NONLINEAR ACOUSTIC AND MAGNETOACOUSTIC WAVES IN MEDIA WITH ISENTROPIC INSTABILITY

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Three models of nonequilibrium media are discussed: the stationary nonequilibrium gas with the exponential relaxation model, the heat-releasing gas with the generalized heat-loss function, and the heat-releasing plasma in a magnetic field. It is shown that under conditions of the acoustic (isentropic) instability of these media, evolution of nonlinear acoustic and magnetoacoustic waves is well described by the generalized nonlinear acoustic equation.
Keywords: negative bulk viscosity, solitary pulse

1. Introduction

We present a brief overview of the general properties of the acoustic wave amplification in three different models of the media with heating and cooling processes. Based on the gas-dynamics or magnetogasdynamics equations, we obtain the generalized nonlinear acoustic (magnetoacoustic) equations and show that they have similar forms but with different nonlinearity, dispersion and dissipation coefficients. Using these equations, we show the disintegration of weak shock waves into a sequence of the self-sustained acoustical (magnetoacoustic) pulses. Parameters of these pulses are found analytically. We show the strong dependence of their amplitudes on the ratio of plasma pressure to magnetic pressure.

2. Acoustical disturbances in the stationary nonequilibrium heat-releasing media with the exponential relaxation model

The system of equations describing the dynamics of the gas-dynamic perturbations in the stationary vibrational excited gas with the exponential relaxation model has the form:

$$P = \frac{\rho T}{M}, \quad \frac{d\rho}{dt} + \rho\frac{\partial \mathrm{v}}{\partial x} = 0, \quad \rho\frac{d\mathrm{v}}{dt} = -\frac{\partial P}{\partial x} + \frac{4}{3}\eta\frac{\partial^2 \mathrm{v}}{\partial x^2},$$

$$C_{V\infty}\frac{dT}{dt} + \frac{dE_v}{dt} - \frac{T}{\rho}\cdot\frac{d\rho}{dt} = \frac{\chi m}{\rho}\frac{\partial^2 T}{\partial x^2} + \Gamma - I,$$

$$\frac{dE_v}{dt} = \frac{E_e - E_v}{\tau_v(T,\rho)} + \Gamma. \tag{1}$$

In Eq. (1), E_V is the vibrational molecular energy, E_e is its equilibrium value, τ_V is the vibrational relaxation time, and Γ is the power of an external heat source (in particular, electric pumping in the discharge, chemical or optical pumping), sustaining the nonequilibrium degree $S_0 = (E_{v0} - E_{e0})/T_0 = \Gamma\tau_{v0}/T_0$; v, T, ρ, P are, respectively, the velocity, temperature, density, and pressure. I= Γ is the heat loss, m is the molecular mass, $d/dt = \partial/\partial t + v\partial/\partial x$, η is the shear viscosity coefficient, χ is the thermal conductivity.

Linearizing (1) for small acoustic perturbations, we obtain that the acoustical increment has the simple form:

$$\alpha = \frac{\omega^2[\xi(\omega)+\mu]}{2\rho_0 c_{Snd}^3(\omega)}, \tag{2}$$

where ξ is the second (bulk) viscosity coefficient, $\mu = 4\eta/3 + \chi\,\mathrm{Re}(1/C_V - 1/C_P)$, c_{Snd} is the sound speed. The general condition of acoustically instability is

$$\xi(\omega) + \mu < 0. \tag{3}$$

Here,

$$C_V = \left(\frac{\partial u}{\partial T}\right)_V = C_{V\infty} + \left(\frac{\partial E_v}{\partial T}\right)_V = \frac{C_{V0} - i\omega\tau_{v0}C_{V\infty}}{1 - i\omega\tau_{v0}}, \tag{4}$$

$$C_P = \left(\frac{\partial h}{\partial T}\right)_P = C_{P\infty} + \left(\frac{\partial E_v}{\partial T}\right)_P = \frac{C_{P0} - i\omega\tau_{v0}C_{P\infty}}{1 - i\omega\tau_{v0}} \tag{5}$$

are the complex heat capacities of the relaxing medium at constant volume and at constant pressure; u, $h = u + P/\rho$ are the internal energy and enthalpy of the medium per one molecule; $C_{V0} = C_{V\infty} + C_v + S_0\tau_T$, $C_{P0} = C_{P\infty} + C_v + S_0(\tau_T + 1)$ are the low-frequency heat capacities at constant volume and constant pressure in the vibrationally excited gas; $T_0, \rho_0, E_{v0}, E_{e0}$ are the stationary values; $C_v = dE_{e0}/dT_0$; $\tau_T = \partial \ln\tau_{v0}/\partial \ln T_0$; $\tau_{v0} = \tau_v(T_0, \rho_0)$; $C_{V\infty}$ and $C_{P\infty}$ are the frozen (high-frequency) heat capacities.

The second viscosity and the sound velocity have the usual forms for media with a single relaxation process:

$$\xi = \frac{\xi_0 C_{V0}^2}{C_{V0}^2 + \omega^2\tau_{v0}^2 C_{V\infty}^2},\ \xi_0 = \frac{C_{V\infty}\tau_{v0}(c_\infty^2 - c_0^2)\rho_0}{C_{V0}}, \tag{6}$$

$$c_{Snd} = \sqrt{\frac{C_{V0}^2 c_0^2 + \omega^2\tau_{v0}^2 C_{V\infty}^2 c_\infty^2}{C_{V0}^2 + \omega^2\tau_{v0}^2 C_{V\infty}^2}}. \tag{7}$$

Here, ξ_0 is the low-frequency second viscosity coefficient; $c_\infty^2 = \gamma_\infty T_0/m$ and $c_0^2 = \gamma_0 T_0/m$ are high-frequency and low-frequency sound speeds, respectively; $\gamma_\infty = C_{P\infty}/C_{V\infty}$, $\gamma_0 = C_{P0}/C_{V0}$ are the frozen and equilibrium adiabatic indexes, respectively; m is the molecular mass.

The second viscosity coefficient (6) is negative under the condition $S_0 > S_{th} = C_v/(C_{V\infty} - \tau_T)$. This condition corresponds to the positive feedback between the acoustical perturbation and nonequilibrium heating, i.e. nonequilibrium heating increases in compression regions and decreases in rarefaction regions of acoustical perturbation. Such a medium becomes acoustically active (the well-known Rayleigh instability criterion).

Acoustic instability is stabilized as a result of non-linear transfer of energy from low-frequency unstable modes to stable high-frequency modes. Nonlinear evolution of acoustic disturbance up to the second order of smallness is described by the generalized nonlinear acoustic equation (GNAE), which we obtained in the following form [1]:

$$C_{V\infty}\tau_{v0}(\tilde{\rho}_{tt} - c_\infty^2\tilde{\rho}_{xx} - c_\infty^2\Psi_\infty\tilde{\rho}_{xx}^2 - \frac{\mu_\infty}{\rho_0}\tilde{\rho}_{xxt})_t + \\ + C_{V0}(\tilde{\rho}_{tt} - c_0^2\tilde{\rho}_{xx} - c_0^2\Psi_0\tilde{\rho}_{xx}^2 - \frac{\mu_0}{\rho_0}\tilde{\rho}_{xxt}) = 0. \quad (8)$$

Here,

$$\Psi_0 = \left[\frac{S_0\tau_T(1+S_0)}{C_{P0}C_{V0}} + \frac{1+2C_{V0}}{2C_{V0}} - \frac{S_0(1+S_0)^2}{2C_{P0}C_{V0}^2}\tau_{TT}\right], \quad (9)$$

$$\Psi_\infty = \frac{(\gamma_\infty+1)}{2},\ \tau_{TT} = \frac{T_0^2}{\tau_{v0}}\frac{\partial^2\tau_{v0}}{\partial T_0{}^2},\ \mu_0 = \frac{4\eta}{3} + \chi M(\frac{1}{C_{V0}} - \frac{1}{C_{P0}}),$$

$$\mu_\infty = \frac{4\eta}{3} + \chi M(\frac{1}{C_{V\infty}} - \frac{1}{C_{P\infty}}),\ \tilde{\rho} = \frac{\rho-\rho_0}{\rho_0}.$$

For $S_0 = 0$, Eq. (9) leads to $\Psi_0 = (\gamma_0+1)/2$. Equation (8) is valid for media with the small dispersion coefficient $d = (c_0^2 - c_\infty^2)/c_\infty^2 << 1$. It is the generalized acoustical equation of relaxing media as it describes acoustical perturbations independently of their spectrum.

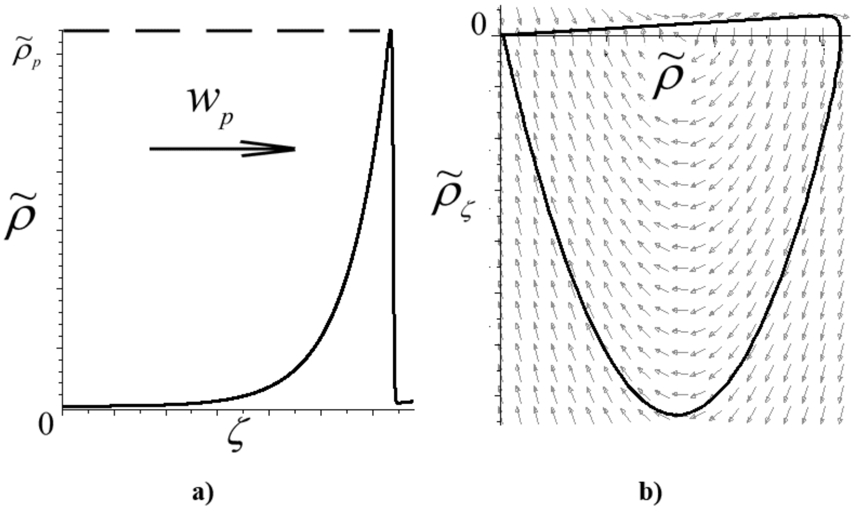

Figure 1: (a) Structure of the self-sustained pulse. (b) – separatrix loop which corresponds to the self-sustained pulse in the phase plane.

In case of instability (3), according to Eq. (8), the weak shock waves with amplitude $\rho_i \le m/(2\Psi_\infty - \Psi_0)$ and localized perturbations disintegrate into a series of self-sustained wave solitary pulses shown in Fig. 1. For $\mu_\infty \to 0$, the shape, amplitude and speed of pulses are determined in the forms:

$$\tilde{\rho}(\tilde{\zeta}) = \begin{cases} \rho_p \exp\left[\frac{(\tilde{\zeta}-\zeta_0)\Psi_0}{2\Psi_\infty}\right] & \tilde{\zeta} \le \zeta_0 \\ 0 & \tilde{\zeta} > \zeta_0 \end{cases}$$

$$\rho_p = \frac{2w_p}{\Psi_\infty} = \frac{2d}{(2\Psi_\infty - \Psi_0)};\qquad w_p = \frac{d\Psi_\infty}{(2\Psi_\infty - \Psi_0)}. \quad (10)$$

3. Acoustical disturbances in the heat-realising media with the generalized heat-loss function

In this section, we show that an acoustic instability can lead to the formation of self-sustained solitary pulses in another heat-realising media model (without relaxation). Initial system of equations (1) changes to:

$$P = \frac{\rho T}{M}, \quad \frac{d\rho}{dt} + \rho\frac{\partial \mathrm{v}}{\partial x} = 0, \quad \rho\frac{d\mathrm{v}}{dt} = -\frac{\partial P}{\partial x} + \frac{4}{3}\eta\frac{\partial^2 \mathrm{v}}{\partial x^2},\ P = \frac{k_B \rho T}{m},$$

$$C_{V\infty}\frac{dT}{dt} - \frac{T}{\rho}\cdot\frac{d\rho}{dt} = \frac{\chi m}{\rho}\frac{\partial^2 T}{\partial x^2} - \Im(T,\rho). \tag{11}$$

Here, generalized heat-loss function $\Im = I - \Gamma$ depends on temperature and density. The condition for isentropic (acoustic) instability is

$$\frac{c_{V\infty} m\rho_0 \Im_{0\rho}}{k_B T_0} + \Im_{0T} \equiv \frac{\rho_0 \Im_{0\rho}}{(\gamma_\infty - 1)T_0} + \Im_{0T} < 0, \tag{12}$$

where γ is the adiabatic index (the ratio of specific heats). Similar to (3), inequality (12) coincides with the negative bulk viscosity existence [2]. For system (11),

$$\xi_0 = \frac{\rho_0 \tau_0 c_{V\infty}(c_\infty^2 - c_0^2)}{c_{V0}} = \frac{P_0 \Gamma_0 \tau_0 \left(c_{V\infty} m\rho_0 \Im_{0\rho}/k_B + T_0 \Im_{0T}\right)}{T_0^2 \Im_T^2}, \tag{13}$$

$$c_\infty = \sqrt{\frac{c_{P\infty} k_B T_0}{m c_{V\infty}}} = \sqrt{\gamma_\infty \frac{k_B T_0}{m}}, \tag{14}$$

$$c_0 = \sqrt{\frac{c_{P0} k_B T_0}{m c_{V0}}} = \sqrt{\gamma_0 \frac{k_B T_0}{m}} = \sqrt{\frac{(T_0 \Im_{0T} - \rho_0 \Im_{0\rho})}{T_0 \Im_{0T}}\frac{k_B T_0}{m}}. \tag{15}$$

In Eq. (13), $\tau_0 = k_B T_0 / \Gamma_0 m$ is the characteristic time of heating, $\Gamma_0 = \Gamma(\rho_0, T_0)$ is the heating rate in the stationary medium, $c_{P\infty} = c_{V\infty} + k_B/m$, $c_{V0} = k_B T_0 \Im_{0T}/m\Gamma_0$, $c_{P0} = k_B(T_0\Im_{0T} - \rho_0\Im_{0\rho})/m\Gamma_0$, $\Im_{0T} = (\partial\Im/\partial T)_{\rho=\rho_0, T=T_0}$, $\Im_{0\rho} = (\partial\Im/\partial\rho)_{\rho=\rho_0, T=T_0}$.

Nonlinear evolution of acoustic disturbance up to the second order of smallness, small dissipation and dispersion coefficients is described still by the same GNAE (8), but with different coefficients:

$$\Psi_0 = \frac{2\gamma_0 - 1}{\gamma_0} - \frac{1}{T_0 \Im_{0T}}\left[\frac{T_0^2 \Im_{0TT}(\gamma_0 - 1)^2}{2\gamma_0} + \frac{\rho_0^2 \Im_{0\rho\rho}}{2\gamma_0} + \frac{T_0 \rho_0 \Im_{0\rho T}(\gamma_0 - 1)}{\gamma_0}\right], \quad \Psi_\infty = \frac{(\gamma_\infty + 1)}{2} \tag{16}$$

$$\Im_{0TT} = \left(\frac{\partial^2 \Im}{\partial T^2}\right)_{\rho=\rho_0, T=T_0}, \quad \Im_{0\rho T} = \left(\frac{\partial^2 \Im}{\partial\rho\partial T}\right)_{\rho=\rho_0, T=T_0}, \quad \Im_{0\rho\rho} = \left(\frac{\partial^2 \Im}{\partial\rho^2}\right)_{\rho=\rho_0, T=T_0}$$

$$\mu_0 = \frac{4\eta}{3},\ \mu_\infty = \frac{4\eta}{3} + \chi M\left(\frac{1}{C_{V\infty}} - \frac{1}{C_{P\infty}}\right).$$

4. Magnetoacoustic disturbances in the heat-realising media with the generalized heat-loss function

Let's consider more complicated case of heat realising plasma in the magnetic field. The medium under investigation is assumed to be fully ionized and electrically neutral. The influence of viscosity and thermal conduction is neglected. With the foregoing as background, the initial system of equations can be written in the following vector form:

$$\frac{\partial \vec{B}}{\partial t} = rot\left[\vec{V} \times \vec{B}\right] + \frac{c^2}{4\pi\sigma}\Delta B,\ div\vec{B} = 0,\ \rho\frac{d\vec{V}}{dt} = -\nabla P - \frac{1}{4\pi}\vec{B} \times rot\left[\vec{B}\right],\ \frac{\partial \rho}{\partial t} + div\rho\vec{V} = 0,$$

$$C_{V\infty}\frac{dT}{dt} - \frac{k_Б \cdot T}{m\rho}\cdot\frac{d\rho}{dt} = -\Im(\rho,T) + \frac{j^2}{\sigma},\ P = \frac{k_Б T\rho}{m},\ \vec{j} = \frac{c}{4\pi}\nabla \times \vec{B}. \qquad (17)$$

In system (17), ρ, T, P are density, temperature and pressure, respectively; $\vec{V}, \vec{B}, \vec{j}$ are velocity vector, magnetic field vector and current density vector, respectively; $k_Б$ is the Boltzmann constant; m is the mean particle mass; c is the light speed in vacuum; σ is the electric conductivity coefficient.

The magnetic field vector with absolute value B_0 lies in x-z plane oriented at angle θ to z-axis. We investigate wave propagation only along z-axis and neglect variation of perturbation along x- and y- axis.

Increment (2) changes to

$$\alpha = \frac{\omega^2\left\{\frac{\xi(\omega)}{\rho_0}\left(c_{f,s}^2 - c_a^2\cos^2\theta\right) + \frac{c^2}{4\pi\sigma}\left(c_{f,s}^2 - c_{Snd}^2\right)\right\}}{2c_{f,s}^3\left(2c_{f,s}^2 - \left(c_a^2 + c_{Snd}^2\right)\right)},$$

where

$$c_{f,sl}^2(\omega) = 0.5\left(c_a^2 + c_{Snd}^2(\omega)\right) \pm 0.5\sqrt{\left(c_a^2 + c_{Snd}^2(\omega)\right)^2 - 4c_{Snd}^2(\omega)c_{az}^2},\ c_a = \sqrt{B_0^2/4\pi\rho_0},\ c_{az}^2 = c_a^2\cos^2\theta.$$

The condition of magnetoacoustic amplification corresponds to

$$\xi_0 < -\frac{c^2\rho_0}{4\pi\sigma}\frac{\left(c_{0f,sl}^2 - c_0^2\right)}{\left(c_{0f,sl}^2 - c_{az}^2\right)}, \qquad \xi_0 = \frac{\tau_0\rho_0 C_{V\infty}\left(c_\infty^2 - c_0^2\right)}{C_{V0}}, \qquad (18)$$

where

$$c_{0f,sl}^2 = 0.5\left(c_a^2 + c_0^2\right) \pm 0.5\sqrt{\left(c_a^2 + c_0^2\right)^2 - 4c_0^2 c_{az}^2},\ c_{\infty f,sl}^2 = 0.5\left(c_a^2 + c_\infty^2\right) \pm 0.5\sqrt{\left(c_a^2 + c_\infty^2\right)^2 - 4c_\infty^2 c_{az}^2}.$$

Here, $c_{0f,sl}$, $c_{\infty f,sl}$ are low- and high-frequency magnetoacoustic wave speeds of fast (*f*) and slow (*sl*) waves.

Similar to the previous two models discussed in sections 2 and 3, nonlinear evolution of acoustic disturbances up to the second order of smallness, small dissipation and dispersion coefficients are described by same GNAE (8), but with different coefficients [3,4]

$$\mu_\infty = \frac{c^2}{8\pi\sigma}\frac{\left(c_{\infty f,s}^2 - c_\infty^2\right)}{\tau_0 c_{\infty f,sl}^2\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)}; \quad \mu_0 = \frac{c^2}{8\pi\sigma}\frac{\left(c_{0f,s}^2 - c_0^2\right)}{\tau_0 c_{\infty f,s}^2\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)},$$

$$\Psi_\infty = \frac{\gamma_\infty + 1}{2}\frac{c_\infty^2}{c_{\infty f,s}^2}\frac{\left(c_{\infty f,s}^2 - c_{az}^2\right)}{\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)} + \frac{3\left(c_{\infty f,s}^2 - c_\infty^2\right)}{2\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)}, \qquad (19)$$

$$\Psi_0 = \frac{c_0^2(2\gamma_0 - 1)\left(c_{0f,s}^2 - c_{az}^2\right)}{c_{0f,s}^2\gamma_0\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)} + \frac{3\left(c_{0f,s}^2 - c_0^2\right)}{2\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)} - \frac{c_0^2\left(c_{0f,s}^2 - c_{az}^2\right)}{2\Im_{L0T}\gamma_0 c_{0f,s}^2\left(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2\right)}\left(\Im_{L0\rho\rho} + \Im_{L0TT}(\gamma_0 - 1)^2 + 2\Im_{L0\rho T}(\gamma_0 - 1)\right) \quad (20)$$

The coefficients μ_∞ and μ_0 determine wave dissipation caused by the finite electrical conductivity in the high- and low-frequency range of the spectrum, respectively.

In the case of absence of the magnetic field, expressions (19), (20) for high-frequency and low-frequency nonlinearity coefficients equal (16).

The self-sustained pulse amplitude ρ_p depends on slope angle θ and type of magnetoacoustic mode (fast or slow). Moreover, there is the strong dependence ρ_p on the magnetic field. In Fig .2, the typical dependence ρ_p on plasma beta $\beta_P = 8\pi P_0 / B_0^2$ is shown.

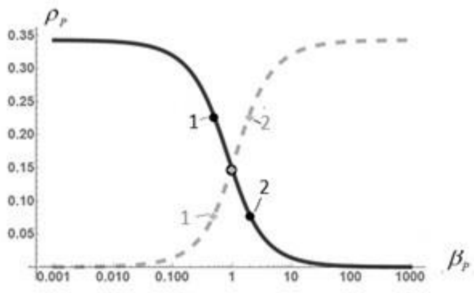

Figure 2: The dependence of the amplitude of the slow (solid line) and fast (dashed line) magnetoacoustic self-sustained pulses on plasma beta. Points 1, 2 correspond β_P =0.5 and 2, respectively.

As it can be seen from Fig. 2, an increase of the external magnetic field (plasma beta decreases) reduces the amplitude of fast wave MHD and increases the amplitude of slow waves MHD.

An amplitude of magnetic field perturbations in the self-sustained pulse equals

$$\frac{\overline{B}_x}{B_0} = -\frac{c_{\infty f,s}^2 \sin\theta}{(c_{az}^2 - c_{\infty f,s}^2)}\rho_p .$$

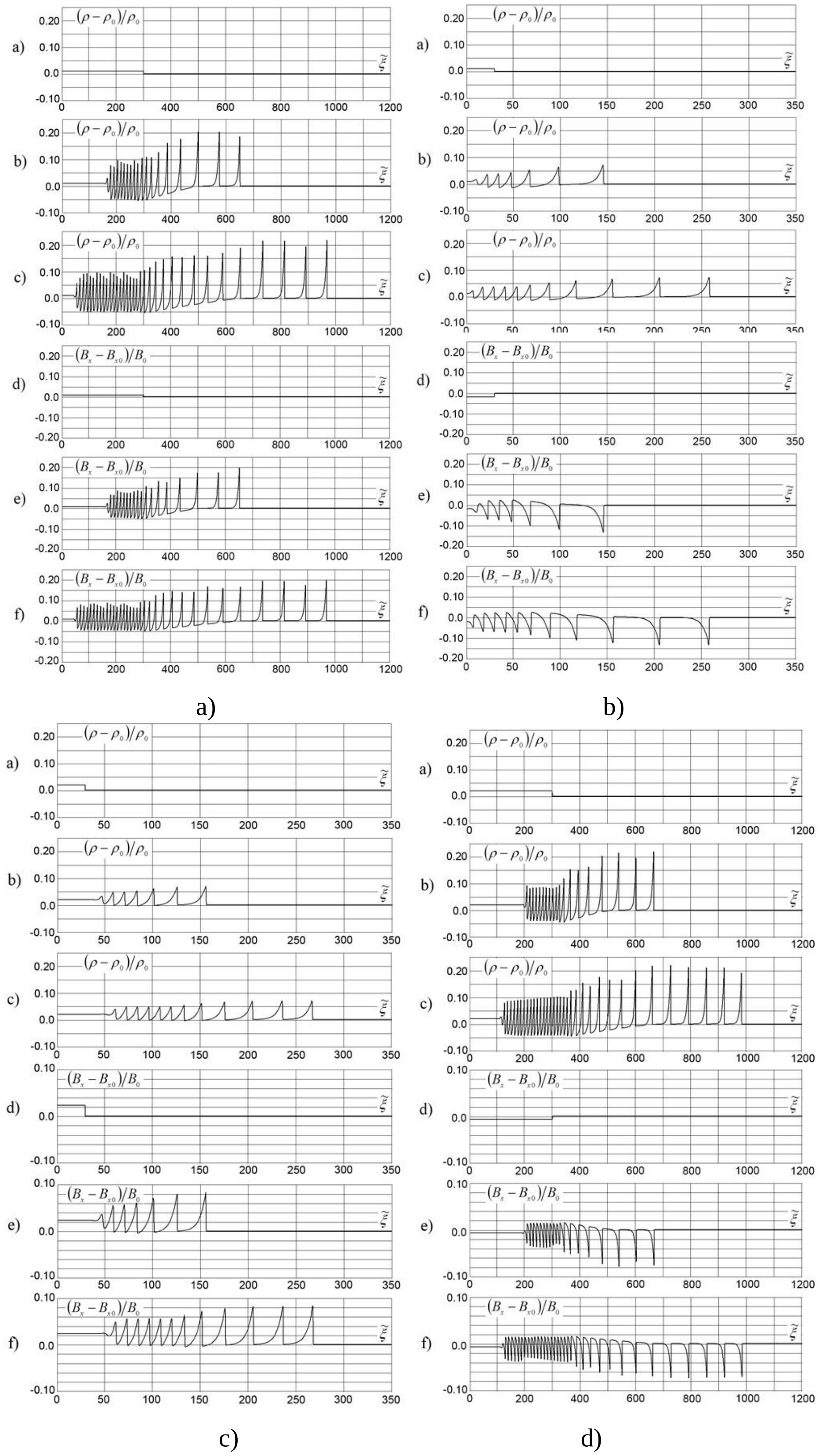

Figure 3: The disintegration of the fast (a),(c) and slow (b),(d) magnetoacoustic waves in the self-sustained pulses in different time moments. $\beta_P = 2$ (a) ,(b) ; $\beta_P = 0.5$ (c), (d); $\theta = \pi/4$.

Results of disintegration of stepwise perturbations for different β_P are shown in Fig. 3. Parameters of numerical simulations have been chosen in the region of magnetoacoustic instability (18).

5. Conclusion

We have shown that GNAE has the same form in three substantially different models of the heat releasing gas. GNAE predicts the disintegration of weak shock waves into a sequence of self-sustained pulses under condition of isentropic (acoustic) instability. Parameters of the self-sustained pulse are determined analytically. Our analysis shows that this decay becomes noticeable in rather long acoustically unstable media. Such media can be both laboratory and natural. For example, trains of propagating fast and slow magneto-acoustic waves have been recorded in the solar corona [5, 6]. The solar corona is an example of a heat-generating medium, which implements various types of thermal instability, including isentropic one [7-9]. Another example of the natural medium with possibility of the isentropic instability is the interstellar gas. Here, the fibrous structure of shock waves is associated with the formation of self-sustained structures in thermally unstable media [2, 10].

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