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LOW FREQUENCY SOUND ABSORPTION THROUGH A MUFFLER WITH METAMATERIAL LINING

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Acoustic metamaterials have been under extensive study to understand structural responses to sound waves. Owing to the distinguishing capability of manipulating sound waves, a broad range of applications have been realized for these materials. One such application has been to enhance the potentiality of existing sound absorbing devices. Sub-wavelength absorbers that work on the principles of Acoustic Black Hole effect have been considered to achieve total absorption, at least, in theory. There have been a few numerical and experimental studies backing up this research concept [Mironov and Pislyakov, Sov. Phys. Acoust., 2002]. In this work, a theoretical basis has been established for acoustic wave propagation through a terminating muffling structure that completely absorbs the wave by the black hole effect.

In the present study, the similar concept of metamaterials has been applied to design the lining of an open muffler. The expansion chamber lining, essentially demarcates the muffling section within the narrow axi-symmetric muffler duct. The plane wave radiation incident through the inlet end of the muffler undergoes impedance matching as it traverses through this lined flair of varying wall admittance. The impact of the lining on the sound waves has been comparatively analyzed semi-analytically and numerically. The capacity of this lined muffler to absorb low frequency sound has been calculated for particular lining structures. A good agreement between numerical and analytical results is observed.

With the aim of achieving improved sound attenuation through noisy systems, the use of Acoustic metamaterials shows a promising future. Industrial ducts, vents and mufflers that form a major part of such systems, when augmented with the metamaterials can hopefully yield quieter machineries.

Keywords: Acoustic Metamaterials, Acoustic Black Hole effect, Mufflers, Metamaterial Lining, Sound Absorption

1. Introduction

Metamaterials in general and Acoustic Metamaterials in particular, have opened up new opportunities for manipulating and controlling waves. Metamaterials, unlike naturally occurring materials, can be characterized by patterns formed by repetition of a particular unitary cell, such that, the individual nature of the unit cells and their arrangement governs the overall behaviour of the material. Acting upon waveforms by changing its resolution, suppressing or diverting their path of propagation, the Acoustic Metamaterials have added a distinguishable perspective to the way many sensing, masking, absorbing and other such devices have been perceived. By being comparably cost effective, light weight and efficacious, the capability of these artificial materials have been tested to tackle several challenges.

Focusing on sound absorbing devices, a major concern has been the absorption of low frequency noise. Its ability to spread for long distances has been observed to have an adverse impact on human health. The Acoustic Metamaterials are capable of totally attenuating noise at low frequencies. For instance, devices comprising of graded sub-structures working on the principle of the Acoustic Black Hole (ABH) effect are currently being researched upon for their likeliness to act as sub-wavelength absorbers. Numerous research papers [1] reporting ABH based sound attenuation, have been concerned with shear waves or Rayleigh - Lamb waves travelling over surfaces such as single dimensional plates and wedges, multi-dimensional pits, indentations, beams, omni-directional absorbers [2] and more recently, the closed termination tubular structures [3].

In the present paper, ABH principle has been applied to design and analyze an open expansion muffling section. The impact of low frequency sound absorption through such a section has been augmented with the consideration of an internal metamaterial lining. The semi analytical model has been numerically validated to understand the effectiveness of the contribution of metamaterial lining to open-ended muffling systems.

2. Theoretical Background

The application of the ABH principle to an enclosed medium within a terminating structure has been probably for the first time, demonstrated by Mironov and Pislyakov [3]. In theory, they establish that a plane wave propagating through the terminating narrow tubular structure interacts with the varying wall admittance, such that its speed decreases as it approaches the tapering end, where it finally stops and loses its energy by accumulation. Their concept claims that such structures described by power law functions, can be represented as ABHs, as they undergo total absorption of sound using majorly non-absorbing materials. To validate the functionality of such structures, considering both the exclusion and inclusion of absorbing materials, A. Azbaid, et al [4] have carried out experiments, which validate the theory. However, for the case of a realistic model, the modest presence of absorbing materials towards the termination, where most of the energy accumulation takes place, would certainly improve the performance. In addition, Oriol Guasch et al [5] have analyzed the performance of this ABH based structure, numerically using the transfer matrix method (TMM). They conclude that transfer matrices might not be able to satisfactorily describe the lateral cavities as; in order to obtain better results the presence of a lossy medium is critical. They have also; much recently published a comparative analysis of various geometric parameters using TMM claiming it to be handy to approach realizing ABH [6].

3. The Model

Considering the concluding remarks in support of the terminating tubular structure, the ABH principle has been herein applied to an open-ended muffling structure. With reference to the termination based theoretical model, a general wave equation has been derived, a solution to which has been semi-analytically obtained for particular boundary conditions within established assumptions and approximations. With regards to the cautions suggested by Oriol, et al [5], TMM codes have been formulated to numerically analyze the impact of lining in the presence of losses.

The open chamber concept model is as shown in Fig. (1). It comprises of a narrow axisymmetric muffling section with a progressively increasing flare. The flare construction represents the metamaterial lining within the chamber. The shape consideration of the flare itself has been varied from its absence to being linear and quadratic. The distinguishing features of the present model from the retarding structure include the presence of protruding flare and an open-ended terminus. The presence of metamaterial lining in the form of multiple rings offers the varying wall admittance that plays a key role in the absorption of propagating sound waves. The presence of viscous and thermal losses has been accounted for while analyzing the model. To be consistent with the ABH principle, power law function based flare shapes have been studied.

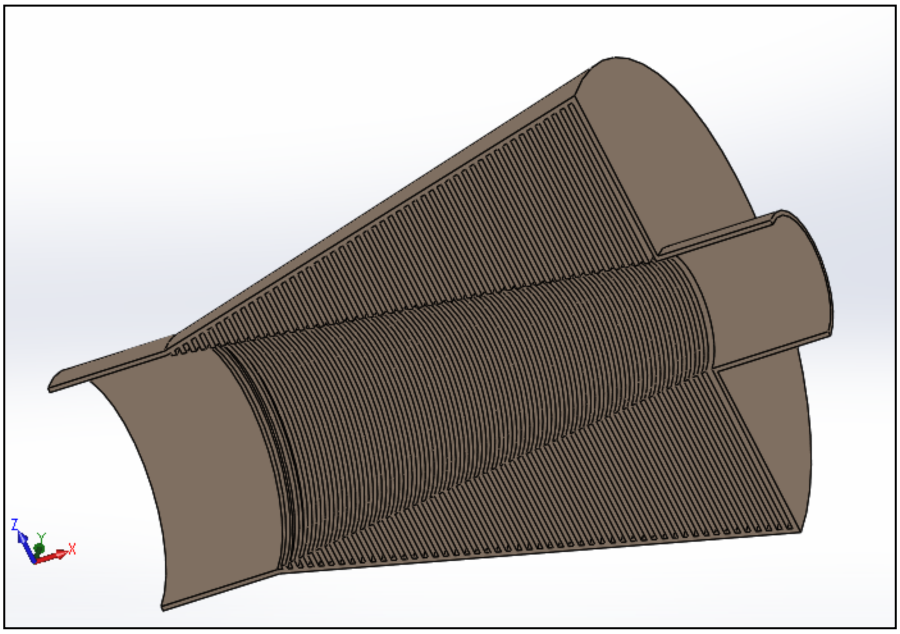

Figure 1: The concept model of muffling section.

4. The Approaches to Modelling

4.1 Semi-Analytical Model

In the previous study [7], a semi analytical approach for the concept model has been established based on the theory developed in [3]. Assuming plane waves propagating through a tube with impeding walls, the general Webster equation has been derived as follows:

$$\frac{d^2p}{dX^2} + 2p'(ln\, r)' + q_L^2\, \zeta\, p = 0 \tag{1}$$

Such that

$$\zeta(X) = 1 + \frac{2iY(X)}{q_r} \tag{2}$$

where,

p - acoustic pressure

$X = x/L$ - dimensionless coordinate

$q_L = k_0 L$ and $q_r = k_0 r$ such that $k_0 = \frac{\omega}{c}$ is the wave number in air

$Y(x) = \frac{v_\perp}{p}$ - wall admittance and $v_\perp$- radial component of particle velocity

The function$\zeta(X)$ as defined in Eq. (2), comprises of $Y(X)$, the spatially varying wall admittance. Taking into account the presence of visco-inertial and thermal exchanges between air and the rings, $Y(X)$ can be expressed as,

$$Y(X) = \left.\frac{v_\perp}{p}\right]_r = \frac{\rho_0 c}{iz_w}\frac{\left(J_0'(q_{wr}) - \frac{J_0'(q_{wR})}{Y_0'(q_{wR})}\, Y_0'(q_{wr})\right)}{\left(J_0(q_{wr}) - \frac{J_0'(q_{wR})}{Y_0'(q_{wR})}\, Y_0(q_{wr})\right)} \tag{3}$$

where,

$q_{wr} = k_w r$ and $q_{wR(X)} = k_w R(X)$ and subscript w implies wall of the internal structure,

J_0, Y_0 - Bessel functions of order zero and prime denotes derivatives with respect to argument.

The presence of k_w and z_w in Eq. (3) are suggestive of the presence of losses.

As per the desired flare shape to be considered, the variations of the outer and inner flare radii can be used to define a spatial function, which shall be used to obtain particular solution.

In the presence of losses, solving any further analytically would be a cumbersome task and thus a numerical approach has been chosen.

4.2 Numerical Approach

Realizing the concept model numerically, the entire chamber has been sliced into repetitive units based on the number of internal rings to be considered as in Fig (2). Each unit is such that it is a union of the slit part and the ring part together with its corresponding main cylindrical pore. The ring part of the unit is made up of its main pore segment while the slit half along with its main pore segment has been further referred to as the cavity. The internal and external radii and other geometric features such as the ring and slit thickness have been defined. Similar to the method followed in another study [5], transfer matrices have been constructed, for each such section, which are then varied spatially over the span of the entire muffling section to obtain the overall matrix.

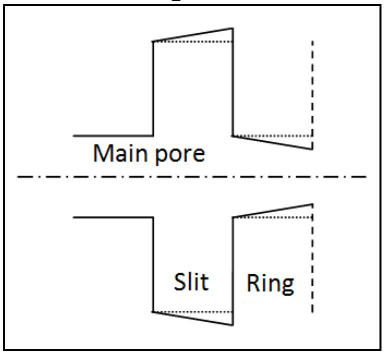

Figure 2: The singular unit for transfer matrix consideration.

The unitary transfer matrix constitutes the relation between the acoustic pressure to the particle velocity at both sides in the presence of the viscous and thermal exchanges.

For the slit part of the unit, the complex density governed by the viscous forces and the compressibility governed by the thermal exchanges have been individually derived and then combined to form the complex wave number and the impedance to account for the losses, as detailed for the analysis of slit-like pore [8].

$$\rho_w = \frac{\rho}{F_v} \tag{4}$$

$$C = \gamma\left(1 - \frac{F_t(\gamma - 1)}{\gamma}\right) \tag{5}$$

Using Eq. (4) and Eq. (5), the complex wave number and the impedance are calculated as,

$$k_{slit} = k_0\sqrt{C\,\rho_w} \tag{6}$$

$$z_{slit} = z_0\sqrt{\frac{\rho_w}{C}} \tag{7}$$

where,

ρ_w- complex density, ρ – density of air and C - complex compressibility
F_v and F_t- viscous and thermal forces per unit volume of the element respectively [8]
γ- adiabatic constant
$z_0 = \rho_0 c$ - impedance of the air

The matrix characterized by the cavity has been derived as a combination of the lossy medium coupled with the influence of the normalized wall admittance as per Eq. (3). Thus accounting for the wall admittance in combination with complex parameters of the slits,

$$\gamma_{norm} = i\left(\frac{z_0}{z_{slit}}\right)\left(\frac{J_1(k_{slit}\, r) - \dfrac{J_1(k_{slit}\, R)}{H_1(k_{slit}\, R)}\, H_1(k_{slit}\, r)}{J_0(k_{slit}\, r) - \dfrac{J_1(k_{slit}\, R)}{H_1(k_{slit}\, R)}\, H_0(k_{slit}\, r)}\right) \tag{8}$$

The complex wave number and impedance of the entire cavity can be now obtained as,

$$k_{cav} = k_0\sqrt{1 + \frac{2i\gamma_{norm}}{k_0\, r}} \tag{9}$$

$$z_{cav} = \frac{z_0}{\sqrt{1 + \dfrac{2i\gamma_{norm}}{k_0\, r}}} \tag{10}$$

As follows from Eq. (1) and Eq. (2), here,

k_{cav} and z_{cav} - complex wave number and impedance of the cavity respectively

The cavity transmission matrix for a slit section [9] can thus be formed as,

$$A_{cav} = \begin{bmatrix} cos(k_{cav}\, 2x) & -iz_{cav}\, sin(k_{cav}\, 2x) \\ -\dfrac{i\, sin(k_{cav}\, 2x)}{z_{cav}} & cos(k_{cav}\, 2x) \end{bmatrix} \tag{11}$$

The ring half of the unit, which is essentially the main pore, has been described by the Johnson-Champoux-Allard-Lafarge (JCAL) model [8,10] to account for the visco-inertial and the thermal losses.

$$\rho_w = \rho\alpha_\infty\left(1 + \frac{\sigma}{-i\omega\alpha_\infty\rho}\sqrt{1 + \frac{-i\omega}{\omega_b}}\right) \tag{12}$$

$$C = \gamma - \frac{\gamma - 1}{1 + \dfrac{\eta}{-i\omega\sqrt{Pr}\rho k'}\sqrt{1 - i\sqrt{Pr}\left(\dfrac{\omega}{\omega'}\right)}} \tag{13}$$

where,

Pr - Prandtl Number σ - air flow resistivity α_∞- tortuosity

ω_b and ω' are defined based on the parameters of the cylindrical pore geometry [8]

k_{ring} and z_{ring}have been formulated by substituting Eq. (12) and Eq. (13) into Eq. (6) and Eq. (7) respectively to obtain the ring transmission matrix for the ring section [9] as,

$$A_{ring} = \begin{bmatrix} cos\left(k_{pore}\, h\right) & -iz_{pore}\, sin\left(k_{pore}\, h\right) \\ -\dfrac{i\, sin\left(k_{pore}\, h\right)}{z_{pore}} & cos\left(k_{pore}\, h\right) \end{bmatrix} \tag{14}$$

The TMM of the single unit is then calculated as $A = A_{ring} \times A_{cav}$

Finally, combining this along with expansion and contraction matrices, as necessary, the transmission matrix for the unit is iterated to completely define the overall matrix of the model.

5. Discussion

The model outlined above has been applied study the impact of the lining in the presence and absence of the external flare. In addition, a comparison has also been made between the linear and the quadratic external flare shapes. The observations for a particular case, referring to Fig.(3), have been compiled and presented for transmission loss and absorption coefficient. The dimensions are as follows:

$r = 0.05\, m; r_{min} = \frac{r}{2}; R = 2r; L = 0.2\, m; h = 0.002\, m; x = 0.001\, m; N = 50$, where r_{min} is minimum radius of the tube, $R$is the maximum external radius, h is thickness of the rings and x is half distance between them.

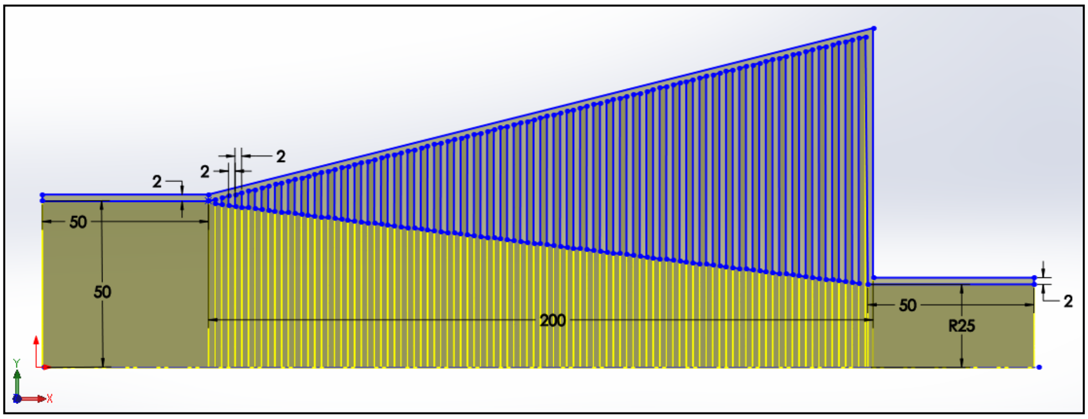

Figure 3: The geometry of the structure shown for half cut section of the muffler.

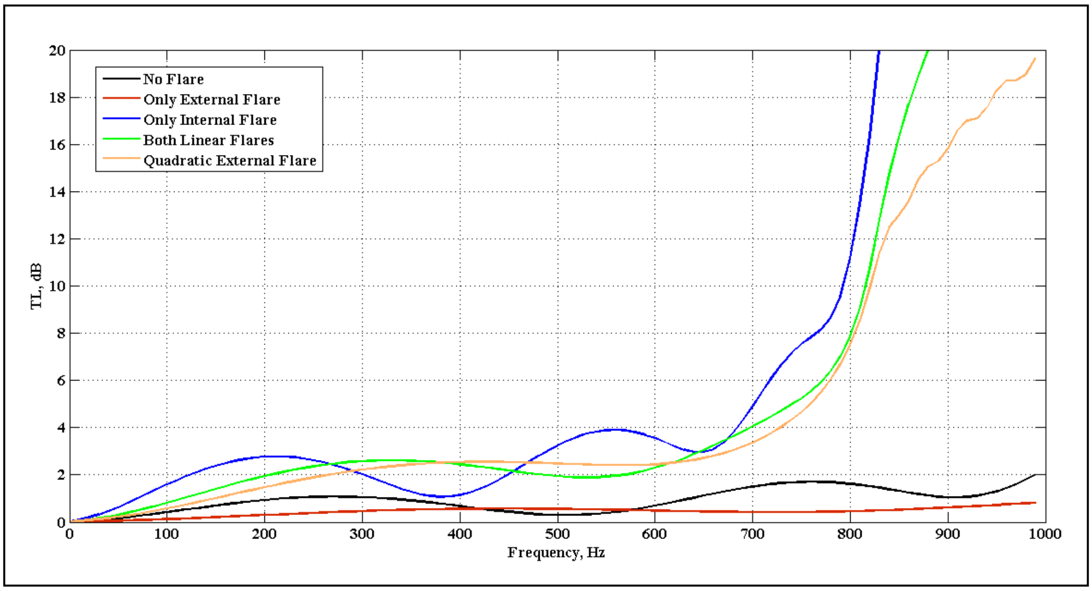

Figure 4: The transmission loss (dB) comparison**.**

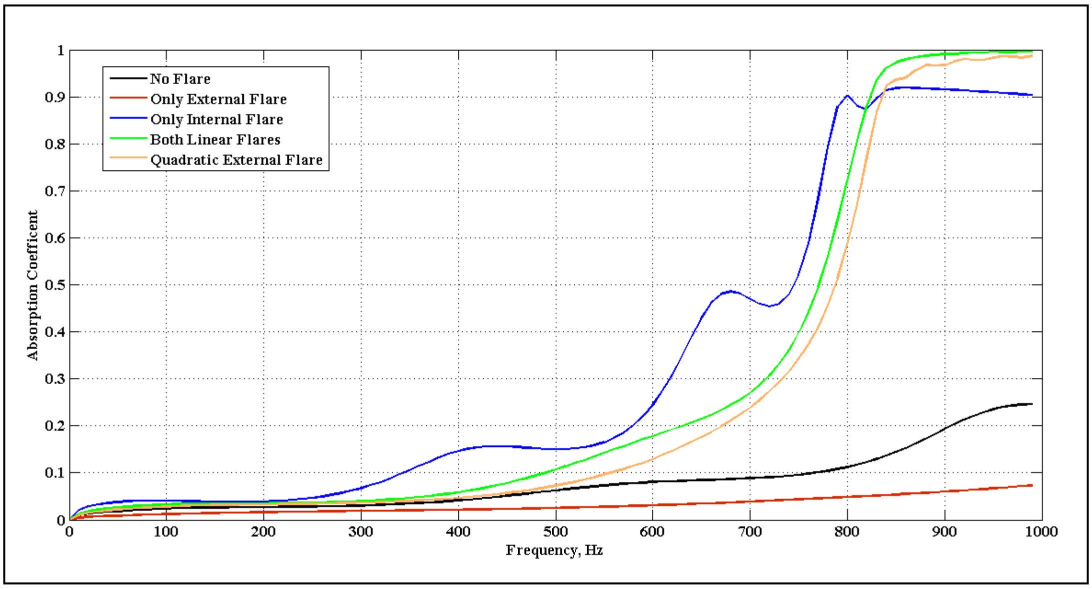

Figure 5: The absorption coefficient comparison**.**

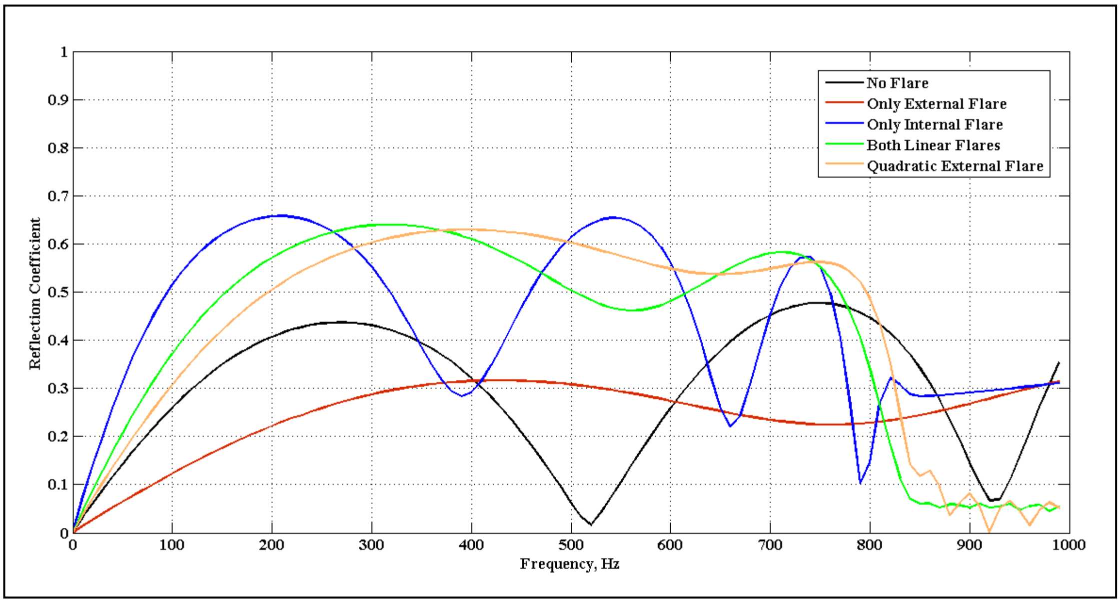

Figure 6: The reflection coefficient comparison**.**

From the comparative study, impact of the flare can be clearly understood. Fig. (4) and Fig. (5) show the transmission loss and absorption coefficient when the model is in transmission regime respectively. Fig. (6) shows the reflection coefficient variation. The absence and presence of flare section have been compared. It is evident that the presence of only an external flare has trivial impact quite similar to the case of no flare. The presence of an internal flare on the other hand, imparts a very strong boost to the coefficients. Of the three cases, where the internal flare is present, the case where only the internal flare is present, shows high values over a narrow range of frequency as compared to the case where the linear internal flare is balanced with the linear or quadratic external flare. This suggests the highly sensitive nature of the internal flare radius. Among the linear and the quadratic external flare, the quadratic is a steadier performer. This supports the theory which states that a power-law variation has higher prominence over the approximated linear variation.

6. Conclusion and Future Scope

The presence of flare in a muffler has higher performance capability than the simple expansion chamber design, has been validated. The comparisons have been presented in terms of the transmission loss, absorption and reflection coefficient plots. In addition, the plots include a comparison between the linear versus the quadratic outer flare shaped muffler.

The outcome, in terms of the transmission loss and the absorption and reflection coefficient plots in the transmission regime have been studied to appropriately conclude that although the linear flare is more effective in the lower frequency zone, the quadratic flare case is definitely recommended for a stable performance.

As this study is a part of an ongoing research work at the University of Salford, the upcoming milestones include the application specific optimization of the concept model. This would be carried out in terms of a plurality of experiments designed using the FEM based solver which will be further validated with the real model built using the rapid prototyping tools.

7. Acknowledgement

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REFERENCES

1 V. V. Krylov, Acoustic black holes: Recent developments in the theory and applications, *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **61** (8), 1296-1306, (2014).

2 O. Umnova, B. Zajamsek, Omnidirectional graded index sound absorber,*Proceedings of the Acoustics 2012 Nantes Conference*,Nantes, France, 23-27 April, (2012.)

3 M. A. Mironov and V. V. Pislyakov, One Dimensional acoustic waves in retarding structures with propagation velocity tending to zero, *Acoustics Physics*, **48** (3), 347–352, (2002).

4 A. El-Ouahabi, V. V. Krylov and D. J. O'Boy, "Investigation of the acoustic black hole termination for sound waves propagating in cylindrical waveguides", *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, **250** (6), 636-645, (2015).

5 O. Guasch, M. Arnela, P. Sanchez-Martin, Transfer matrices to analyze the acoustic black hole effect in duct terminations, *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, 2431-2439, (2016)

6 O. Guasch, M. Arnela, P. Sanchez-Martin, Transfer matrices to characterize linear and quadratic acoustic black holes in duct terminations, *Journal of Sound and Vibration*,**395**, 65-79, (2017)

7 N. Sharma, O. Umnova, A. T. Moorhouse, Analysis of a low frequency muffler based on the acoustic black hole effect, *Proceedings of the Institute of Acoustics*, Acoustics 2016, **38** (1), (2016)

8 M. C. Wright, M. Ed., *Lecture Notes on the Mathematics of Acoustics*, Imperial College Press, (2005).

9 M. L. Munjal,, *Acoustics of Ducts and Mufflers*, Ed, John Wiley & Sons, (2014)

10 P. Leclaire, O Umnova, T. Dupont, R. Panneton, Acoustical properties of air-saturated porous material with periodically distributed dead-end pores, *Journal of Acoustic Society of America*, **137**(4), 1772-1782, (2015).