

CONFORMING RECTANGULAR AND TRIANGULAR PLATE-BENDING ELEMENTS

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1. INTRODUCTION

The finite element displacement method should be capable of describing arbitrarily-shaped, plane boundaries in order to represent configurations like rocket fins and plates with irregular cut outs. Triangular, unlike rectangular elements, can realistically approximate such geometries but they suffer from the disadvantage that their orientation can have a significant effect on accuracy. Cowper et al¹ compare previously available rectangular and triangular plate-bending elements and conclude that their conforming triangular element NRCD is superior. (Conformity in this context means that transverse displacements and slopes are continuous between elements.) However, a conforming rectangular element, UM6, is developed in this paper and shown to be better than NRCD for dynamic problems. UM6 has the additional advantage that it conforms with NRCD so that a combination of these two elements is capable of describing irregularly shaped boundaries. But the combination is only useful if it is more accurate

than using triangular elements alone. This is shown to be the case for the particular example of a rocket fin idealised by a cantilevered triangular plate in bending.

2. RESULTS

The rectangular element UM6, having the polynomial displacement function shown in Figure 1, is formulated to be conformable with triangular element NRCD. Accuracy and convergence characteristics of UM6 were evaluated by comparing the computed natural frequencies and normal modes of simply supported and clamped rectangular plates with exact and/or with the results of Cowper et al's triangular and Mason's rectangular elements.² Figure 3 shows the comparison for a simply supported plate having a sidelength ratio of 40:27. It can be seen from this figure that Mason's 24 degree of freedom rectangular element is more accurate than the other elements for higher natural frequencies, at low constrained degrees of freedom but its rate of convergence to the exact solutions is inferior. Figure 3 also demonstrates the previously known fact that the accuracy of the finite element approximation using NRCD triangular elements depends upon their arrangement. UM6, however, is not only more accurate than either of the two arrangements shown, but converges more rapidly to the exact solutions.

A mesh composed of different combinations of UM6 and NRCD elements is employed in the dynamic analysis of the cantilevered triangular plate shown in Table 1. It is found that as the ratio of the number of rectangular to triangular elements increases the natural frequencies of the plate decrease and tend to the experimental values given in reference 3.

3. CONCLUSIONS

A rectangular plate bending element UM6 has been developed which proves to be generally more accurate and possesses superior convergence characteristics to previously formulated rectangular and triangular elements. UM6 has the additional advantage that it conforms with a triangular element and the resulting combination can realistically describe arbitrarily-shaped, plane boundaries.