flexibly supported rigid foundation block.

2. Notation

$\boldsymbol{A}$	transformation matrix with elements a_{ij}
$\boldsymbol{C}$	foundation damping matrix
$\boldsymbol{c}$	diagonal modal damping matrix with diagonal elements c_k
$\boldsymbol{f}$, $\boldsymbol{F}$	vector of forces acting on foundation, vector of complex amplitudes thereof
$\boldsymbol{K}$	foundation stiffness matrix
$\boldsymbol{k}$	diagonal modal stiffness matrix with diagonal elements k_k
$\boldsymbol{M}$	foundation mass matrix
$\boldsymbol{m}$	diagonal modal mass matrix with diagonal elements m_k
m	number of measurement data speed sets
n	number of foundation degrees of freedom
$\mathbf{Q}$	vector of complex amplitudes of modal displacements of the foundation
$\boldsymbol{x}$, $\mathbf{X}$	vector of foundation displacements, vector of complex amplitudes thereof
$\boldsymbol{\Phi}$, $\boldsymbol{\Phi}_k$	foundation modal matrix with elements ϕ_{ij}, k^{th} column vector of $\boldsymbol{\Phi}$

$\boldsymbol{\lambda}$	diagonal matrix of foundation eigenvalues with diagonal elements λ_k
Ω	excitation frequency, rotor speed
ω_k	k^{th} undamped natural frequency of foundation = $\sqrt{\lambda_k}$
$\boldsymbol{\xi}$	diagonal normalised modal damping matrix with diagonal elements ξ_k
ζ_k	k^{th} modal damping ratio

3. Theory

For an unbalanced RBFS such as schematically shown in Figure 1, the system excitation is synchronous with excitation frequency Ω. The equations of motion of a general n DOF foundation may be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}\,, \tag{1}$$

where ***M, C*** and ***K*** are n by n symmetric matrices. The elements of $\boldsymbol{x}$ are the n independent displacements chosen to coincide with convenient measurement locations which include the excitation force application points. The elements of $\boldsymbol{f}$ are the external excitation forces, acting at selected locations (for the foundation in fig. 1 these would be the forces transmitted to the foundation at the bearing supports). Assuming periodic response with fundamental frequency Ω, one can write [4]

$$-\Omega^2\boldsymbol{M}\boldsymbol{X} + i\Omega\boldsymbol{C}\boldsymbol{X} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{F}\,. \tag{2}$$

The elements of $\boldsymbol{X}$, viz. $X_1, X_2, \ldots, X_n$, are obtained from foundation motion measurements whereas the elements of $\boldsymbol{F}$, viz. $F_1, F_2, \ldots, F_n$, are calculated from the rotor model, the rotor unbalance and rotor and foundation motion measurements at the bearing stations [7]. Letting

$$\boldsymbol{X} = \boldsymbol{\Phi}\boldsymbol{Q}\,, \tag{3}$$

and multiplying through by $\boldsymbol{\Phi}^T$, eqn (2) becomes

$$\left[-\Omega^2\boldsymbol{m} + i\Omega\boldsymbol{c} + \boldsymbol{k}\right]\boldsymbol{Q} = \boldsymbol{\Phi}^T\boldsymbol{F}\,. \tag{4}$$

Defining the transformation matrix $\boldsymbol{A}$ as

$$\boldsymbol{A}^T = \boldsymbol{\Phi}^{-1}, \tag{5}$$

eqn (4) can also be written as

$$\left[-\Omega^2\boldsymbol{I} + i\Omega\boldsymbol{\xi} + \boldsymbol{\lambda}\right]\boldsymbol{A}^T\boldsymbol{X} = \boldsymbol{m}^{-1}\boldsymbol{\Phi}^T\boldsymbol{F}\,. \tag{6}$$

Eqn (6) yields the n identification equations (k=1,…n)

$$\left(-\Omega^2 + i\Omega\xi_k + \lambda_k\right)\sum_{j=1}^{n} a_{jk}X_j - \sum_{j=1}^{n}\phi_{jk}F_j / m_k = 0\,. \tag{7}$$

Since X_j and F_j are complex quantities, eqn (7) actually embodies the two equations

$$\Omega\xi_k\sum_{j=1}^{n} a_{jk}X_j^I = \left(\lambda_k - \Omega^2\right)\sum_{j=1}^{n} a_{jk}X_j^R - \sum_{j=1}^{n}\phi_{jk}F_j^R / m_k \tag{8}$$

and $$-\Omega\xi_k \sum_{j=1}^{n} a_{jk} X_j^R = \left(\lambda_k - \Omega^2\right)\sum_{j=1}^{n} a_{jk} X_j^I - \sum_{j=1}^{n} \phi_{jk} F_j^I / m_k \,, \tag{9}$$

where the superscripts R and I denote real and imaginary parts.

The parameters to be identified in the k^{th} identification equation now are: $\lambda_k = k_k/m_k = \omega_k^2$, ξ_k, m_k and a_{jk} ($j = 1,..,n$). The ϕ_{jk} ($j= 1,..,n$) are automatically identified once $\boldsymbol{A}$ has been fully identified. Because the mode shape elements are relative values, so are the $a_{jk,}$ and one can assign an arbitrary value to any one of the a_{jk} values. Hence, the number of unknown parameter values per identification equation is $(n+2)$; and, depending on the solution approach, the minimum number of speed data sets needed to solve the resulting simultaneous equations, obtained by substituting for Ω and the corresponding X_j and F_j into eqns (8) and (9), is also at most $(n+2)$. These simultaneous equations are nonlinear, and an effective solution strategy is required to find the above parameters. Once found, one has, in effect, obtained an equivalent foundation. Thus, one can find $\boldsymbol{M}$ from

$$\boldsymbol{M} = \boldsymbol{AmA}^T \,, \tag{10}$$

with similar expressions for $\boldsymbol{K}$ and $\boldsymbol{C}$.

4. Solution strategies

The first solution approach was that adopted in ref. [5]. It involved further manipulation of eqns (8) and (9) to eliminate ξ_k, giving an alternative set of k identification equations (k=1,….,n)

$$\left(\lambda_k - \Omega^2\right)\sum_{j=1}^{n} a_{jk}\left(X_j^R + S_k X_j^I\right) - \sum_{j=1}^{n} \phi_{jk}\left(F_j^R + S_k F_j^I\right)/ m_k = 0 \tag{11}$$

where $$S_k = \sum_{j=1}^{n} a_{jk} X_j^I \Big/ \sum_{j=1}^{n} a_{jk} X_j^R \,. \tag{12}$$

Note that eqn (11) is valid for all ξ_k. Hence, one can find all the other parameters first. However, eqn (11) is strongly nonlinear and an iterative approach was adopted. Assuming an initial $\boldsymbol{A}$ matrix, one can evaluate the S_k and the ϕ_{jk} at any speed for which measurement data are available, so that the evaluation of eqn (11) for m speeds ($m \geq 7$), for some arbitrary guessed value for λ_k, results in m homogeneous linear equations in the seven unknowns a_{jk} ($j=1,\ldots.6$) and m_k. Least squares regression was used to reduce the number of equations to seven. Nontrivial solutions exist only when the determinant of the resulting coefficient matrix is zero; and this will (presumably) occur only when the guessed value for λ_k corresponds to an eigenvalue. Numerically, this is tantamount to finding the λ_k at which the determinant has a minimum. Having found the $\lambda_{k,}$, one can find the a_{jk} for each mode in turn, again using eqn (11), but now assuming any one of the a_{jk} to be unity and solving the resulting non homogeneous simultaneous linear equations in six unknowns, again using least squares regression if $m>6$. Once the a_{jk} have been found for each mode, one has an updated $\boldsymbol{A}$ matrix and the process is repeated, until there is no significant change in successive updated $\boldsymbol{A}$ matrices. Here, the iterations were continued until there was no change in the fifth significant digit in any of the a_{jk} or the $m_{k.}$ Having found the above parameters for all k modes, one can then determine the parameters ξ_k. Several ways for finding these are possible. The procedure adopted was to recognise that with the already identified modal parameters one can decouple the equations of motion, evaluate the mobility functions for the k modes and then find the damping for the k^{th} mode by plotting the mobility function in the complex plane [8]. Though simple in concept, there is no guarantee that such a simple iteration of the matrix $\boldsymbol{A}$ will converge to the correct solution or converge at all; and various constraints or inner iteration loops were found to be necessary to achieve convergence. Also, the better the initial choice of $\boldsymbol{A}$, the greater the likelihood of, and the faster the convergence.

Here the initial ***A*** was chosen as the approximate solution obtained when solving for the parameters using eqn (7), ignoring the fact that the ϕ_{jk} are actually functions of ***A*** and assuming zero damping.

The second solution approach concentrates on using eqns (8) and (9) to solve for the parameters. Assuming that the ϕ_{jk} $/m_k=b_{jk}$ are independent unknowns, the evaluation of eqns (8) and (9) for m speeds ($m \geq 7$), for some arbitrary guessed values for λ_k and ξ_k, results in 2m homogeneous linear equations in at most twelve unknowns a_{jk} $(j=1,....6)$ and b_{jk} $(j=1,....6)$. Least squares regression is used to reduce the number of equations to as many equations as there are unknowns. Nontrivial solutions exist only when the determinant of the resulting coefficient matrix is zero. Numerically this is tantamount to finding the minimum of the determinant which is a function of the two variables λ_k and ξ_k. On finding λ_k and ξ_k, one can find the a_{jk}, again using eqns (8) and (9) , but now assuming any one of the a_{jk} to be unity and solving the resulting non homogeneous simultaneous linear equations in at most eleven unknowns, again using least squares regression. Once the a_{jk} have been found, one has found the ***A*** matrix. The evaluation of the m_k values is then reasonably straightforward. Note that this approach does not involve iteration of the ***A*** matrix but is likely to be more prone to round off errors as one needs to evaluate the determinants of larger matrices.

5. Numerical experiments

The same damped flexibly supported rigid foundation block that was previously identified in ref. [5] was selected to evaluate the simplified identification procedure. This foundation has exactly six DOF. Figure 2 shows the measurement locations on the upper surface of the block, allowing for the application of the external force ***f*** in the x_2 and x_5 directions at the connection point C_1 and in the x_3 and x_6 directions at the connection point C_2. The block mass is 502.49 kg. With respect to the centre of mass, the connection points C_1 and C_2 are at (-L/2, H/2, 0) and at (3L/8, H/2, 0) respectively.

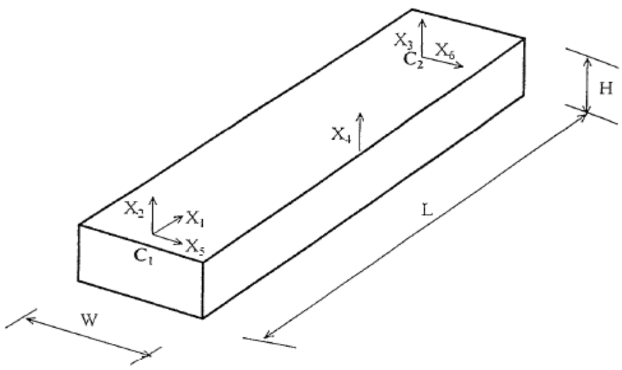

Figure 2: Measurement locations and directions (W = 317.5mm, H = 158.75mm, L = 1270mm).

The foundation support stiffnesses and locations as well as rotor and bearing details for the RBFS in Figure 1 are as in ref. [7]. Hence, for specified foundation modal damping ratios, one can evaluate for this foundation the ***M***, ***C, K***,***Φ, λ*** and ***m*** directly; and for specified unbalance, one can evaluate the steady state system response of the RBFS in Figure 1. Using in-house software, the response was calculated over the speed range of 300 to 1450 rad/s in steps of 50 rad/s with unbalances of $U_1 = U_5 = 10^{-4}$ kg.m, $U_2 = U_4 = 10^{-5}$ kg.m and $U_3 = 10^{-6}$ kg.m,. The input data 'measurements' were then the response amplitudes ***X*** and the force amplitudes ***F*** at these speeds, giving 24 input data sets. Identifications were carried out using both of the solution approaches described in Section 4, with the 'measurements' truncated to either 5 or 2 significant digits. The 5 digit input data served to evaluate the validity of the identification procedures and to define the achievable accuracy of the adopted computational procedure by minimising the effect of measurement and round off errors. The 2 digit input data better reflected attainable field measurement accuracy.

Table 1: Actual and identified undamped natural frequencies, damping ratios and modal masses (a – actual; 1 – first solution approach [5]; 2 – second solution approach)

Mode	1	2	3	4	5	6
ω_a (rad/s)	765.0	626.7	543.5	1259.	823.6	1002.
$\zeta_a \times 10^3$	8.000	7.000	11.00	10.00	12.00	9.000
m_a (kg)	386.4	3.293	22.13	286.3	54.30	.07426
ω_1 (rad/s)	770.1	628.0	543.0	1260.	823.6	1004.
$\zeta_1 \times 10^3$	8.183	5.112	10.93	9.941	10.35	8.638
m_1 (kg)	382.7	3.431	17.68	291.6	51.76	.01862
ω_2 (rad/s)	765.4	625.7	546.9	1258.	822.2	1003.
$\zeta_2 \times 10^3$	8.508	8.433	11.86	10.04	11.95	8.630
m_2 (kg)	390.6	3.066	19.11	286.6	52.95	.07046

6. Results and discussion

The identified parameters obtained with both solution approaches using 5 digit input data agreed with the actual parameters to four significant digits apart from minor deviatons in the fourth digit in a few of the modal masses and a few of the damping ratios. This proved the soundness of the identification procedures in principle. Table 1 compares the identified undamped natural frequencies, modal masses and modal damping ratios obtained with both solution approaches with the actual ones using 2 digit input data. Agreement is fair. Particularly disturbing are the 0.7% error in the natural frequency of the first mode and the 27% error in the modal damping ratio of the second mode when using the first approach; and the 0.6% error in the natural frequency of the third mode when using the second approach. Further work is apparently warranted to minimise build up of round off errors. Space restrictions do not allow for the display of the identified modal matrix elements. However, the agreement between the actual modal matrix and identified modal matrix when using 2 digit data accuracy was again fair, regardless of the solution approach.

The effect of these errors on the suitability of the now obtained equivalent foundations can be seen in Figures 3, 4 and 5, where the predicted unbalance response amplitudes at the left end, quarter way along and halfway along the rotor are compared with the actual ones. The actual responses and those obtained using 5 digit input data are indistinguishable. The agreement between the actual responses and those obtained using 2 digit input data are not quite so good, there being errors of around 4% at some of the peaks or troughs. It appears that the identification accuracy using the far simpler second solution approach is as good as that obtained with the first approach.

7. Summary of conclusions

The proposed identification techniques are valid in principle and were correctly implemented, for when the input data is accurate to 5 digits, there is excellent agreement between the actual and identified foundation modal parameters; and the corresponding equivalent foundation can accurately reproduce the response of an unbalanced RBFS over the speed range of interest.

When the input data is accurate to 2 digits the agreement between the actual and identified foundation modal parameters is not quite so good; and the corresponding equivalent foundations cannot reproduce accurately all the response values of an unbalanced RBFS over the speed range of interest, predicating the need for further investigations to minimise the effect of round of errors.

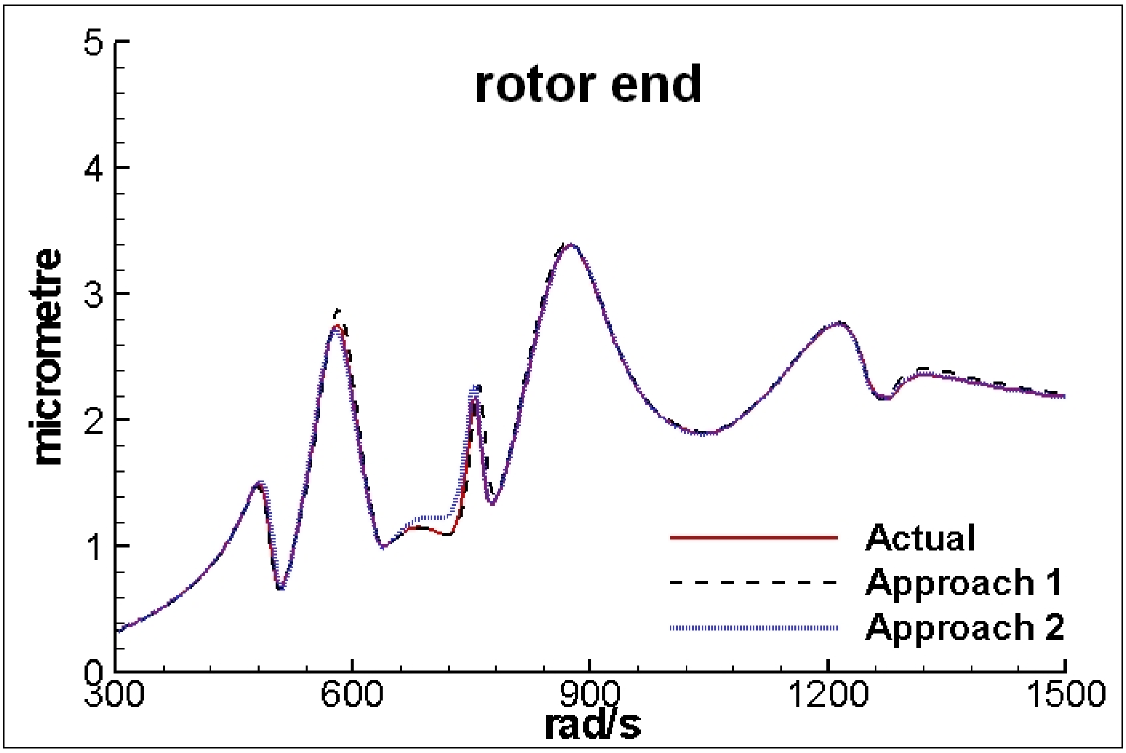

Figure 3: Comparison of unbalance responses at left end of rotor

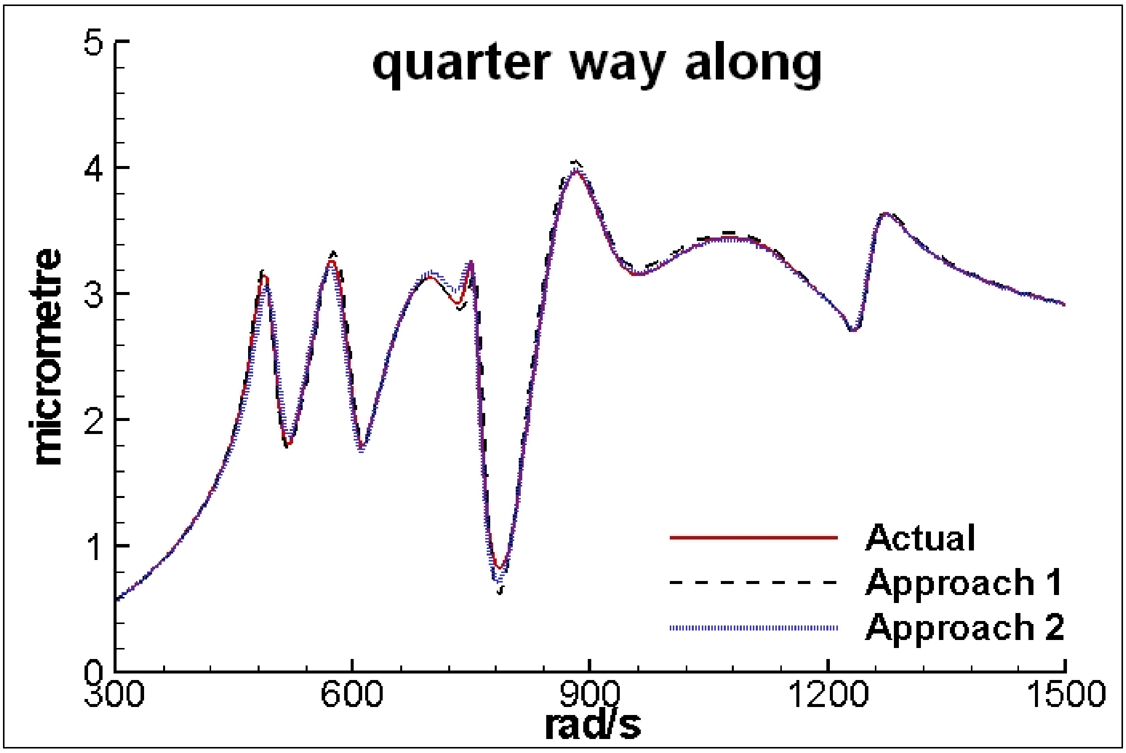

Figure 4: Comparison of unbalance responses quarter way along from left end of rotor

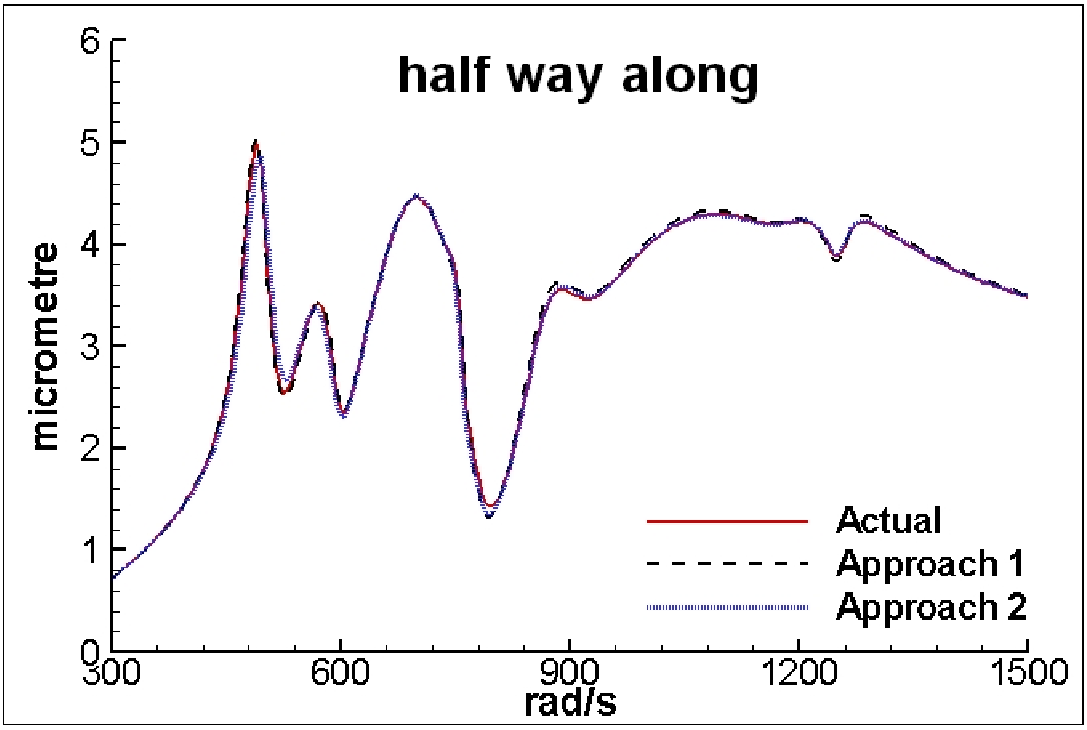

Figure 5: Comparison of unbalance responses half way along the rotor.

It appears that the identification accuracy using the simpler second solution approach, which does not involve iteration, is as good as that obtained with the first approach.

Once the solution procedures are improved so as to achieve good identification with 2 digit input data, the proposed procedure promises to be applicable in the field as it can utilise directly measurements available from existing monitoring instrumentation.

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