RAINBOW-TRAPPING ABSORBERS FOR TRANSMISSION PROBLEMS: BROADBAND AND PERFECT SOUND ABSORBING PANELS.

Noé Jimenez, Vicent Romero-García, Vincent Pagneux, and Jean-Philippe Groby
Laboratoire d'Acoustique de l'Université du Maine, UMR6613 CRNRS, Le Mans, France

email: *Jean-Philippe.Groby@univ-lemans.fr*

Broadband and perfect sound absorption by subwavelength panels for transmission problems is reported. The asymmetric panels are composed of a periodic array of open waveguides loaded by Helmholtz resonators (HRs) with slightly different dimensions along the structure depth. In each waveguide, the deepest resonator generates a low cut-off frequency, reducing drastically the transmission. The geometry of the preceding HR is designed to possess a slightly higher resonance frequency and is tuned to match the structure impedance with the surrounding one, thanks to the critical coupling condition. Therefore, reflection vanishes and the structure becomes critically coupled to the incident wave, resulting in perfect sound absorption. This process is repeated by adding HRs, whose resonance frequency is slightly higher than the preceding one, to each waveguide. The last added HR fixes the high cut-off frequency of the perfect absorption band in such a way that slow sound condition is achieved within each waveguide over a broadband frequency range. We experimentally, theoretically and numerically report perfect sound absorption from 350 to 1000 Hz for a transparent open panel composed of 9 resonators with a total thickness of 11.4 cm, i.e., 10 times smaller than the wavelength and covering almost two octaves.

Keywords:

1. Introduction

Wave manipulation by metamaterials has been extensibly studied during these last decades because of their exotic properties that have been observed using electromagnetic [1], elastic [2] or acoustic [3] waves. Most of these phenomena arise from the singular propagation conditions at selected frequencies as observed in optics [1], elastodynamics [4] or acoustics [5]. For audible sound waves, the selective chromaticism of most of the studied metamaterials limits their practical applications: the audible frequency band covers more than ten frequency octaves, while in contrast, visible light spectrum covers less than one octave.

The design of efficient and thin materials for sound absorption is a major topic in acoustics. Strong dispersion due to local resonances induces slow sound propagation [6]. Using slow sound, deep-subwavelength thickness absorbing structures can be designed [7, 8, 9, 10, 11]. To achieve perfect absorption, the structure should not scatter waves, i.e., all eigenvalues of the scattering matrix must vanish at the same frequency. This generally occurs when the intrinsic losses of the system exactly compensate the energy leakage at resonances of the structure [12]. When this condition is fulfilled the system is critically coupled with the exterior medium and perfect absorption is observed. Two strategies can be followed to achieve perfect absorption in transmission problems: either making use of degenerate resonators by exciting monopolar and dipolar resonances at the same frequency [13] or breaking of the symmetry of the material by making use of double-interacting resonators [14].

To our knowledge, broadband and perfect acoustic absorption by subwavelength thickness structures has not been reported yet in transmission problems. In this work, we tackle this problem making use of asymmetric propagation in panels composed by monopolar resonators with graded dimensions, namely *rainbow-trapping absorbers*.

2. System description and modeling

The structures are composed by a L-thick rigid panel periodically perforated by variable square cross-section waveguides and loaded by N HRs of different dimensions, as shown in Figs.1 (a, b). Each waveguide is therefore divided in N segments of length a^n, height h_3^n and width h_1^n. A HR is located in the middle of each waveguide section. The *rainbow-trapping absorber* is shown in Fig.1 (a) and is composed of $N = 8$ HRs. The subwavelength absorber is designed to reach broadband perfect absorption.

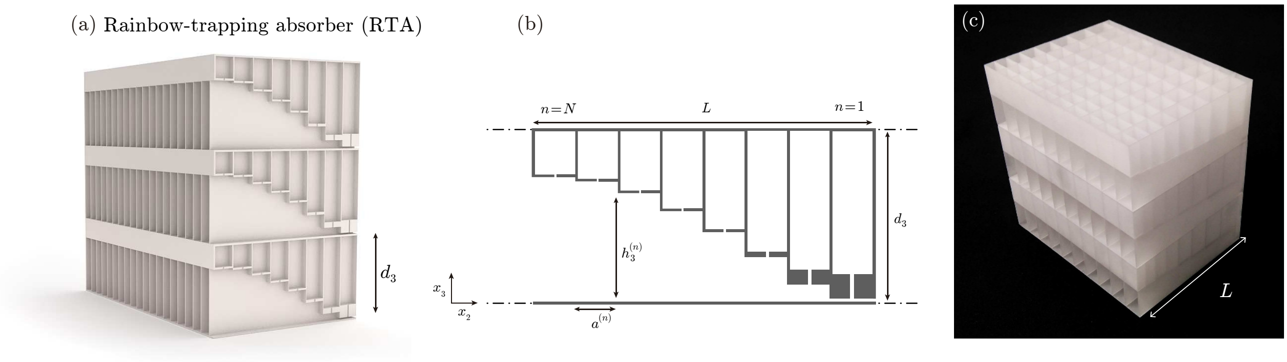

Figure 1: (a) Conceptual view of a rainbow trapping absorber (RTA) with $N = 8$ HRs. (b) Scheme showing the geometrical variables for RTA panels. (c) Photograph of the sample containing 10×3 unit cells.

The thermo-viscous losses were accounted for by using the effective parameters, i.e., the complex and frequency dependent density and bulk modulus, given in [15]. Both structures were theoretically modelled by using the transfer matrix method (TMM) and analyzed by using the scattering matrix, $\mathbf{S}$. While the transfer matrix, $\mathbf{T}$, relates the sound pressures, p, and normal acoustic particle velocities, v_x, on the beginning and at the end of the panel, the scattering matrix, $\mathbf{S}$, relates the amplitudes of the incoming waves to the system with those of the out-coming waves. The poles and zeros of the eigenvalues of the $\mathbf{S}$-matrix in the complex-frequency plane provide information on totally absorbed incident waves [16], because the elements of the $\mathbf{S}$-matrix are the reflection coefficients from both sides of the structure, R^+ and R^-, and the transmission coefficient, T. The asymmetric absorption coefficients are calculated as $\alpha^+ = 1 - |R^+|^2 - |T|^2$ for the positive x-axis ingoing waves and $\alpha^- = 1 - |R^-|^2 - |T|^2$ for the negative x-axis ingoing waves. In order to validate the analytical models we use a numerical approach based on the Finite Element Method (FEM) using COMSOL Multiphysics 5.2™.

3. Sample design and experiments

The geometrical parameters of both structures were tuned using optimization methods (sequential quadratic programming (SQP) [17]). In the case of the rainbow trapping absorber ($N = 9$), the cost function was $\varepsilon_{\mathrm{RTA}} = \int_{f_1}^{f_N} |R^+|^2 + |T|^2 df$, i.e., to maximize the absorption in a broad frequency bandwidth. We selected from $f_1 = 300$ to $f_N = 1000$ Hz. In the case of the RTA the length of the panel was constrained to $L = 11.4$ cm, i.e. a panel 10 times thinner than the wavelength at 300 Hz. The sample was 3D printed by means of stereo-lithography techniques using a photosensitive epoxy polymer (Accura 60®, 3D Systems Corporation, Rock Hill, SC 29730, USA). The transmission, reflection and absorption were measured in a impedance tube with square cross-section of side 15 cm. In the experiments, the amplitude of the acoustic source was low enough to consider negligible the contribution of the nonlinearity of the HRs.

4. Rainbow-trapping absorbers (RTA)

The idea is to create a frequency-cascade effect and critically coupled several resonators, thus, generating a rainbow-trapping effect. The process is as follows. First, we tune the last resonator in the waveguide to produce a band-gap above a frequency f_{gap}. Thus, above its resonance transmission vanishes. Second, a resonator is placed in the preceding segment of the waveguide, with slightly higher resonance frequency, f_{N-1}. The geometries of this resonator and the section of the waveguide are tuned to impedance match the system at this frequency. Therefore, the reflection vanishes and a peak of perfect absorption is achieved. This HR also reduces the transmission at even higher frequencies. Then, the process can be repeated by extending the waveguide with more segments, each one with a tuned HR with slightly higher resonance frequency. The last HR also matches the impedance of the metamaterial with the exterior medium.

A set of 9 resonators were tuned with resonance frequencies ranging from 330 to 917 Hz. The manufactured sample is shown in Fig. 1 (c). Figures 2 (a-b) show the absorption, reflection and transmission of the device calculated with the TMM, FEM and measured experimentally. The last resonator presents a resonance frequency of $f_{\mathrm{gap}} = 259$ Hz, causing the transmission to drop. As a result of the cascade process, the impedance of the structure in the working frequency range is matched with the exterior medium while the transmission vanishes. As a consequence, a flat and quasi-perfect absorption curve is observed by the RTA in this frequency range. The representation of the corresponding eigenvalues of the **S**-matrix, namely $\lambda_{1,2} = T \pm \sqrt{R^{+}R^{-}}$, in the complex frequency plane is shown in Fig. 2 (c-d). We can see even under the constraints imposed by the metamaterial manufacturing, that all the $N-1$ zeros are located very close to the real axis and that the zeros of λ_1 are at the same frequencies as λ_2. In the designed system, not all the zeros are located exactly on the real axis, but as long as the zeros are broad (note the logarithmic color scale in Fig. 2 (c-d)) they overlap producing quasi-perfect sound absorption in a frequency band from 300 to 1000 Hz for a panel 10 times thinner than the wavelength, i.e. $L = 11.4$ cm.

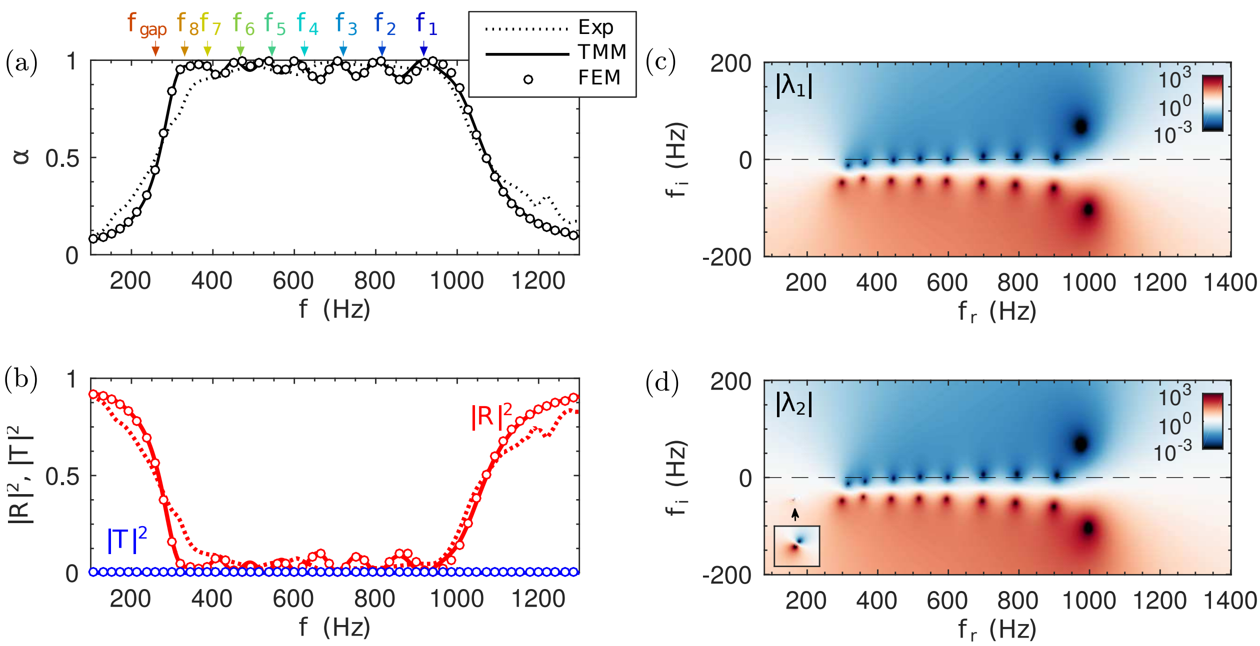

Figure 2: (a) Absorption obtained by using the TMM (continuous line), FEM simulations (circles) and measured experimentally (dotted line). (b) Corresponding reflection (red curves) and transmission (blue curves). (c-d) Complex frequency representation of the eigenvalues of the scattering matrix, $\lambda_{1,2}$. Colormap in $10\log_{10}|\lambda|^2$ scale.

5. Conclusions

We reported perfect acoustic absorption over a broad frequency band by subwavelength thickness panels in transmission problems using the concept of rainbow-trapping effect. Flat and perfect absorption over a frequency range from 300 to 1000 Hz, i.e., almost two octaves, using a rainbow-trapping absorber ten times smaller than the incoming wavelength and composed by 9 resonators is proposed. We showed that to obtain broadband and perfect absorption in transmission problems, three conditions must be fulfilled: first, the zeros of the scattering matrix must be at the real axis; second, the zeros of both eigenvalues, $\lambda_{1,2}$, must be at same frequencies; third, the zeros must be broad to overlap in frequency. These three conditions are mandatory to maximize the broadband absorption of the absorbers, and were in fact minimized by the optimization process.

While the current configuration using Helmholtz resonators presents potential applications managing acoustic waves in civil, automotive or aerospace engineering, the metamaterials presented here paves the way to new investigations by using rainbow-trapping effect produced with either resonators as membranes or poroelastic plates,

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REFERENCES

1. Zheludev, N. I. and Kivshar, Y. S. From metamaterials to metadevices, *Nature materials*, **11** (11), 917–924, (2012).

2. Christensen, J., Kadic, M., Kraft, O. and Wegener, M. Vibrant times for mechanical metamaterials, *Mrs Communications*, **5** (03), 453–462, (2015).

3. Cummer, S. A., Christensen, J. and Alù, A. Controlling sound with acoustic metamaterials, *Nature Reviews Materials*, **1**, 16001, (2016).

4. Ding, Y., Liu, Z., Qiu, C. and Shi, J. Metamaterial with simultaneously negative bulk modulus and mass density, *Physical review letters*, **99** (9), 093904, (2007).

5. Yang, Z., Mei, J., Yang, M., Chan, N. and Sheng, P. Membrane-type acoustic metamaterial with negative dynamic mass, *Phys. Rev. Lett.*, **101** (20), 204301, (2008).

6. Santillán, A. and Bozhevolnyi, S. I. Acoustic transparency and slow sound using detuned acoustic resonators, *Phys. Rev. B*, **84** (6), 064304, (2011).

7. Leclaire, P., Umnova, O., Dupont, T. and Panneton, R. Acoustical properties of air-saturated porous material with periodically distributed dead-end poresa), *J. Acoust. Soc. Am.*, **137** (4), 1772–1782, (2015).

8. Groby, J.-P., Huang, W., Lardeau, A. and Aurégan, Y. The use of slow waves to design simple sound absorbing materials, *J. Appl. Phys.*, **117** (12), 124903, (2015).

9. Groby, J.-P., Pommier, R. and Aurégan, Y. Use of slow sound to design perfect and broadband passive sound absorbing materials, *J. Acoust. Soc. Am.*, **139** (4), 1660–1671, (2016).

10. Jiménez, N., Huang, W., Romero-García, V., Pagneux, V. and Groby, J.-P. Ultra-thin metamaterial for perfect and quasi-omnidirectional sound absorption, *Applied Physics Letters*, **109** (12), 121902, (2016).

11. Jiménez, N., Romero-García, V., Pagneux, V. and Groby, J.-P. Quasiperfect absorption by subwavelength acoustic panels in transmission using accumulation of resonances due to slow sound, *Phys. Rev. B*, **95**, 014205, (2017).

12. Romero-García, V., Theocharis, G., Richoux, O., Merkel, A., Tournat, V. and Pagneux, V. Perfect and broadband acoustic absorption by critically coupled sub-wavelength resonators, *Sci. Rep.*, **6**, 19519, (2016).

13. Yang, M., Meng, C., Fu, C., Li, Y., Yang, Z. and Sheng, P. Subwavelength total acoustic absorption with degenerate resonators, *Appl. Phys. Lett.*, **107** (10), 104104, (2015).

14. Merkel, A., Theocharis, G., Richoux, O., Romero-García, V. and Pagneux, V. Control of acoustic absorption in one-dimensional scattering by resonant scatterers, *Appl. Phys. Lett.*, **107** (24), 244102, (2015).

15. Stinson, M. R. The propagation of plane sound waves in narrow and wide circular tubes, and generalization to uniform tubes of arbitrary cross-sectional shape, *J. Acoust. Soc. Am.*, **89** (2), 550–558, (1991).

16. Romero-García, V., Theocharis, G., Richoux, O. and Pagneux, V. Use of complex frequency plane to design broadband and sub-wavelength absorbers, *The Journal of the Acoustical Society of America*, **139** (6), 3395–3403, (2016).

17. Powell, M. J., (1978), A fast algorithm for nonlinearly constrained optimization calculations. *Numerical analysis*, pp. 144–157, Springer.