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Some kinematic considerations of tone generation in axial turbomachinery.

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Introduction

Kinematic considerations apply severe restrictions to the acoustic frequencies and modes which are possible from many processes in turbomachines. The best known kinematic analysis is due to Tyler and Sofrin (1). They showed for the interaction at the n-th harmonic between B rotor blades and V stator vanes that the pressure signal is of the form

$$p(\theta, t) = \sum_{m=-\infty}^{\infty} B a_m \exp\{i(m\theta - nB\Omega t + \phi_m)\}$$

where the acoustic mode order is restricted to values satisfying $m = nB + kV$, k being an arbitrary integer. For negative values of k it is possible for m to be much smaller than nB and the acoustic pressure pattern can rotate faster than the rotor. This has explained the propagation along inlet and exhaust ducts of the tone from rotor-stator interactions when the rotor tip speed is subsonic.

Tones at blade passing frequency due to the interaction of the rotor with inflow distortion tend nowadays to be a more serious problem. This was sufficiently like the interaction of a rotor with stator wakes that it seemed natural to identify the distortion order, ν, with the number of stators, V. With Tyler and Sofrin's result this lead to the intuitively unreasonable conclusion that a low order distortion could always produce a propagating mode (because of the arbitrary integer, k). This also disagreed with the result of Barry and Moore (2), who considered the distortion to modulate the steady blade force. This discrepancy is easily cleared up by the analysis given below using a rotating co-ordinate system, but it brings out an inherent assumption used by Tyler and Sofrin which allowed them to have on analysis for generation either on the rotor or on the stator.

An analysis in rotating co-ordinates is then applied to the interaction of a sound wave with a moving rotor. This is probably the cause of the sum and difference tones usually found in the noise spectrum from a multistage compressor or turbine.

Rotor-distortion tone generation

Suppose that a B bladed rotor (angular velocity Ω) interacts with a distorted inflow, one component of which is denoted by $A\exp(\pm i\nu\theta)$, ν being the order of distortion. In a frame of reference moving with the rotor the frequency experienced due to this one component is $\nu\Omega$ and the pressure signal sensed by an observer at θ_R in the moving frame of reference due to any one blade can be written

$$p(\theta_R,t) = \sum_{m=-\infty}^{\infty} a_m \exp\left\{i(m\theta_R - \nu\Omega t + \phi_m)\right\}$$

where m is the order of the acoustic modes.

The blades are assumed identical, each producing the same pressure signal but shifted in phase. The resultant pressure from all the blades can then be written

$$p(\theta_R,t) = \sum_{q=0}^{B-1}\sum_{m} a_m \exp\left\{i\left[m(\theta_R - \tfrac{2\pi}{B}q) - \nu\Omega(t + \tfrac{2\pi}{B}\tfrac{q}{\Omega}) + \phi_m\right]\right\}$$

The summation over q leads to:

$$p(\theta_R,t) = \sum_{m=-\infty}^{\infty} B a_m \exp\left\{i\left(m\theta_R - \nu\Omega t + \phi_m\right)\right\}$$

if $(m+\nu)/B$ =n, an integer

and $p(\theta_R,t) = 0$ if $(m+\nu)/B \neq$ an integer

The pressure can be referred to a stationary frame of reference, θ, by $\theta = \theta_R + \Omega t$ which, together with the restriction $m = nB - \nu$ leads directly to

$$p(\theta,t) = \sum_{m'=-\infty}^{\infty} a_{m'} \exp\left\{i(m'\theta - nB\Omega t + \phi_{m'})\right\}$$

m' is used to signify that only values satisfying $m = nB - \nu$ are admissible. This equation is identical to that obtained by Tyler and Sofrin, except that the expression for the modes, $m = nB - \nu$, does not contain the arbitrary integer k, and this confirms the result of Barry and Moore.

One can see from this the significance of the integer k obtained by Tyler and Sofrin. They considered the interaction 'events' in stationary co-ordinates and it is only possible to represent generation on a rotor in stationary co-ordinates if the event has infinitesimal circumferential width. This is equivalent to representing the interaction by an array of delta functions (infinitesimal width but finite integrated amplitude), and it is well known that the harmonic analysis of such an array contains all the harmonics of the spacing at equal amplitude. The integer k is then the harmonic of the distortion pattern.

One can represent a continuous distortion by a continuous distribution of delta functions. If ψ is used to denote the instaneous rotor position and θ the position of a stationary observer, the pressure due to an element of a v-th order distortion of width $\delta\psi$ can be written

$$p^{\psi}(\theta,t) = \sum_{m=-\infty}^{\infty} a \exp\left\{i\left[m(\theta-\psi) - nB\Omega(t - \tfrac{\psi}{\Omega}) + \phi_m\right]\right\} \exp(\pm i\nu\psi)\,\delta\psi$$

The resultant pressure is the integral of this around the circumference and in general this tends to zero. For $m - nB \pm \nu = 0$, however, it integrates to a form essentially identical to that found using a rotating frame of reference.

Knowing the order of distortion it is possible to say at what rotor tip relative Mach number cut-off will occur for a particular harmonic nB. Since the cut-off ratio more or less determines where the far-field radiation peaks, one can say from the field shapes (knowing blade speed and number) what order of distortion is important. From this one can arrive at a good idea of the relevant source of distortion. Because the order of distortion needed to produce a propagating tone varies with the tip Mach number, it follows that the type and origin of the distortion dominating the tone generation will also vary with tip speed.

The interaction of a sound wave with a moving rotor

Suppose there is a sound wave of frequency ω and circumferential order h propagating along a duct in which there is a rotor with B blades and an angular velocity Ω. In a frame of reference moving with the rotor the frequency sensed by the rotor is ($\omega - \Omega h$) which is designated ω'.

When the acoustic wave strikes a blade it causes the blade to radiate, but the exact interaction may be complicated. For the present it suffices to assume scattering occurs with sound in all possible modes being produced by each blade. Then the pressure sensed by a moving observer at θ_R due to one blade is of the form

$$p(\theta_R, t) = \sum_{m=-\infty}^{\infty} a_m \exp\{i(m\theta_R - \omega' t + \phi_m)\}$$

Each blade produces an identical distribution but shifted in phase, and the resultant pressure is the sum from all B blades,

$$p(\theta_R, t) = \sum_{q=0}^{B-1} \sum_m a_m \exp\left\{i\left[m\left(\theta_R - \frac{2\pi}{B} q\right) - \omega'\left(t - \frac{2\pi q}{B\,\omega'/h}\right) + \phi_m\right]\right\}$$

The summation over q leads to

$$p(\theta_R, t) = \sum_{m=-\infty}^{\infty} B a_m \exp\{i(m\theta_R - \omega' t + \phi_m)\}$$

if $(-m+h)/B = n$ = n an integer

and $p(\theta_R, t) = 0$ if $(-m+h)/B \neq$ an integer

The pressure signal in stationary co-ordinates $\theta = \theta_R + \Omega t$ is, on introducing $\omega' = \omega - \Omega h$ and the restriction on the modes $m =$

$$p(\theta, t) = \sum_{m'=-\infty}^{\infty} B a_{m'} \exp\{i(m'\theta - (\omega \pm nB\Omega)t + \phi_{m'})\}$$

where m' indicates that only modes satisfying $m = h \pm nB$ are admissible. The two signs, although strictly unnecessary from the restriction to integer values of n above, are used to draw attention to the sum and difference tones produced by the interaction.

Consider the simplest case of two rotors on one shaft. If the wave incident on rotor two originates from rotor one interacting with a distortion of order v, it can be seen at once that $h = nB - v$ and the circumferential phase velocity of the incident wave is $n_1 B_1 \Omega/(n_1 B_1 - v)$. The interaction of this wave with the second rotor gives frequencies $(n_1 B_1 \pm n_2 B_2)\Omega$ with circumferential phase velocities $(n_1 B_1 \pm n_2 B_2)\Omega/(n_1 B_1 \pm n_2 B_2 - v)$. From this it can be seen that the sum tones always have a lower phase velocity than the incident wave, whilst the difference tone can have a very much higher phase velocity. The cut-off ratio is directly proportional to the circumferential phase velocity.

References

(1) J.M. Tyler and T.G.Sofrin. SAE Trans. 70, 1962

(2) B. Barry and C.J. Moore Jnl. Sound Vib, 17, 1971