ULTRASONIC SCATTERING FROM CYLINDRICAL AND SPHERICAL SHELLS

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1. INTRODUCTION

Let us consider a thin empty spherical shell submerged in a boundless fluid medium. The plane harmonic pressure wave strikes the shell and is scattered by it. The motion of the shell is described by the equations of linear elasticity theory, and that of the liquid is governed by the Helmholtz equation. It is a two-dimensional steady-state problem. The secondary field of acoustic pressure, which is scattered by the shell, is analyzed. Using the method of separation of variables, one can obtain the exact solution in a series form [1]. It is valid for arbitrary values of spherical coordinates r, θ. For the sake of simplicity we shall examine the pressure at a fixed point of observation, situated in the Farfield of the backscattering ($r/a = 10^4$, $\theta = \pi$). We shall consider a typical case in hydroelasticity, the case of an aluminium shell immersed in water:
aluminium: $\rho_1 = 2.79 \cdot 10^3$ kg/m^3, c_ℓ = 6380 m/s. c_t = 3100 m/s,
water: $\rho = 1 \cdot 10^3$ kg/m^2, c =1470.

The relative thickness of the shell $h = 1-b/a = 1/32$. The calculation is carried out in the range $0 \leq x = ka \leq 400$ with the computation step $\ell_x = 10/256$. Here, a and b are the outer and inner radii of the shell, respectively; $k = \omega/c$ is the wave number in the liquid. Qualitatively, the form function is similar to that presented in [2, Fig.1b]. The plots of the resonance components of the partial modes are computed. The single-type resonances are joined into families. In the considered range, the incident wave generates in the shell the following peripheral waves: A, S_0, A_1, S_1. At bigger values of x, the Lamb-type waves of higher orders S_l and A_l (l = 2,3,4,...) will be generated. For a more thick-walled shell, say at $h \sim 1/10$, the incident wave will also generate the A_0 wave.

2. RESULTS

The extrema of the form function curve correspond to the resonance frequencies of the partial modes. As a rule, the resonances of peripheral waves with high Q-factor or amplitude are well-observed on the form function curve. When the resonance has both of these properties, its chance of being distinguished on the form function is enhanced. Usually, as frequency increases, the form function curve becomes more discontinuous (cut). This becomes particularly apparent in the vicinity of the cut-off frequencies of the Lamb-type waves, after which the resonances of the newly generated wave can be observed.

The resonances of the bending wave A can be clearly observed in the strong bending domain. In the example considered, this is the range $20 < x < 60$. The resonances with $33 < n < 61$ are the most clearly observable ones. The influence of the S_0 wave can be observed in the range of $0 < x < 210$ (at $0 \leq n \leq 59$), i.e. up to the cut-off frequency of the A_1 wave ($\bar{x} = \bar{2}12.11$). the successive resonances of this wave can be observed only in superposition with the A_1 wave resonances. At small n ($n < 30$), the latter have a small amplitude, but possess a rather high Q-factor. One can see the influence decreases as x and n increase. At $x > 385$, one can see on the form function curve a high and broad "splash". It is caused by the resonances of the S_1 wave. The cut-off frequency of this wave is $x = 423.75$. With n increasing, the first 50 resonance frequencies of this wave move along the x-axis from the right to the left, and the succeeding ones, as usual, from the left to the right. On the form function curve, the influence of the resonances of this wave can be marked up to the cut-off frequency of the S_2 wave ($x = = 436.37$).

We shall use the following notation in the description of the modal resonances: x_n is the resonance frequency, ζ_n is the amplitude of the modal resonance component at the resonance frequency, q_n is the width (on the x-axis) of the partial mode resonance curve at its half-amplitude level. The smaller q_n is, the higher is the Q-factor.

The S_0 wave. Generally speaking, with n increasing, the resonance amplitude slowly grows, changing from $\zeta_1 = 0.9139$ to $\zeta_{100} = 1.3312$. At $1 < n < 15$, with n increasing, the Q-factor quickly grows; after this, at $15 < n < 75$, it slowly decreases. The Q-factor, for example, may be characterized with $q_{100} = 21.6$.

The A wave. At $26 < n < 36$, with n increasing, the resonance amplitude quickly grows from $\zeta_{26} = 0.0065$ to $\zeta_{36} = 4.7035$. At $n > 36$, the amplitude decreases slowly to $\zeta_{51} = 4.1104$. The Q-factor changes in the same way. The procedure of resonance scattering theory allows us to separate the resonances only for $n \leq 48$.

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The A_1 wave. With n increasing, the resonance amplitude slowly grows from $\zeta_1 = 0.0042$ to $\zeta_{100} = 0.9770$. At $1 < n < 20$, with n increasing, the Q-factor is almost constant. With further increase in n, it slowly decreases: $q_{60} = 6.4$, $q_{80} = 9.2$, $q_{100} = 11.6$.

The S_1 wave. With n increasing, the resonance amplitude slowly grows from $\zeta_1 = 0.0142$ to $\zeta_{98} = 0.8682$. The Q-factor is small at $n < 12$. With n increasing, it grows: $q_{40} = 14$, $q_{60} = 11$, $q_{80} = 5.4$.

The resonance frequencies and amplitudes of the partial modes are given in Tables 1 and 2. We did not round-off here the resonance positions, and give them with two significant digits after the decimal point for the purpose of actual comparison, although the computation step is $\zeta_x = 10/256$.

It s easy to find the phase and group velocities of a peripheral wave when the position of the resonance is known. The resonance frequency of a peripheral (running) wave coincides with the resonance frequency of a partial mode (standing wave). The resonance takes place when exactly (n+1/2) wavelengths fit the meridian circle length.

$$2\pi a = (n+1/2)\lambda_{nl}. \qquad (1)$$

Here n defines the ordinal number of resonance and determines its family (the type of the peripheral waves). From condition (1) the phase c^{ph} and group c^{gr} velocities may be found

$$c^{ph}(x_{nl}) = cx_{nl}/(n+1/2), \quad c^{gr}(x_{nl}) = c(x_{(n+1)l} - x_{nl}). \qquad (2)$$

The dispersion curves of the peripheral waves generated in shells of different thicknesses can be compared when the dependence y(z) is calculated. Here y and z are nondimensional values

$$y = c^{ph}/c_t, \quad z = k_t d \qquad (3)$$

where c_t is the velocity of the transverse wave in the linear elasticity theory, $z = (1/2)\ (c/c_t)hx$, $k_t = \omega/c_t$ and $d = (1/2)$ (a-b). On the z-y plane the resonances of some other order n but for different dispersion curves l, are situated on lines which pass through the origin. The angle between the line corresponding to the n-th resonance and the z-axis is arctan $2\ [(n+1/2)h]^{-1}$.

The resonance frequencies of partial modes can be found approximately from the dispersion equations of model problems, namely, describing Lamb-type waves in a plane elastic layer. For the S_0 and S_1 waves they are found from equation E = 0[4, eqn 29], and for the A_1 wave from the equation F = 0[4,eqn 29]. In these equations instead of y, the relation

$$y_* = 2(1-h/2)\ z\ [(n+1/2)h]^{-1} \quad (4)$$

should be used. (In the model problem, it is assumed that the S_0, S_1 and A_1 waves are propagating on the middle surface of the shell). For the A wave, we should consider the problem concerning the waves in a plane elastic layer, one side of which is in contact with the liquid and the other is free, as the model problem. The resonance frequencies are found from the equation $E(F+\Psi) + F(E+\Psi) = 0$ [4, eqn 28] in which, instead of y (3), the relation

$$y_* = 2z\ [(n+1/2)h]^{-1} \quad (5)$$

should be introduced. (In the model problem, it is supposed that the A wave is propagating on the surface separating the elastic body and the liquid.) In contrast with the model problems concerning waves in a plane layer, here the dispersion equations must be solved for every n (n = 1,2,3,...) value. The resonance frequencies found from the dispersion equations for model problems are determined as $x_{nl}*$ and presented in Tables 1 and 2. There they are denoted by x_n* because l is evident from the table caption. The comparison of the results shows that at not very small values of n, the coincidence is rather good. Qualitatively it can be described as: the long wave ($\lambda \sim 4a$) strongly "feels" the curvature of the scatterer; the wave with wavelength comparable with the typical size of the scatterer ($\lambda \sim a$) is not very "sensitive" to the curvatures; the short wave ($\lambda \sim a/2$) is almost not sensitive to the curvature. For small n the accuracy of determination of the resonance frequencies can be improved by using the procedure outlined in [5,6]. In the case of the A wave, which is generated in the shell by the incident wave, over a rather narrow x range and at low x values, the analytic formulae of resonance scattering theory may be used [3].

The acoustic spectrogramme can be constructed using the resonance frequencies of the partial modes. In [7-9] a proposal is made to use it as an acoustic signature. Although direct methods of determining the resonance frequencies have been elaborated [10-15], not all the resonances are apparent on the form function. Particularly, this is related to resonances with low Q-factors, which overlap in frequency. As a rule, the neighbouring resonances are in anti-phase. The amplitude of superposition of resonances with low Q-factors is smaller, and sometimes essentially smaller than every summand. The extrema of this superposition do not coincide with the resonance frequencies. In such a case the extrema of the form function curve are also shifted with respect to the resonance frequencies. Sometimes, this shift attains (1/2) $(x_{n+1}-x_n)$. In the example considered, the S_0 wave has these properties at $35 < n < 70$, the A_1 wave at $25 < n < 95$, and the S_1 wave at $1 < n < 80$.

3. CYLINDRICAL SHELLS

The scattering process is similar to that described above when the plane acoustic wave strikes a circular cylindrical shell at normal incidence. In this case the resonance amplitudes of partial modes are considerably smaller, because in the cylindrical case the source is a point, not a circle. The formulae (1)-(5) are valid, but n should be inserted into them instead of (n+1/2). In the spherical case the extra 1/2 in (n+1/2) is caused by focussing. Approximate asymptotic formulae for the positions of the resonance frequencies at small values of n are given in [6]. They are checked when scattering on a cylindrical shell with h = 1/10 is considered.

When a plane acoustic wave falls on circular cylindrical shell at oblique incidence, the phase velocity of the peripheral wave can be found from

$$c^{ph}(x_{nl}) = c(x_{nl}/n) \cdot (1 + [(x_{nl}/n)\sin \alpha]^2)^{-1/2} \qquad (6)$$

which follows from evident equality

$$(w_{nl}/c^{ph})^2 = [(w_{nl}/c)\sin \alpha]^2 + (n/a)^2. \qquad (7)$$

Here α is an angle between the direction of the incident wave propagation, and the normal to the longitudinal axis of the shell.

At oblique incidence, the estimation of the positions of the resonance frequencies of the peripheral waves S_0, S_1 and A_1 can be found rather exactly from the dispersion equations of the Lamb-type waves in a plane "dry" layer [16]. These equations may be obtained from the equations E = 0 and F = 0, respectively, by changing y for y_0, where

$$y_0 = (a_1 [1 + (a_2nh/a_1z)^2])^{-1} \qquad (8)$$

$$a_1=(c_t/c)\sin \alpha, \quad a_2=(1+b/a)^{-1}. \qquad (9)$$

In the case of the A wave, the estimation of the resonance frequencies' positions can be found from the equation $E(F+\Psi) + F(E+\Psi) = 0$ in which y_0 (9) is inserted with $a_2 = 1$.

The obliquely incident wave also generates shear peripheral waves T_l ($l = 0,1,2,...$) in a cylindrical shell. The resonance frequencies of these waves can be found approximately from the equations

$$shp_t = 0, \quad chp_t = 0 \tag{10}$$

for the symmetric and antisymmetric waves, respectively. In equations (10) the notation $p_t = (z/y_0)(1-y_0^2)^{1/2}$ is used.

The resonance frequencies of the T_0 wave are given by the formula

$$z_n = a_2nh(1-a_1^2)^{-1/2}, \tag{11}$$

and those of the T wave by the formula

$$z_n = (\pi/2)\,(\,[1 + [(2/\pi)a_2nh]^2]\,.\,(1-a_1^2)^{-1})^{1/2}. \tag{12}$$

The comparison of the exact (computed according to the procedure of resonance scattering theory) and the approximate values of the resonance frequencies shows the efficiency of the proposed formulae for the S_0, A, A_1, T_0 and T_1 waves [16] in the case of scattering by an aluminium shell with h = 1/32 immersed in water at three different angles of incidence: 0^o, 5^o and 10^o.

On the z-y plane, both for shells with h =1/32 and thicker shells with h = 1/10, the dispersion curves of the phase velocities of the peripheral waves practically coincide for spherical and cylindrical (in case of normal incidence) shells. They do not differ from the corresponding dispersion curves in the case of a plane layer. This concerns both the domains of the dispersion curves at small n values (in the vicinity of the cut-off frequencies) and the points of intersection of the dispersion curves of the peripheral waves S_l and A_l (l = 2,3,4).

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TABLE 1

A				S_0			
n	ζ_n	x_n	x_n*	n	ζ_n	x_n	x_n*
31	1.2321	23.87	23.36	1	0.9139	6.25	5.61
32	0.6392	25.20	24.69	2	0.9947	10.00	9.36
33	1.1969	26.48	26.03	3	1.0285	13.59	13.11
34	2.3386	27.81	27.38	4	1.0441	17.23	16.84
35	3.7359	29.14	28.74	5	1.0524	20.50	20.59
36	4.7035	30.47	30.11	6	1.0474	24.61	24.33
37	4.5803	31.80	31.48	7	1.0555	28.28	28.07
38	4.6195	33.09	32.85	8	1.0606	31.99	31.81
39	4.5477	34.38	34.21	9	1.0610	35.70	35.54
40	4.5369	35.66	35.57	10	1.0177	39.41	39.27
41	4.4816	36.95	36.91	20	1.0454	76.41	76.38
42	4.4499	38.20	38.25	30	1.0800	112.89	112.89
43	4.4102	39.45	39.57	40	1.0931	148.33	148.38
44	4.3731	40.70	40.88	50	1.1086	182.19	182.29
45	4.3378	41.95	42.17	60	1.1323	213.71	213.85
46	4.3054	43.16	43.44	70	1.1655	241.91	242.11
47	4.2797	44.41	44.69	80	1.2103	266.02	266.21
48	4.2437	47.50	-	90	1.2666	285.74	285.96
49	4.2099	46.99	47.14	100	1.3312	301.91	302.12
50	4.1687	48.32	48.34				

TABLE 2

A_1

n	ζ_n	x_n	x_n^*
1	0.0042	212.11	212.09
2	0.0204	212.27	212.25
3	0.0486	212.50	212.48
4	0.0780	212.81	212.79
5	0.1022	213.20	213.17
6	0.1215	213.67	213.63
7	0.1369	214.22	214.17
8	0.1574	214.80	214.78
9	0.1750	215.51	215.46
10	0.1942	216.25	216.21
20	0.3606	227.34	227.30
30	0.5004	243.79	243.72
40	0.6137	263.98	263.87
50	0.7047	286.64	286.49
60	0.7785	310.82	310.68
70	0.8395	335.90	335.76
80	0.8913	361.29	361.23
90	0.9364	386.56	386.64
100	0.9770	411.41	411.64

S_1

n	ζ_n	x_n	x_n^*
1	0.0142	423.75	423.69
2	0.0523	420.70	420.68
10	0.1011	415.51	415.49
15	0.1511	410.31	410.31
20	0.2022	405.51	405.53
25	0.2542	401.25	401.29
30	0.3068	397.66	397.68
35	0.3597	394.73	394.78
40	0.4126	392.62	392.65
45	0.4650	391.33	391.37
50	0.5166	391.02	391.04
55	0.5667	391.76	391.76
60	0.6148	393.63	393.63
65	0.6602	396.80	396.78
70	0.7026	401.37	401.32
75	0.7412	407.42	407.33
80	0.7759	415.00	414.86
85	0.8065	424.06	423.87
90	0.8331	434.53	434.29
95	0.8560	446.25	445.95