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INTERACTION OF SOUND WITH JETS

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The general phenomenon in which we are interested is the interaction of sound with jets, or more precisely the interaction of sound with the vortex sheet separating two fluids in relative motion.

Three particular cases will be discussed. The first is that of an infinite plane vortex sheet and a line source of sound parallel to it and orthogonal to the direction of flow. The second is a two dimensional jet bounded by plane vortices, again with a line source, and finally an infinite cylindrical jet containing a point source. In each case the Mach number of the flow is M and we assume a harmonic source having time dependence $\exp(i\omega t)$ or $\exp(ikat)$ with $\omega = ka$.

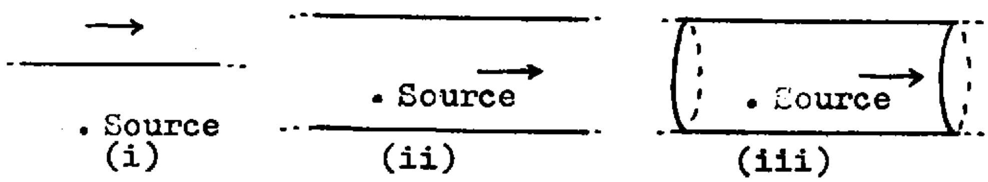

Fig. 1

A detailed analysis for the plane vortex sheet has been published for the case $M<1$ (Jones & Morgan, Proc. Camb. Phil. Soc. 1972 72, 465) and a paper dealing with $M>1$ is in Press. Because of the relative simplicity of the problem the main features of the mathematical treatment and difficulties will be reviewed first for this case.

The mathematical form of all three problems is to solve:

$$\begin{array}{lll} \text{In the flow} & \nabla^2\phi - \left(ik + M\frac{\partial}{\partial x}\right)^2\phi & = 0 \\ \text{In ambient fluid} & \nabla^2\phi + k^2\phi & = \delta(z - z_0) \end{array} \qquad (1)$$

together with the appropriate linearised boundary conditions. (In this case the source is assumed to lie in the stationary fluid)) A formal solution to this problem can in each case be obtained in a straightforward way using Fourier-Laplace transforms. This corresponds to taking a solution of the form

$$\phi_A = \int_C \Phi(u, y)\, e^{-ikux}\, du \qquad (2)$$

where for k real and positive the contour C is $(-\infty, \infty)$ modified to avoid branch cuts in the complex u-plane in a way which makes ϕ_A satisfy the radiation conditions at infinity, i.e. makes ϕ_A an outgoing wave.

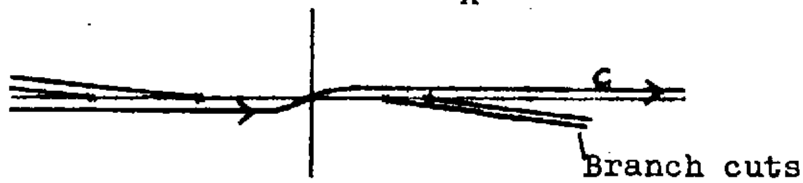

Fig.2. Contour in complex u-plane

The kernel Φ can be found uniquely once all the boundary conditions are satisfied, and the corresponding field ϕ_A seems well behaved, being finite everywhere and decaying in the correct way at infinity. It is not however the solution to our problem though this is far from being immediately obvious. Trouble only arises when attention is turned from the case of an harmonic source to that of an impulsive source $\delta(t)$ at time t=0. The solution p_A to the impulsive problem is given by a simple Fourier transform

$$p_A(t) = \int_{-\infty}^{\infty} \phi_A(\omega) e^{i\omega t} d\omega \qquad (3)$$

This can be evaluated exactly for the infinite sheet and the result is that $p_A(t)$ is non-zero for all negative times. This clearly violates causality and so if we accept that the physical system must be causal p_A and ϕ_A cannot be correct. It may be worth noting that the non-causal part of p_A corresponds to a part of ϕ_A which seems quite inoccuous since it decays exponentially with distance. It therefore satisfies Sommerfeld's radiation conditions, and we conclude that these conditions are not sufficient to make a solution causal.

The only way open to us to alter the solution ϕ_A is to add onto it any solution of the homogeneous problem, that is the problem with no source term present. Such homogeneous solutions correspond to poles of the integrand $\Phi(u;y)$. For the infinite sheet these are at the zeros of

$$\Lambda(u) \equiv (1-Mu)^2(1-u^2)^{1/2} + ((1-Mu)^2-u^2)^{1/2} \qquad (4)$$

There are only two of these for M<2, at u_o and u_o^*, shown in Fig.3.

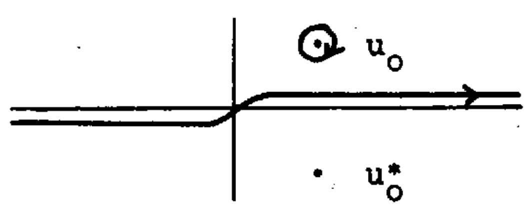

Fig. 3.

It turns out that a homogeneous solution given by integrating around the small circle about u_o shown above in the formula (2) must be added to make the solution causal. A solution associated with u_o^* could also be rejected on physical grounds as it would increase exponentially with distance from the vortex sheet.

This one pole u_o of the kernel completely alters the physical field. The addition of the extra homogeneous solution introduces an instability wave, which is associated with the Helmholtz instability of the vortex sheet. This wave decays exponentially with distance from the sheet but increases exponentially with distance downstream (and so violates the Sommerfeld conditions). It appears in consequence to be confined to the downstream sector illustrated in Fig.4. This sector always makes an angle of 45° with the vortex sheet in the still medium and an angle $\beta(M) < 45°$ in the moving one. As M increases β decreases until at $M=2\sqrt{2}$ $\beta=0$ and the instability no longer exists.

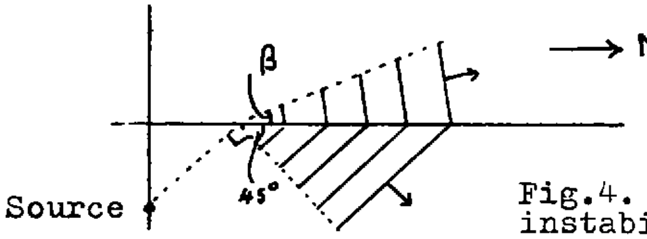

Fig.4. Sector of instability wave.

Much more can be said about the structure of the remainder of the sound field but in this paper we will restrict our attention to these instability waves which in many practical situations seem likely to have a dominant influence.

When we come to the cases of the jets much of the basic outline presented above remains true, though greater difficulties are encountered in the details. For the cylindrical jet, for example, in the region outside the jet the kernel takes the form

$$\Phi = \sum_{-\infty}^{\infty} \Phi_n = \sum_{-\infty}^{\infty} \frac{e^{in\theta}\, H_n^{(1)}(kvr)\, J_n(kwr_0)\,(1-Mu)}{kr_1\left[(1-Mu)^2 v H_n^{(1)\prime}(kvr_1) J_n(kwr_1) - w H_n^{(1)}(kvr_1) J_n'(kwr_1)\right]} \quad (5)$$

where v and w are functions of u. In this case it is impossible to find the exact solution to the impulsive source problem in order to discover whether or not the initial choice of contour of integration gives a causal solution. It is also impossible to find exactly the positions of all the poles of the integrand. It is easily shown however that each term in the sum (5) has an infinite number of poles, and these may be roughly located in the limits kr_1 very small and very large, where r_1 is the radius of the jet. For example we have for $kr_1 \gg 1$ and $-M > -(1-M^2)$ the distribution shown in Fig. 5.

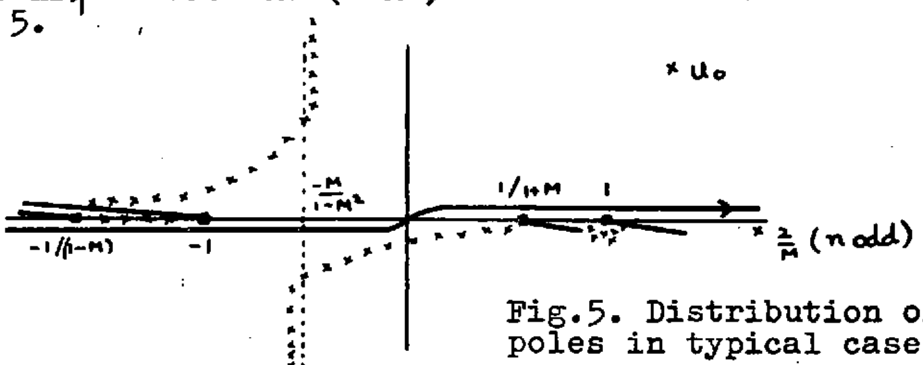

Fig.5. Distribution of poles in typical case.

Tam (J.Fl.Mech. 1971 46,747) who has considered the instability of a cylindrical jet used a very crude approximation to the kernel which shares with the form (5) only the property of having a pole near u_0 (for large kr_1). His kernel in fact has only two poles - at u_0 and u_0^* which leaves no difficulty in choosing u_0 as the one contributing the instability wave.

The earlier work on the single vortex sheet shows that causality is the surest guide to whether or not a pole gives a contribution. An alternative form of this criterion is needed now however since the exact impulsive solution is not available. This is chosen to be that $\phi_A(k)$ can not be causal if it is singular in the lower half of the complex k-plane. This is a valid condition since the behaviour of $\phi_A(k)$ is such as to allow the contour of integration in (3) to be closed in the lower half of the k-plane when $t<0$. So $p_A(t)$ for $t<0$ is the sum of the contributions of the integrand at any singularities of $\phi_A(k)$. It is possible to show exactly that Φ_n has only one pole in the lower k-plane, which proves that ϕ_A can not give a causal solution. It is then only a matter of showing that the singularity in $\phi_A(k)$ is removed by altering the contour C of Fig. 2 to include a contribution from a single pole - which is close to u_o for high frequencies. An interesting conjecture suggested by our experience but which has not been proved for all frequencies is that a pole gives an instability wave if and only if it is in the first quadrant of the u-plane. Since poles in the other quadrants would give unphysical waves increasing exponentially upstream or away from the sheet this seems plausible.

Coming now to a brief description of the instability waves themselves we find little new in the high frequency case. As might be expected they continue to be restricted to a downstream sector which for both jets makes an angle of 45° with the direction of flow, at large distances from the jet.

In the low frequency limit however, when the wavelength becomes large compared with the width of the jet differences emerge. It is convenient to distinguish the symmetric from the asymmetric part of the sound field. For the cylindrical jet the symmetric part is the term with n=0 in (5), independent of θ. For the two dimensional jet it is half the field produced by having two symmetrically placed sources. In either case the asymmetric part of the field is the remainder. The results can then be summarised in the table below.

Cylindrical jet	2-Dimen.[1] jet	Symmetric	Pole at u=	Angle ψ of sector
x		Yes	1/M	$\to 0^{\circ}$
	x	Yes	1/M	$\to 0^{\circ}$
x		No	(1+i)/M	$45^{\circ} < \psi < 90^{\circ}$
	x	No	$(M^2 kh)^{-1/3} e^{i\pi/3}$	60°

Table1. Low frequency behaviour.

Notice that the symmetric part of the instability wave is hardly present in the external medium for very low frequency for either jet, since the angle tends to zero. The asymmetric field for the two dimensional jet is confined to a sector with angle 60° for all M, but the corresponding angle for the cylindrical jet increases with M. It is given exactly by

$$\sin(\psi) = \left(\left(\left((4+M^4)^{\frac{1}{2}}+M^2-2\right)/2M^2\right)^{\frac{1}{2}}\right), \qquad (6)$$

and is approximately 45° when M=0, 50° when M=1 and increases up to 90° as M tends to infinity. It is important to notice that the low frequency instabilities exist for all M, though the high frequency ones are cut off for $M=2\sqrt{2}$.

It seems then that to prevent a jet amplifying sound it must have a velocity nearly three times that of sound (to eliminate high frequencies) and the sound source has to be symmetric (to reduce the low frequencies).