DYNAMIC PLASTIC BUCKLING OF STIFFENED CYLINDRICAL SHELLS

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Introduction

In an attempt to clarify the phenomenon of dynamic plastic buckling, the response of a simple model has been examined in References [1] and [2] which also contain citations to other dynamic plastic buckling studies on various structural members. More recently, the dynamic plastic buckling of a stringer-stiffened cylindrical shell impacted axially has been examined in Reference [3] using a perturbation method of analysis. This note contains further numerical results and observations for this particular case.

Theoretical Details

A perturbation method of analysis was developed in Reference [3] for a stringer-stiffened cylindrical shell with an attached mass M which travels with an initial velocity V and strikes a rigid wall. The solution consists of two parts: dominant or axisymmetric uniform behaviour and perturbed or axially varying behaviour. The displacements which characterise dynamic plastic buckling develop from any initial imperfections which are present in the initial geometry of the shell. Further details of the theoretical procedure are presented in Reference [3].

If a stiffened cylindrical shell of mean radius a and length L has an initial radial displacement imperfection field $\overline{w}'$ which consists of an infinite number of components ($\overline{w}_n'$) each one of which varies with $\sin(n\pi x/L)$, where x is an axial coordinate, then it transpires that the buckled profile is characterised by a critical mode number n^c and the dimensionless perturbed radial displacement ($\delta_n c = w_n' c/a$) in the critical mode at the cessation of dominant motion is

$$\delta_n c = E_n c(0) a_n c , \tag{1}$$

where $a_n c = \overline{w}_n' c/a$ and $E_n c(0)$ is a displacement amplification function. The displacement amplification function is a function of $\alpha = h/a$ (h is cylindrical shell thickness), $\beta = a/L$, $\overline{\sigma} = 1 + M/m$ (m is mass of cylindrical shell and stiffeners), $\overline{\lambda} = E'/\sigma_y$ (E' is tangent modulus and σ_y is yield stress), $K = \rho V^2/\sigma_y$ (ρ is density of material), $A^* = A/bh$ (A is cross-sectional area of a stiffener, $b = 2\pi a/S$, S is number of stiffeners), $e^* = e/a$ (e is eccentricity of a stiffener measured from the mid-surface of a cylindrical shell), and $I^* = I/a^2bh$ (I is second moment of area of a stiffener about the mid-surface of a cylindrical shell).

Numerical Results

The curves in Figure 1 demonstrate the growth of the dimensionless perturbed radial displacements ($\delta_n c$) with dimensionless velocity (squared) (K) which is obtained using equation (1) and the numerical results in Figure 6 of Reference [3] for two values of the dimensionless initial radial imperfections ($a_n c$)

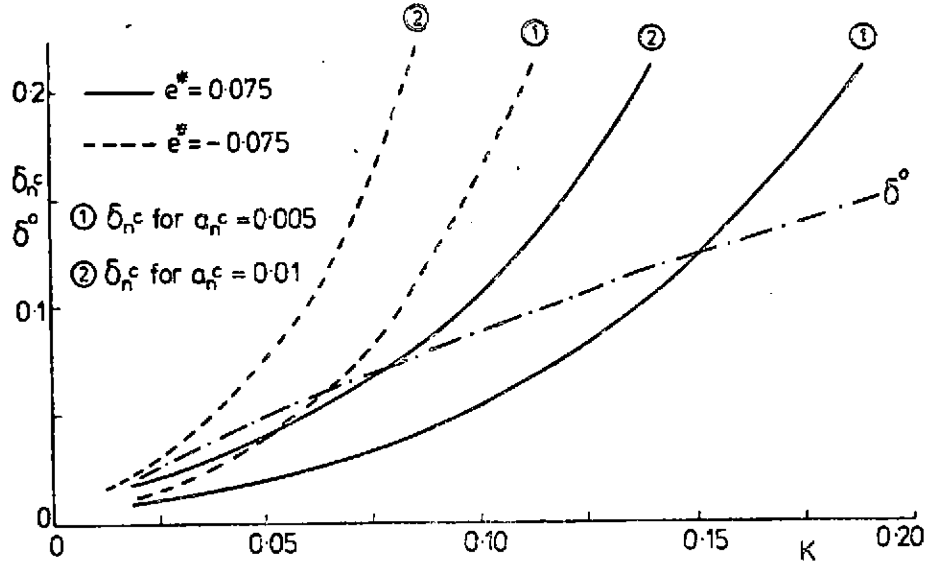

FIGURE 1. GROWTH OF RADIAL DISPLACEMENTS WITH K FOR $\alpha = \beta = 0{\cdot}1$, $A^* = 0{\cdot}25$, $I^* = 0{\cdot}001458$, $\bar{\lambda} = 3$, $\bar{\sigma} = 4{\cdot}5$ AND $S = 10$.

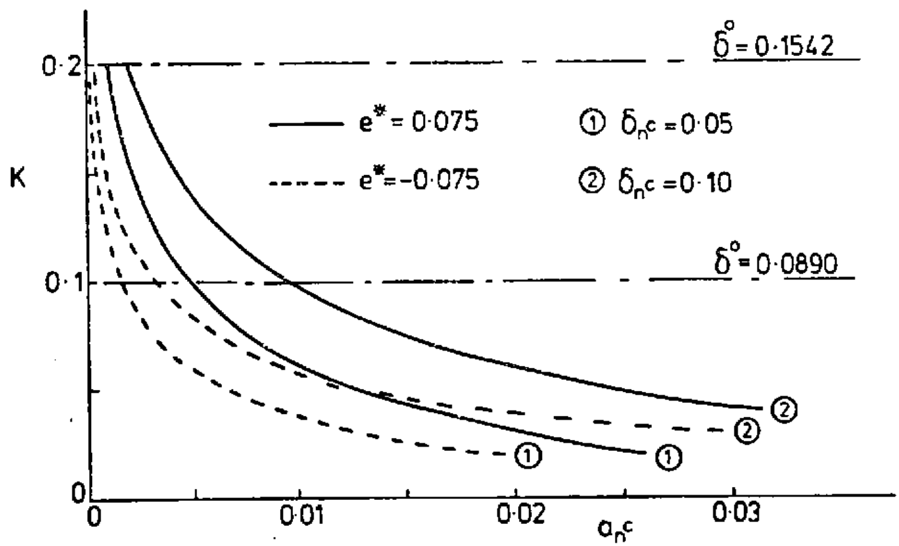

FIGURE 2 ISO-DAMAGE CURVES FOR $\alpha = \beta = 0{\cdot}1$, $A^* = 0{\cdot}25$, $I^* = 0{\cdot}001458$, $\bar{\lambda} = 3$, $\bar{\sigma} = 4{\cdot}5$ AND $S = 10$.

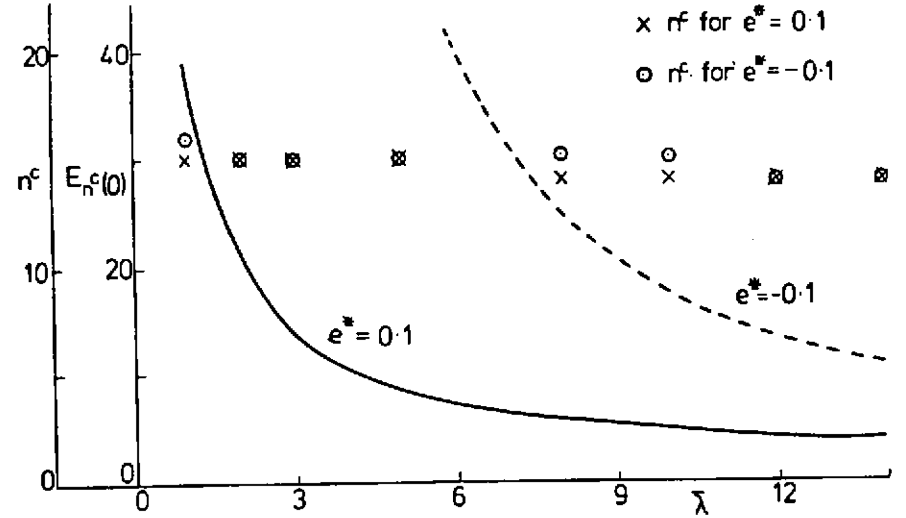

FIGURE 3. VARIATION OF CRITICAL MODE NUMBER (n^c) AND DISPLACEMENT AMPLIFICATION FUNCTION ($E_{n^c}(0)$) WITH $\bar{\lambda}$ FOR $\alpha = \beta = 0{\cdot}1$, $A^* = 0{\cdot}5$, $I^* = 0{\cdot}0054$, $\bar{\sigma} = 4{\cdot}5$, $K = 0{\cdot}2$ AND $S = 16$.

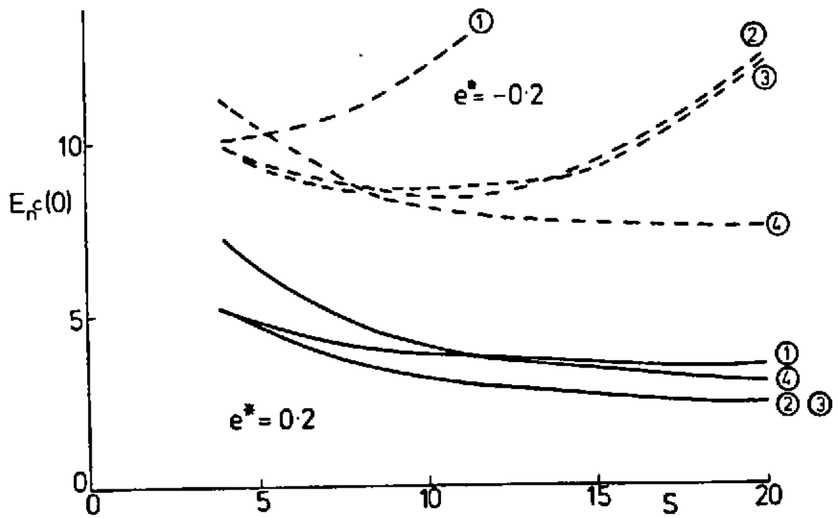

FIGURE 4. $E_{n^c}(0)$ VERSUS NUMBER OF RECTANGULAR STIFFENERS (S) FOR CONSTANT A, $\alpha = 0{\cdot}2$, $\beta = 0{\cdot}1$, $\bar{\lambda} = 3$. ① $\bar{\sigma} = 4{\cdot}5$, $K = 0{\cdot}2$, ② $\bar{\sigma} = 4{\cdot}5$, $\bar{\sigma}(1+A^*)K = 1{\cdot}08$, ③ $(1+A^*)\bar{\sigma}K = 1{\cdot}08$, M = CONSTANT, $A^* = 0{\cdot}2$ AT $S=4$ ④ $(1+A^*)\bar{\sigma}K = 0{\cdot}99$, M = CONSTANT, $A^* = 0{\cdot}1$ AT $S=4$.

lying in the critical mode. The dimensionless dominant displacement (δ^o) is also shown for comparison purposes. These numerical results are replotted as iso-damage curves in Figure 2 which clearly demonstrate the deleterious influence of initial imperfections in the critical mode.

It is evident from the numerical results in Figure 3 that the displacement amplification function and therefore the perturbed radial displacement according to equation (1) is sensitive to the material parameter ($\bar{\lambda}$) while the critical mode number is not.

The variation of $E_nc(0)$ with number of stringer stiffeners (S) is shown in Figure 4 which is prepared using data from Figure 14 in Reference [3] together with some new numerical results. The cross-sectional area of a stiffener (A) is identical on each of the curves ① to ④, while the initial kinetic energy is constant on each of curves ② to ④. In addition, the end mass M is constant along curves ③ and ④. The stiffeners on curve ④ have one half the cross-sectional area of those on curve ③ at a corresponding value of S.

It is evident from Figures 1 to 4 (for stringers with rectangular cross-sections) and other numerical results in Reference [3] that it is more efficient to place stiffeners on the outside ($e^* > 0$) of a cylindrical shell impacted axially than on the inside surface ($e^* < 0$). It is also interesting to observe that curves ② and ③ in Figure 4 for inside stiffeners ($e^* = -0.2$) have a minimum and therefore suggest an optimum design with approximately 10 to 12 inside stringers. However, the initial momentum of the system increases 7.16% and 3.98% with S along curves ③ and ④, respectively. Furthermore, these optimum designs for inside stiffeners have greater potential growth of initial radial displacement imperfections than any of the cases with outside stiffeners in Figure 4 regardless of the value of S.

The sensitivity of the theoretical analysis to $\bar{\sigma}$ was also explored for the particular case in Figure 3 with $\bar{\lambda} = 3$. It transpires for $e^* = 0.1$ that $E_nc(0)$ varies from 13.67 to 13.77 as $\bar{\sigma}$ increases from 1.5 to 10 and K is reduced from 0.6 to 0.09 in order to maintain a constant initial kinetic energy. $E_nc(0)$ increases from 147.5 to 151.9 when $e^* = -0.1$, and the critical mode number is 15 for all the calculations regardless of the sign of e^*.

Finally, it may be shown that the theoretical results are also valid for a stationary stiffened cylinder struck by a mass M travelling with an initial velocity V. In this case $\bar{\sigma}$ must be interpreted as M/m.

References

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