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CLUSTERIZATION FOR DERIVING EIGENPAIRS OF A VIBRO-ACOUSTIC RECTANGULAR CAVITY

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Considerable efforts have been made with a view to deriving the eigenpairs of a strongly coupled rectangular cavity comprising a flexible panel and five rigid walls, which has frequently been employed as a target model for discussing transmission loss of a cavity-backed panel. The eigenpairs of even such a simplistic cavity however have not been found for decades. The reason is obvious: a spatial boundary condition of a distributed parameter system made it difficult to solve the eigenvalue problem. Just recently, the authors succeeded in deriving the eigenpairs of the strongly coupled rectangular cavity by introducing a cluster functions that may tackle the spatial boundary condition. As a result of coupling, two kinds of acoustic modes are found to appear; standing wave mode and evanescent mode. The former is generated by sound reflections in the cavity, whereas the latter by coupling effect. It is likely that the dimensions of a characteristic matrix of a coupled cavity increase because of a distributed parameter system being dealt with. This paper then presents a clusterization method in order to reduce the computation burden. For this purpose, it is shown that the characteristic matrix of a cavity is dominated by a coupling coefficient matrix. Inasmuch as coupling is strictly selective, the coupling matrix is found to be expressed in a form consisting of four independent cluster matrices. It is shown that the eigenpairs of the coupled cavity may then be obtained by aggregating eigenpairs of each cluster.

1. Introduction

The term "coupling" indicates interference between a structural and an acoustic field of a cavity, resulting in the shift of the eigenpairs of uncoupled system dynamics. Depending on the degree of coupling, cavity systems can be classified into two categories: a weakly coupled cavity system (or a modally coupled cavity) system and a strongly coupled cavity system.

A weakly coupled cavity system often introduced in sound transmission control problems [for instance, 1,2] is based on the modal coupling theorem established under the assumption that the fluid medium is non-dense and the cavity walls not "thin." The characteristic of this system is that the eigenfunctions of a coupled system remain the same as those of an acoustically rigid walled cavity, while only the eigen-frequencies of the cavity change.

When cavity walls become thin and the cavity gap shallow, the assumption of a modal coupling is no longer valid; thus, such a case falls into the second category, i.e., a strongly coupled cavity system. Considerable efforts have been made in literature to derive the exact solution of coupled rectangular cavity system that comprises five rigid walls and a flexible panel. Dowell and Voss [3] expressed the sound pressure acting on a cavity-backed panel as a linearized form of Bernoulli's equation; Pretlove [4], in an attempt to spatially match the structural and acoustic mode shapes, introduced a cosine series expansion for simulating the cavity-backed panel deflection which is originally expressed as a sine-sine function, however, convergence of the method was not shown; Tanaka *et al.* [5-7] presented a set of cluster functions falling on the category of essentially degenerate eigenfunctions possessing the same eigen-frequency in common, deriving explicitly the eigenpairs of a strongly coupled cavity. Employing the cluster function, Tanaka *et al.* [8, 9] succeeded in deriving the eigenpairs of a strongly coupled eigenpairs, clarifying its fundamental properties.

This paper begins by overviewing the eigenpairs derivation of a strongly coupled rectangular cavity comprising five rigid walls and one flexible panel which Tanaka *et al*. derived [8, 9]. The basic characteristics of the eigenpairs of a strongly coupled cavity are then deciphered with a particular emphasis on an evanescent cavity mode emerging as a result of strongly coupling between an acoustic field and a structural field. It is likely that the dimensions of a characteristic matrix of a coupled cavity increase because of a distributed parameter system being dealt with. With a view to overcoming this problem, this paper then presents a clusterization method which may reduce the computation burden. It is shown that the characteristic matrix of a cavity is dominated by a coupling coefficient matrix. Inasmuch as coupling is strictly selective, the coupling matrix is found to be expressed in a form consisting of four independent cluster matrices. It is then shown that the eigenpairs of the coupled cavity may be obtained by merely aggregating eigenpairs of each cluster.

2. Eigenpairs of a strongly coupled rectangular cavity and clusterization

2.1 Overview of eigenpairs derivation of a strongly coupled rectangular cavity

Consider a rectangular cavity comprising a flexible panel placed on the top and five acoustically rigid walls as shown in Fig.1

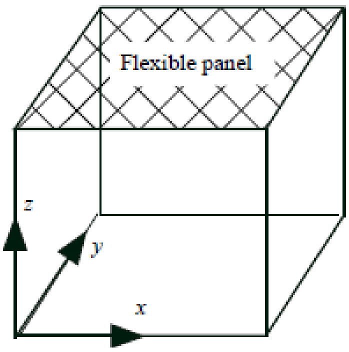

Figure 1: Rectangular cavity model with a flexible panel on the top

Sound wave equation in the cavity may then be written as

$$c^2\nabla^2\bar{\phi}\,(x,y,z)+\bar{\varpi}^{\,2}\bar{\phi}\,(x,y,z)=0 \tag{1}$$

where c, $\bar{\phi}$, $\bar{\varpi}$ denote the sound speed, velocity potential and eigenvalue after coupling. Note that a bar in the expression implies parameters after coupling. Since the above is a homogeneous equation so that as a whole the solution to Eq.(1) may not be written using an expansion theorem, however we dare to introduce the expansion theorem using a cluster function $\tilde{\phi}_i$ as

$$\bar{\phi}\,(x,y,z)=\sum a_i\tilde{\phi}_i(x,y,z) \tag{2}$$

where the cluster function satisfies the following homogeneous sound wave equation

$$c^2\nabla^2\tilde{\phi}_i(x,y,z)+\bar{\varpi}^2\tilde{\phi}_i(x,y,z)=0 \qquad \forall i \tag{3}$$

Equation (3) denotes that the cluster function is an essentially degenerated eigenfunction with the common eigenvalue $\bar{\varpi}$, and hence the superposition of the cluster functions satisfies the homogeneous sound wave equation. The cluster function is further decomposed to

$$\tilde{\phi}_i(x,y,z) = \psi_i(x,y)\eta_i(z) \tag{4}$$

where

$$\psi_i(x,y) = \cos\frac{l_i\pi}{L_x}x\cos\frac{m_i\pi}{L_y}y\ \ (l_i, m_i = 0,1,2,3...) \tag{5}$$

$$\eta_i(z) = \cos\gamma_{l_i m_i} z \tag{6}$$

whereby the sound wave equation may be rewritten as

$$\left(\frac{l_i\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2 + \gamma_{lm}^2 = \frac{\varpi^2}{c^2} \tag{7}$$

Equation of motion of a panel may be written as

$$D\nabla^4\overline{v}\ (x,y) - \varpi^{\ 2}\rho h\overline{v}\ (x,y) = \varpi^{\ 2}\rho_a\overline{\phi}\ (x,y,z)\Big|_{z=L_z} \tag{8}$$

where D, $\overline{v}$, ρ , h , ρ_a are flexural rigidity, surface velocity, density of the panel, thickness of the panel, and air density, respectively. The above is intrinsically homogeneous equation, however because of a coupling effect, the panel behaves in a manner that is subjected to sound pressure from inside the cavity which acts and an external force, hence Eq.(8) may then be expressed using an expansion theorem,

$$\overline{v}\ (x,y) = \sum_{i=1}^{n} b_i\varphi_i(x,y) \tag{9}$$

where φ_i is the *in vacuo i*th vibration modal function that satisfies

$$D\nabla^4\varphi_i(x,y) - \omega_i^{\ 2}\rho h\varphi_i(x,y) = 0 \tag{10}$$

and where ω_i is the associated eigen frequency.

Next, spatial boundary condition is given by

$$v\ (x,y) = \frac{\partial}{\partial z}\overline{\phi}\ (x,y,z)\Big|_{z=L_z} \tag{11}$$

Using the expansion theorem, the equation of motion of a panel may then be described as

$$\sum_{\kappa=1}^{n}\left(\omega_\kappa^2 - \varpi^{\ 2}\right)\rho h b_\kappa\varphi_\kappa(x,y) = \varpi^{\ 2}\rho_a\sum_{\kappa=1}^{m} a_\kappa\tilde{\phi}_\kappa(x,y,L_z) = \varpi^{\ 2}\rho_a\sum_{\kappa=1}^{m} a_\kappa\psi_\kappa(x,y)\eta_\kappa \tag{12}$$

Likewise, the boundary condition may also be written as

$$\sum_{\kappa=1}^{n} b_\kappa\varphi_\kappa(x,y) = \sum_{\kappa=1}^{m} a_\kappa\psi_\kappa(x,y)\eta_\kappa{}'(L_z) \tag{13}$$

In order to exclude the dependency of location in Eq.(13), it is common practice to multiply the *s*th *in vacuo* eigenfunction of the panel on both sides of Eq.(13), and then integrate over the panel domain, hence

$$b_s\frac{S}{4} = \frac{\rho_a\varpi^2}{\rho h\left(\omega_s^2 - \varpi^2\right)}\sum_{\kappa=1}^{m} a_\kappa\beta_{s\kappa}\eta_\kappa(L_z) \tag{14}$$

Equation (14) may also be expressed in a matrix form as

$$\mathbf{b} = \mathbf{\Lambda}_\omega\mathbf{B}\mathbf{\Lambda}_\eta\mathbf{a} \tag{15}$$

where

$$\mathbf{\Lambda}_\eta = \begin{pmatrix} \eta_1 & & & \mathbf{0} \\ & \eta_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \eta_n \end{pmatrix} \tag{16}$$

$$\mathbf{B} = \begin{pmatrix} \cdots & \vdots & \cdots \\ \cdots & \beta_{ij} & \cdots \\ \cdots & \vdots & \cdots \end{pmatrix} \tag{17}$$

The boundary condition in Eq.(13) may also be written as

$$b_s \frac{S}{4} = \sum_{\kappa=1}^{m} a_\kappa \beta_{s\kappa} \eta_\kappa{}'(L_z) \tag{18}$$

The above equation may then be expressed in a matrix form as

$$\mathbf{b} = \mathbf{B}\mathbf{\Lambda}_{\eta'}\mathbf{a} \tag{19}$$

where a coupling coefficient may be defined as

$$\beta_{s\kappa} = \int \varphi_s(x,y)\psi_\kappa(x,y)dxdy \tag{20}$$

Combining Eq.(15) and Eq.(19), the characteristic matrix equation of a strongly coupled rectangular cavity is produced.

$$\left(\mathbf{\Lambda}_\varpi \mathbf{B}\mathbf{\Lambda}_\eta - \mathbf{B}\mathbf{\Lambda}_{\eta'}\right)\mathbf{a} = \mathbf{0} \tag{21}$$

or

$$\mathbf{M}\mathbf{a} = \mathbf{0} \tag{22}$$

Now that the characteristic equation is obtained in Eq.(22), the next stage is to search eigenvalues such that the determinant $\mathbf{M}$ is zero. Once eigenvalues are searched, the eigenvectors may also be obtained.

2.2 Clusterization for deriving the eigenpairs

Equation (22) may further be written in detail as

$$\begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1m} \\ m_{21} & m_{22} & \cdots & m_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nm} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix} = \mathbf{0} \tag{23}$$

where

$$m_{s\kappa} = \left[\frac{\rho_a \bar{\varpi}^2}{\rho h\left(\omega_s^2 - \bar{\varpi}^2\right)} \eta_\kappa(L_z) - \eta_\kappa{}'(L_z) \right] \beta_{s\kappa} \tag{24}$$

To solve an eigenvalue problem in Eq.(22), the matrix **M** must be square, hence *n* = *m*. The procedure to seek the eigenpairs is straightforward. First, find a frequency that satisfies the determinant of **M** being zero. With the eigenvalue, the eigenvector may then follows.

In general, the dimension of **M** tends to be large due to a distributed parameter system dealt with, hence it is worth simplifying the structure of the eigenvalue problem of a strongly coupled cavity. Note that every term $m_{s\kappa}$ in Eq.(24) contains the coupling coefficient $\beta_{s\kappa}$, hence the property of the matrix **M** is dominated by the coupling coefficient matrix **B**. The coupling coefficient $\beta_{s\kappa}$ indicates the coupling magnitude between the *s*th vibration mode and the *κth* acoustic mode. To further simplify the eigenvalue problem, we are now going to introduce a cluster coupling method.

Assume that all the vibration modes are classified into cluster $\bar{\mathrm{A}}, \bar{\mathrm{B}}, \bar{\mathrm{C}}$ and $\bar{\mathrm{D}}$ while the acoustic cut-on modes are clustered into $\bar{\mathrm{a}}, \bar{\mathrm{b}}, \bar{\mathrm{c}}$ and $\bar{\mathrm{d}}$. Due to the coupling characteristics, the cluster $\bar{\mathrm{A}}$ couples only with $\bar{\mathrm{a}}$, and $\bar{\mathrm{B}}$ for $\bar{\mathrm{b}}$, $\bar{\mathrm{C}}$ for $\bar{\mathrm{c}}$ and $\bar{\mathrm{D}}$ for $\bar{\mathrm{d}}$; e.g. odd/odd structural mode couples only with even/even acoustic mode. Define that

$$n = n_{\bar{\mathrm{A}}} + n_{\bar{\mathrm{B}}} + n_{\bar{\mathrm{C}}} + n_{\bar{\mathrm{D}}} \tag{25}$$

$$m = m_{\bar{\mathrm{a}}} + m_{\bar{\mathrm{b}}} + m_{\bar{\mathrm{c}}} + m_{\bar{\mathrm{d}}} \tag{26}$$

where for instance $n_{\bar{\mathrm{A}}}$ in the above denotes the number of structural modes belonging to the cluster A. Then, due to the properties of a cluster filtering, the matrix **M** in eq.(22) may be partitioned into

$$\begin{pmatrix} \mathbf{M}_{\bar{\mathrm{A}}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{\bar{\mathrm{B}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{\bar{\mathrm{C}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{\bar{\mathrm{D}}} \end{pmatrix} \begin{pmatrix} \mathbf{a}_{\bar{\mathrm{A}}} \\ \mathbf{a}_{\bar{\mathrm{B}}} \\ \mathbf{a}_{\bar{\mathrm{C}}} \\ \mathbf{a}_{\bar{\mathrm{D}}} \end{pmatrix} = \mathbf{0} \tag{27}$$

where

$$\mathbf{M}_{\bar{\mathrm{A}}} \in \boldsymbol{C}^{n_{\bar{\mathrm{A}}} \times m_{\bar{\mathrm{a}}}},\ \mathbf{M}_{\bar{\mathrm{B}}} \in \boldsymbol{C}^{n_{\bar{\mathrm{B}}} \times m_{\bar{\mathrm{b}}}},\ \mathbf{M}_{\bar{\mathrm{C}}} \in \boldsymbol{C}^{n_{\bar{\mathrm{C}}} \times m_{\bar{\mathrm{c}}}},\ \mathbf{M}_{\bar{\mathrm{D}}} \in \boldsymbol{C}^{n_{\bar{\mathrm{D}}} \times m_{\bar{\mathrm{d}}}}$$

Moreover, Eq.(27) may be reduced to

$$\mathbf{M}_{\bar{\mathrm{A}}}\mathbf{a}_{\bar{\mathrm{a}}} = \mathbf{0} \tag{28}$$

$$\mathbf{M}_{\bar{\mathrm{B}}}\mathbf{a}_{\bar{\mathrm{b}}} = \mathbf{0} \tag{29}$$

$$\mathbf{M}_{\bar{\mathrm{C}}}\mathbf{a}_{\bar{\mathrm{c}}} = \mathbf{0} \tag{30}$$

$$\mathbf{M}_{\bar{\mathrm{D}}}\mathbf{a}_{\bar{\mathrm{d}}} = \mathbf{0} \tag{31}$$

It is clear that, from Eq.(28) through Eq. (31), a large dimensioned characteristic equation in (22) is partitioned into four clusters, and hence the burden to search for eigenpairs is significantly allayed.

Moreover, the eigenpairs of a strongly coupled rectangular cavity are independently as well as individually obtained in each cluster, hence the total eigenpairs may be in a form of merely aggregating the eigenpairs of each cluster.

It is also clear that in order for an eigenvalue problem to hold, matrices $\mathbf{M}_{\bar{\mathrm{A}}}$, $\mathbf{M}_{\bar{\mathrm{B}}}$, $\mathbf{M}_{\bar{\mathrm{C}}}$ and $\mathbf{M}_{\bar{\mathrm{D}}}$ need to be square, hence $n_{\bar{\mathrm{A}}} = m_{\bar{\mathrm{a}}}$, $n_{\bar{\mathrm{B}}} = m_{\bar{b}}$, $n_{\bar{\mathrm{C}}} = m_{\bar{c}}$, $n_{\bar{\mathrm{D}}} = m_{\bar{d}}$. Recall that the condition, *n* = *m*, was needed to solve the eigenpairs problem, however, it turns out that it is the necessary condition, and not sufficient one. In other words, $n_{\bar{\mathrm{A}}}$, $n_{\bar{\mathrm{B}}}$, $n_{\bar{\mathrm{C}}}$ and $n_{\bar{\mathrm{D}}}$ need not to be equal. In the case where $n_{\bar{\mathrm{A}}} \neq m_{\bar{\mathrm{a}}}$, for instance, the number of structural mode or acoustic mode should be adjusted such that $\mathbf{M}_{\bar{\mathrm{A}}}$ being square.

3. Numerical analysis

Consider a rectangular cavity comprising flexible panel placed on top and five acoustically rigid walls with the dimension of ($L_x = 0.18m,\ L_y = 0.38m,\ L_z = 0.866m,\ h = 0.8mm$). In this case, clusters $\bar{\mathrm{A}}$, $\bar{\mathrm{B}}$, $\bar{\mathrm{C}}$, $\bar{\mathrm{D}}$ correspond to the cluster of (odd/odd structural modes and even/even acoustic modes), (odd/even structural modes and even/odd acoustic modes), (even/odd structural modes and odd/even acoustic modes) and (even/even structural modes and odd/odd acoustic modes), respectively. First, we need to obtain eigenfrequncies of the vibration mode of a panel and cut on mode of a cavity.

Table 1 shows the modal indices of structural mode and acoustic cut-on mode placed in order of frequency. Consider for instance the case where s = κ = 1, which refers to an odd/odd structural mode, cluster A, and even/even acoustic cut on mode, hence the coupling coefficient β_{11} is non-zero hence ε_{11} is non-zero also. This fact is reflected in the eigenvalue matrix in Eq.(32)

Table 1: modal indices of structural modes and acoustic cut on modes in order of the frequency where s refers to structural mode and κ to acoustic cut on mode

indices	1	2	3	4	5	6	7	8	9	10
s	11	12	13	21	14	22	23	15	24	16
κ	00	01	02	10	11	12	03	13	04	20

$$\begin{matrix} & & 00 & 01 & 02 & 10 & 11 & 12 & 03 & 13 & 04 & 20 \end{matrix}$$

$$\begin{pmatrix} 11 \\ 12 \\ 15 \\ 21 \\ 14 \\ 22 \\ 23 \\ 15 \\ 24 \\ 16 \end{pmatrix} \begin{pmatrix} m_{11} & & m_{13} & & & & & & m_{19} & m_{110} \\ & m_{22} & & & & & m_{27} & & & \\ m_{31} & & m_{33} & & & & & & m_{39} & m_{310} \\ & & & m_{44} & & m_{46} & & & & \\ & m_{52} & & & & & m_{57} & & & \\ & & & & m_{65} & & & m_{68} & & \\ & & & m_{74} & & m_{76} & & & & \\ m_{81} & & m_{83} & & & & & & m_{89} & m_{810} \\ & & & & m_{95} & & & m_{98} & & \\ & m_{10\,2} & & & & & m_{10\,7} & & & \end{pmatrix} \begin{pmatrix} a_1^{00} \\ a_2^{01} \\ a_3^{02} \\ a_4^{10} \\ a_5^{11} \\ a_6^{12} \\ a_7^{03} \\ a_8^{13} \\ a_9^{04} \\ a_{10}^{20} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{32}$$

As a result of properly arranging the order of a raw and a column in the above, we have

$$\begin{pmatrix} m_{11} & m_{13} & m_{17} & m_{110} & & & & & & \\ m_{31} & m_{33} & m_{37} & m_{310} & & & & & & \\ m_{81} & m_{83} & m_{87} & m_{810} & & & & & & \\ & & & & m_{22} & m_{27} & & & & \\ & & & & m_{52} & m_{57} & & & & \\ & & & & m_{10\,2} & m_{10\,7} & & & & \\ & & & & & & m_{44} & m_{46} & & \\ & & & & & & m_{74} & m_{76} & & \\ & & & & & & & & m_{65} & m_{68} \\ & & & & & & & & m_{95} & m_{98} \end{pmatrix} \begin{pmatrix} a_1^{00} \\ a_3^{02} \\ a_9^{04} \\ a_{10}^{20} \\ a_2^{01} \\ a_7^{03} \\ a_4^{10} \\ a_6^{12} \\ a_5^{11} \\ a_8^{13} \end{pmatrix} = \mathbf{0} \tag{33}$$

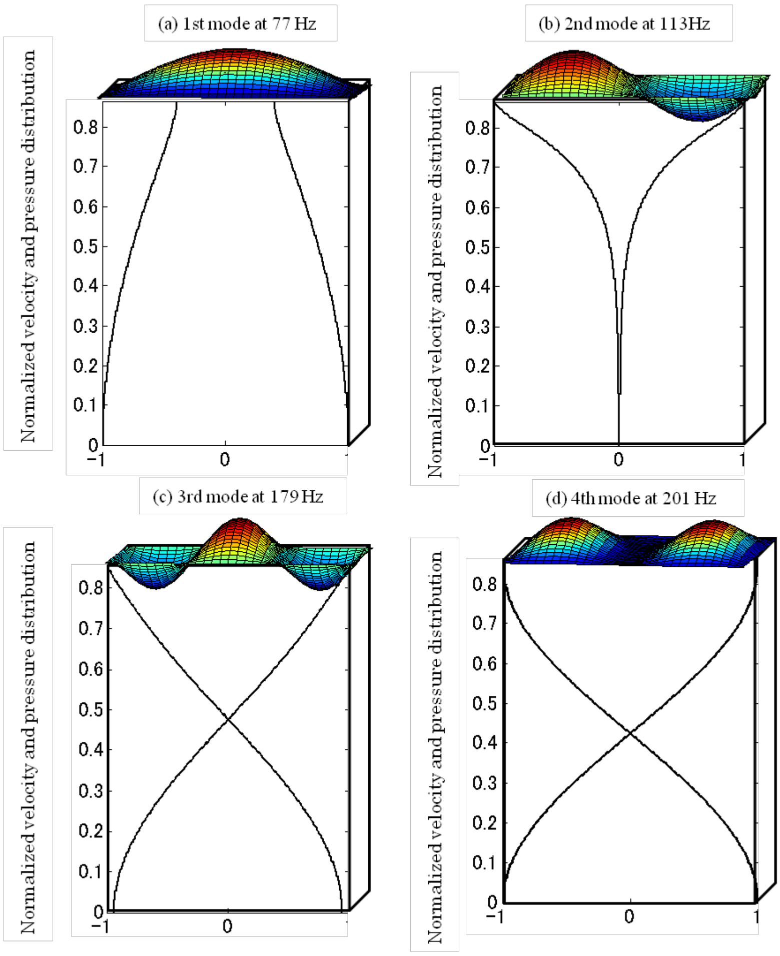

Figure 2: Normalized velocity mode shape of a flexible panel and the acoustic mode shapes in the cavity at the 1st ~ 4th coupled mode [8, 9]

Illustrated in Fig.2 are the normalized velocity mode shapes of a flexible panel of the coupled rectangular cavity [8, 9]. Observe that the structural modal behaviours from the 1st through 3rd mode are dominated by the *in vacuo* mode shape; (1,1), (1,2) and (1,3) mode, respectively. As for the 4th mode, however, the original *in vacuo* structural (2,1) mode is replaced by the deformed (1,3) mode because of the coupling effect. Figure 2 also shows the corresponding acoustic mode shapes along the *z* direction depicted using the expression: in Eq.(2) in the vicinity of $x = L_x / 2$ and $y = L_y / 2$. Unlike the acoustic mode shapes observed in a rigid wall, the mode shapes in the *z* direction appear different. Regarding the first mode at 77 Hz in Fig.2(a), although the rigid wall mode shape is depicted by two parallel straight lines along the *z* axis, the acoustic mode shape after coupling shrinks as *z* increases, leading to that of an open ended cavity at $z = L_z$ as an extreme case. Acoustic mode shapes in Figs.2(c) and (d) are similar to each other, albeit the 4th mode is considerably affected by the coupling effect. Note that among the four in Fig.2, the 2nd acoustic mode shape appears different from the ordinary mode shapes. Such an intriguing mode shape, characteristic of a strongly coupling effect between the vibrational and acoustic fields, emerges when the eigenvalue after coupling becomes smaller than the cut-on frequency.

4. Conclusions

The derivation of the eigenpairs of a strongly coupled rectangular cavity comprising a flexible panel placed on top and acoustically rigid five walls was presented, fundamental properties of the eigenpairs being discussed. It was shown that, as a result of coupling between a structural field and acoustic field in the cavity, two kinds of acoustic modes are found to appear; standing wave mode and evanescent mode. The former is generated by sound reflections in the cavity, whereas the latter by coupling effect. It is likely that the dimensions of a characteristic matrix of a coupled cavity increase because of a distributed parameter system being dealt with. This paper then presented a clusterization method in order to reduce the computation burden on deriving the eigenpairs of a strongly coupled cavity. For this purpose, it was shown that the characteristic matrix of a cavity is dominated by a coupling coefficient matrix. Inasmuch as coupling is strictly selective, the coupling matrix is found to be expressed in a form consisting of four independent cluster matrices. The eigenpairs of the coupled cavity may then be obtained by aggregating eigenpairs of each cluster. Finally, a numerical example was demonstrated, verifying the validity of the proposed clusterization approach.

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