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ACOUSTIC ATTENUATION IN DISSIPATIVE SPLITTER SILENCERS CONTAINING MEAN FLUID FLOW

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1. INTRODUCTION

Splitter type silencers, which incorporate parallel baffles consisting of bulk-reacting porous sound-absorbing material, are very widely used in flow duct attenuators. Design methods for such attenuators must obviously be based on appropriate theoretical models, but there appears to be a total absence of adequate models reported in the literature. Mechel [1,2] reported an analysis of splitter silencers with locally-reacting baffles, but the very important physical effect of wave transmission *through* the baffles is lost by his assumption of local reaction. In reference [1], Mechel states his opinion that a bulk liner of finite length "can be treated only by finite element methods" but, quite clearly, it is possible to use other numerical techniques (such as finite differences) or analytical methods. Tam [3] investigated the intensity distribution in a splitter silencer but, again, he assumed the baffles to be locally reacting. Other, previous, pieces of work on sound transmission in ducts with bulk liners (for example, the papers by Cummings [4] and Wassilieff [5]) have been concerned with ducts lined on either one or two opposite sides and, as we shall see, certain essential features of splitter silencers are inevitably omitted in such analyses. To obtain a complete model of a splitter silencer, one has to account for the sound fields in *all* the baffles, coupled by appropriate boundary conditions.

In this paper, we give an analytical treatment of sound propagation in dissipative duct silencers of the parallel baffle type, without accounting for "end effects" (which are currently being investigated). Mean flow in the air passages is taken into account, as is anisotropy of the absorbent.

Eigen-solutions that satisfy the governing differential equations and prevailing boundary conditions are sought; in these, there is a common wavenumber in all gas-flow passages and in the sound-absorbing baffles. We examine the structure of some of the modes of propagation, their relative axial attenuation rates and the implications of these data in the context of air-flow duct attenuators.

2. GEOMETRY

Figure 1 shows an example of the type of geometry with which this work is concerned. Any variation in the modal sound field along the z-axis will simply involve the incorporation of a cosine function into the transverse eigenfunction; this can be introduced if required (see, for example, the paper by Cummings [4]). The situation considered here involves no z-dependence, and so the geometry is that of Figure 2. The duct consists of two liners - of width h - placed against the walls, and a number of central splitters - of width 2h - positioned a distance *a* apart. There is a uniform mean flow (of Mach number M) which is assumed to be restricted to the airway. The lining may be anisotropic, so two wave numbers, k_a and k_t, and two complex densities, ρ_a and ρ_t, are needed to represent these properties in the axial and transverse directions respectively. The corresponding (real) values in the airway are k_o and ρ_o.

3. THEORY

The acoustic pressure, p_i^*, and the normal particle displacement, u_i^*, vary harmonically with time, and modal solutions of the type

$$p_i^*(x,y_i) = p_i(y_i)\ e^{i(\omega t-\alpha x)} , \qquad (1)$$

$$u_i^*(x,y_i) = u_i(y_i)\ e^{i(\omega t-\alpha x)} , \qquad (2)$$

are sought for each region i. In these expressions, ω is the radian frequency and α the complex axial wave number for a mode. The eigenfunctions p_i must satisfy the wave equation corresponding either to the airway or the liner,

$$\frac{d^2p_i}{dy_i^2} + (k_o - \alpha M)^2 p_i - \alpha^2 p_i = 0 \qquad \text{for even } i, \tag{3}$$

$$\frac{1}{k_t^2}\frac{d^2p_i}{dy_i^2} + \left(1 - \frac{\alpha^2}{k_a^2}\right) p_i = 0 \qquad \text{for odd } i, \tag{4}$$

where (3) is the convected wave equation and (4) is the wave equation for an anisotropic sound-absorbing medium (see reference [5]). The eigenfunctions are expressed in the form

$$p_i = A_i \cos k_i y_i + B_i \sin k_i y_i, \tag{5}$$

in which case equations (3) and (4) yield

$$(k_o - \alpha M)^2 - \alpha^2 - k_i^2 = 0 \tag{6}$$

and

$$1 - \frac{k_i^2}{k_t^2} - \frac{\alpha^2}{k_a^2} = 0. \tag{7}$$

The boundary conditions imposed are zero normal particle displacement on the walls and continuity of pressure and normal displacement on the boundaries between airway and liner, i.e.

$$u_1(0) = 0 \tag{8}$$

$$u_n(d_n) = 0 \tag{9}$$

$$p_i(d_i) = p_{i+1}(0) \tag{10}$$

$$u_i(d_i) = u_{i+1}(0), \tag{11}$$

where d_i is the width of the region i. Equations (8), (9) and (11) may be rewritten in terms of the pressure gradient,

$$\frac{dp_1}{dy_1}(0) = 0, \tag{12}$$

$$\frac{dp_n}{dy_n}(0) = 0, \tag{13}$$

$$\frac{1}{\rho_i}\frac{dp_i}{dy_i}(d_i) = \frac{1}{\rho_{i+1}}\frac{dp_{i+1}}{dy_{i+1}}(0), \quad i=1,\ldots\ n-1, \tag{14}$$

where $\rho_i = \rho_L$ for odd i and $\rho_i = \rho_o(k_o - \alpha M)^2$ for even i. Substituting (5) in these equations then gives

$$A_i \cos k_i d_i + B_i \sin k_i d_i = A_{i+1} \tag{15}$$

$$B_1 = 0 \tag{16}$$

$$-A_n \sin k_n d_n + B_n \cos k_n d_n = 0 \tag{17}$$

$$\frac{k_i}{\rho_i}(-A_i \sin k_i d_i + B_i \cos k_i d_i) = \frac{k_{i+1}\ B_{i+1}}{\rho_{i+1}}. \tag{18}$$

The coefficient B_1 is removed from the formulation to leave 2n-1 equations in 2n unknowns (including α). If, now, the resulting equations are written in matrix form

$$X_{ij}Y_j = 0, \tag{19}$$

where $[Y] = (A_1\ A_2\ B_2\ \ldots\ldots A_n\ B_n)^T$,

then the resulting eigen-problem for eigenvalues α, and eigenfunctions Y_j, is

$$\det(X_{ij}) = 0. \tag{20}$$

4. COMPUTATION

Equation (20) has an infinite number of solutions, and the eigenvalues must be kept in some sort of order so that no values are missed. Another feature of the problem is that the for a duct containing several splitters, an expansion of the determinant in equation (20) gives an expression that is difficult to differentiate analytically, so a method like the Newton-Raphson scheme for finding the roots is ruled out unless numerical differentiation is employed. The method used by the authors was one developed by Muller [6] . It involves fitting a quadratic curve to three guesses of the root and the corresponding values of the left-hand side of equation (20); the roots of the quadratic may then easily be found. One of these roots replaces an initial guess and this process is repeated until a satisfactory degree of convergence is achieved. (Muller's method also has a number of other advantages over the Newton-Raphson method.)

At the lowest frequency of the range in which data were required, a development of the method reported by Cummings [7] was used to find the modal values of α; in this, the bulk acoustic properties of the absorbent were progressively changed from those of the gas to those of the porous medium and the modal wavenumbers tracked during the process. The hard-wall wavenumbers were used as initial values in the iterative procedure, and an extrapolation method involving the fitting of quadratic polynomials to the modal wavenumbers was used during the tracking process, in order to avoid the "jumping" between modes that so often plagues methods of this kind. When the wavenumbers at the lowest frequency had been found, the frequency was increased, and the iterated solutions at a particular frequency used as initial values for the frequency above. A similar tracking method was used in modal tracking in the frequency domain.

5. RESULTS

Measured data were taken in an experimental duct that had a lined test section with one central splitter, 105 mm wide, and two half-thickness linings on the walls. The airway widths were 134 mm. There was no air-flow in the duct. The absorbent was a semi-rigid glass fibre slab with steady flow resistivities (respectively) normal to, and parallel to, the fibres of 12 870 and 6 570 SI rayl/m. The sound source was a loudspeaker array that could generate a series of selected acoustic modes. The bulk acoustic properties of the absorbent were measured in separate experiments.
Figure 3 shows the predicted and measured axial decay rates, in dB/m, for the fundamental mode and the first cross-mode in the duct. Agreement between theory and experiment is acceptably good up to 2 kHz. The most interesting (and disturbing) feature of these results is that the attenuation of the first cross-mode is very little different from that of the fundamental mode from 500 Hz to 1 kHz, and is substantially *less* between 1 and 2 kHz. The implications of this are that the least attenuated mode in an array of splitters may not be the fundamental mode (as is often popularly supposed), but could be a higher order mode in the *entire* splitter system. Therefore, the whole splitter array - and not just one "module" of it - must be taken into account in the analysis. In figure 4, the measured and predicted phase speed of the fundamental mode and the first cross-mode are compared. Again, agreement is generally good, though some discrepancies are noted at low frequencies in the case of the first cross-mode.

Figures 5-6 show comparisons between the predicted and measured transverse sound pressure and phase distribution for the fundamental mode and the first cross-mode at 1 kHz. Again, agreement between prediction and measurement is good. The range of sound level in the fundamental mode is less than 2 dB (note the expanded scale), whereas in the the first cross-mode a deep and sharp minimum in the centre of the duct is interposed between two regions of almost constant sound level. In the latter case, the expected phase change of about 180° occurs in the centre of the duct, whilst in the former case, the angle varies within a range of 60°.

The theoretical model takes account of mean flow, but space does not permit the inclusion of predicted data with flow here. As one would anticipate, the effects of mean flow on the attenuation are much as they are in the case of ducts lined on two opposite sides [4].

6. DISCUSSION

The theoretical model described here will predict the modal sound field, including the axial attenuation rate, for any desired number of acoustic modes in a duct containing an arbitrary number of bulk-reacting sound-absorbent baffles having anisotropic bulk properties, in the presence of a uniform mean airflow. It yields demonstrably accurate predictions, and can be used in conjunction with an appropriate procedure to match the modal expansions for the sound fields in the lined and unlined duct sections, thereby accounting for "end effects" at the terminations of the silencer.

It has been shown here that one has to model the *entire* system in order to be sure of the attenuating characteristics of the silencer. It is *not* enough to take a single module of the silencer as being representative of the whole.

7. ACKNOWLEDGEMENTS

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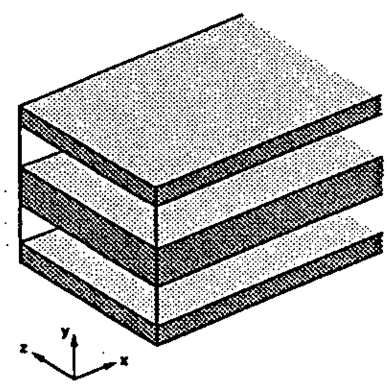

Figure 1. Splitter configuration

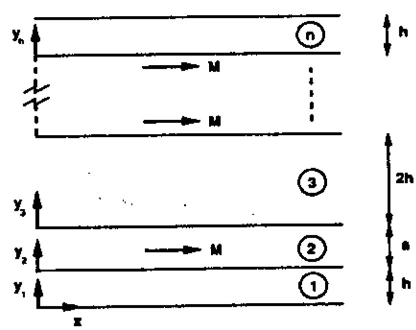

Figure 2. Coordinate systems

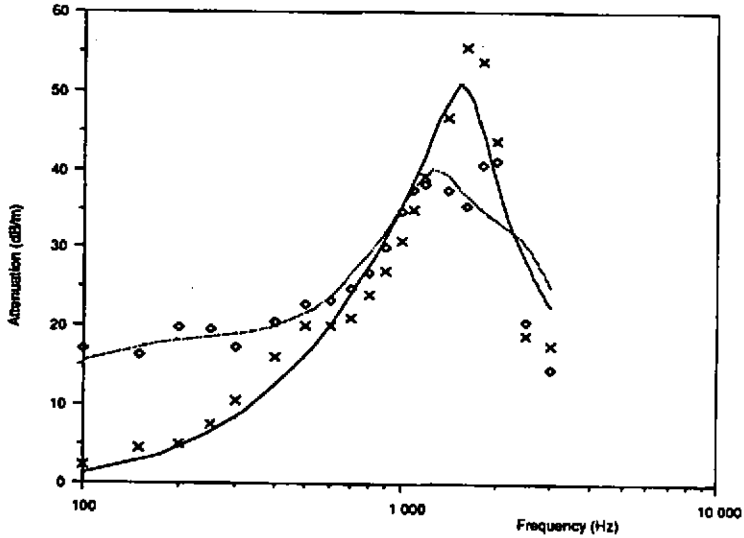

Figure 3. Modal attenuation rate for the fundamental mode (× , measured ; ——, predicted) and the first cross - mode (◇ , measured ; ·········, predicted)

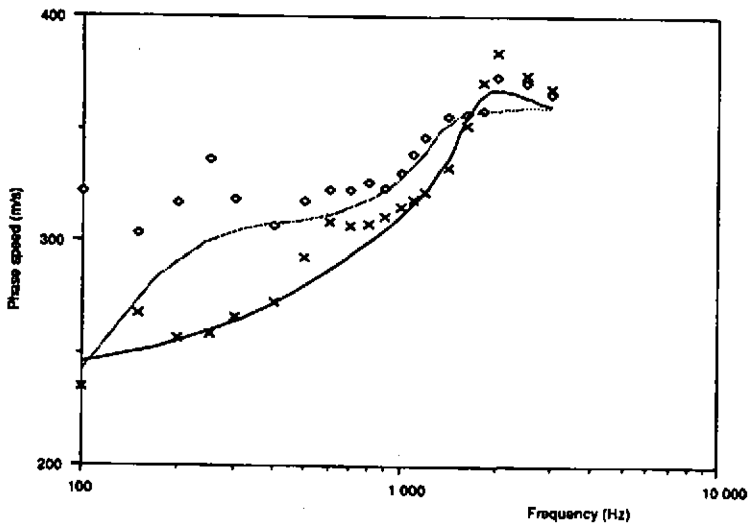

Figure 4. Phase speed of the fundamental mode and first cross - mode (symbols as in figure 3.)

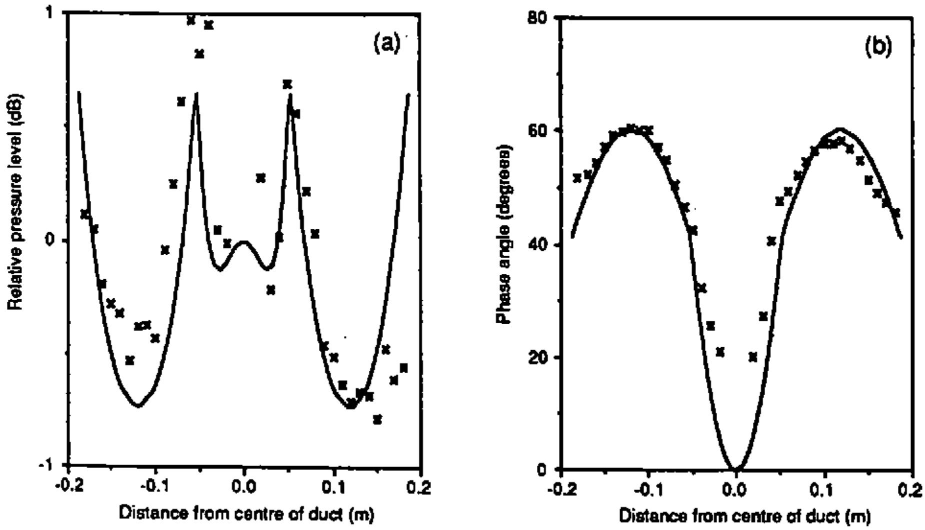

Figure 5. Transverse sound level (a) and phase (b) patterns for the fundamental mode (x , measured; ——— , predicted)

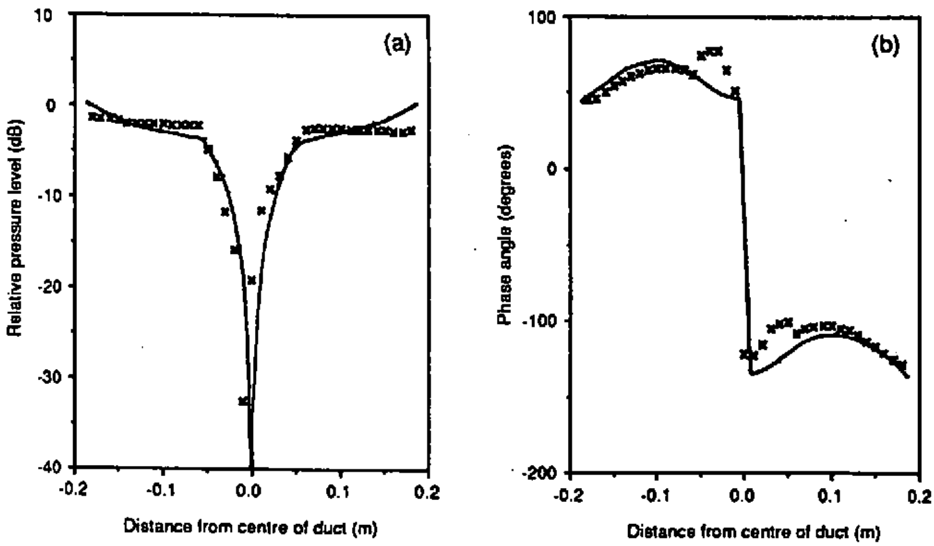

Figure 6. Transverse sound level (a) and phase (b) patterns for the first cross mode (x , measured; ——— , predicted)