

Proceedings of the Underwater Acoustics Group, Institute of Acoustics  
 AIR-BACKED PIEZOELECTRIC DISCS AS WIDE BAND UNDERWATER SOUND SOURCES  
 by

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The broadening of the frequency response of piezoelectric disc transducers radiating into a water load by use of single or multiple acoustic impedance matching layers is well known [1]. The best pulse response is obtained when a zero acoustic reflection coefficient exists over a wide frequency range at the disc-surface/load-medium interface together with the complete absorption of any acoustic energy incident on the backing medium. However, such an arrangement is both difficult to realise and would be of low sensitivity. If the disc is air-backed, then, excepting losses, all the acoustic energy generated eventually enters the load-medium, giving a high sensitivity transducer. The use of transition layers between the disc face and the load medium to improve the pulse response whilst retaining the high sensitivity of an air-backed disc is explored in detail below.

An equivalent circuit representation of the transducer, built around the Mason model [2] was employed to obtain a quantity  $x(t)$ , defined as the inverse Fourier Transform of the complex ratio  $F/V$  (see Figure 1) at discrete time intervals of  $(4f_0)^{-1}$  seconds,  $f_0$  being the piezoelectric disc thickness frequency. The quantity  $x(t)$  is termed the transducer impulse response. Limitations of the equivalent circuit in representing a piezoelectric disc are essentially those listed by Kossoff [1].

The performance of a particular transducer configuration is assessed by comparing its impulse response  $x_2(t)$  with the impulse response of an air-backed piezoelectric disc radiating into an infinite medium of characteristic acoustic impedance equal to that of the disc material; the latter impulse response is here designated the ideal impulse response,  $x_1(t)$ . Quantitative comparisons of performance are available through  $R_{12}$ , the maximum value of the normalised cross correlation function between  $x_1(t)$  and  $x_2(t)$ .

It is well known that an acoustic match between the disc and the load medium is obtained at the same number (N) of frequencies as there are quarter wavelength transition layers if the transition layer acoustic impedances,  $P_i$  are all the geometric mean of those of their two contiguous materials.

That is

$$P_{N-i} = \sqrt{P_{N+1-i} P_{N-1-i}} ; i = 0, N-1 \quad (1)$$

where  $i = 0$  refers to the water load and  $i = N+1$  to the piezoelectric disc:  $P_0 = 1.5 \cdot 10^6$  and  $P_{N+1} = 33.7 \cdot 10^6 \text{ kgm}^{-2}\text{s}^{-1}$

The zero's of the reflection coefficient\* can be made to coalesce at  $f_c$ , the frequency at which the layers are quarter wavelength, if the  $P_i$ 's are selected by what might be termed the Binomial Method:

$$P_{N-i} = P_{N+1-i} (P_0/P_{N+1})^{Y_i} ; i = 0, N-1 ; Y_i = C_i^N / \sum_{i=0}^N C_i^N \quad (2)$$

The  $C_i^N$  are the Binomial coefficients. However, the Geometric Mean and the Binomial Methods of selecting the  $P_i$  are specific instances of a more general procedure. If the magnitude of the reflection coefficient  $\alpha_r$  as a function of frequency  $f$  is constrained to follow the relation [3].

$$\alpha_r = k^2 T_N(x) / [1 + k^2 T_N(x)] \quad (3)$$

where  $T_N(x)$  is the Tchebycheff polynomial of degree N, and

$$x = \cos \left( \frac{\pi}{2} \frac{f}{f_c} \right) / S, \quad k = (R-1) / (2\sqrt{R} T_N(1/S)), \quad R = P_0/P_{N+1}, \text{ then the}$$

Geometric Mean or Binomial Methods result when S or k have particular values. The combinations of  $P_i$  values required for an  $\alpha_r$  behaviour as given by equation (3) are available from expressions in [3] as a function of the fractional bandwidth of  $\alpha_r$  (defined as  $(1 - (2/\pi)\cos^{-1}(S))$ ) for  $N = 2, 3$  and 4. When the  $P_i$  are selected such that  $\alpha_r$  follows equation (3), this procedure is termed Method A.

If an air-backed piezoelectric disc is loaded by an acoustic resistance, then the parameter  $R_{12}$  exhibits a broad maximum as a function

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\* Throughout the paper, this term refers to the acoustic reflection at the radiating surface of the piezoelectric disc.

of this acoustic resistance. Such a result suggests that it might be more desirable to have a finite  $\alpha_r$  at all frequencies rather than actually achieving zeros in  $\alpha_r$  at particular frequencies. To this end, two procedures were pursued.

Firstly, Method B found that combination of the  $P_i$ 's which minimised a quantity  $M$  where

$$M = \sum_{f=0}^{f=2f_0} \left| Z(f, P_1, P_2, P_3, \dots, P_N) - P_{N+1} \right|^2 \quad (4)$$

with  $Z$  as the acoustic impedance loading the disc face.

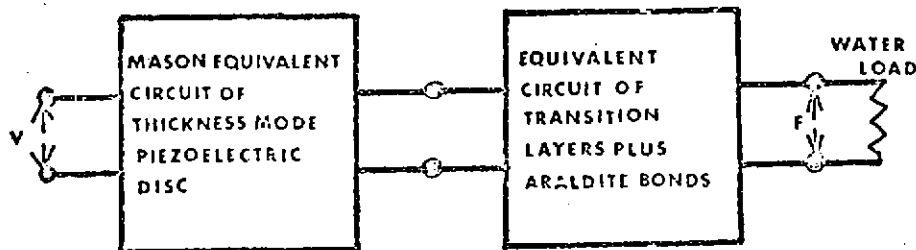
Secondly, Method C found that combination of the  $P_i$ 's which maximised  $R_{12}$  as a function of a new variable  $P$ : the  $P_i$  values corresponding to  $P$  values for this method are found from either equation (1) or (2) with the variable  $P$  in place of  $P_{N+1}$ .

The results obtained from the equivalent circuit are summarised in Figure (2). It is seen that the overall best value of  $R_{12}$  for a constant number of transition layers always results from a choice of  $P_i$  made under Method A. However, of the values of  $N$  investigated neither the Geometrical Mean nor the Binomial method values of  $P_i$  give this best value of  $R_{12}$ , but rather it occurs at a bandwidth value somewhat lower than that associated with the Geometrical Mean method. Of importance in the practical realisation of layered transducers are the broad peaks which occur in  $R_{12}$  as a function of  $P$  (Figure (3)) which, together with the results of Method A (Figure (2)) point to a large degree of flexibility available in the choice of transition layer materials. In Table 1 are listed combinations of the  $P_i$  for the two layer case which produce an  $R_{12}$  value between 0.83 and 0.85 inclusively. When araldite bonds between the various layers of a multilayer transducer are included in the equivalent circuit, no deterioration in  $R_{12}$  occurs if the bond thickness is less than  $(\lambda_c/200)$ .

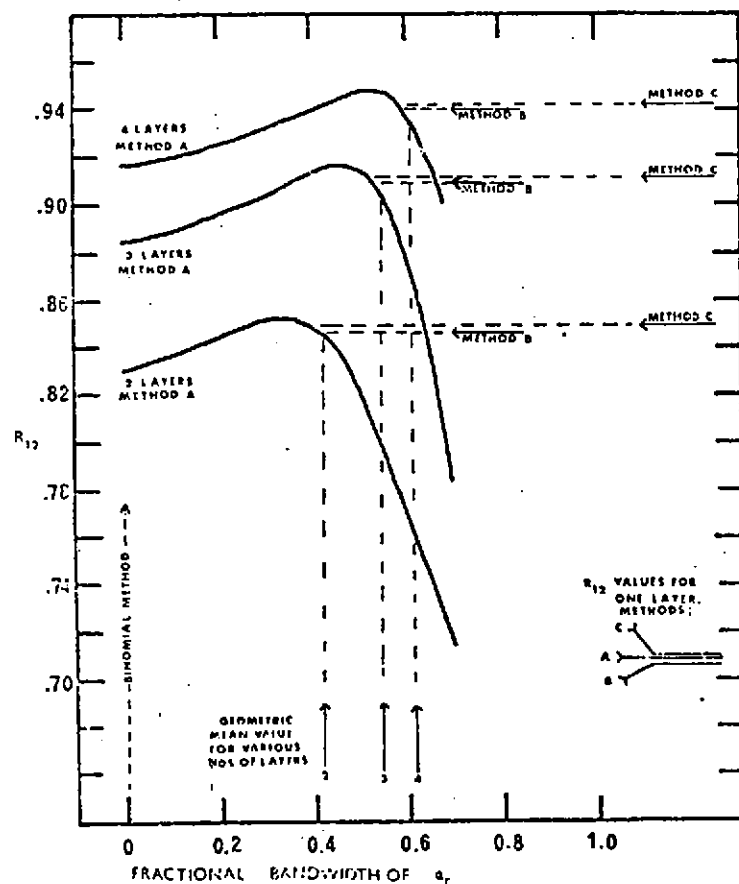
A glance at Table 1 and Figure (2) shows that a transducer with glass and perspex layers should be close to optimum for the two layer case: evolutionary time and frequency responses of such a transducer are given in Figure (4).

#### References

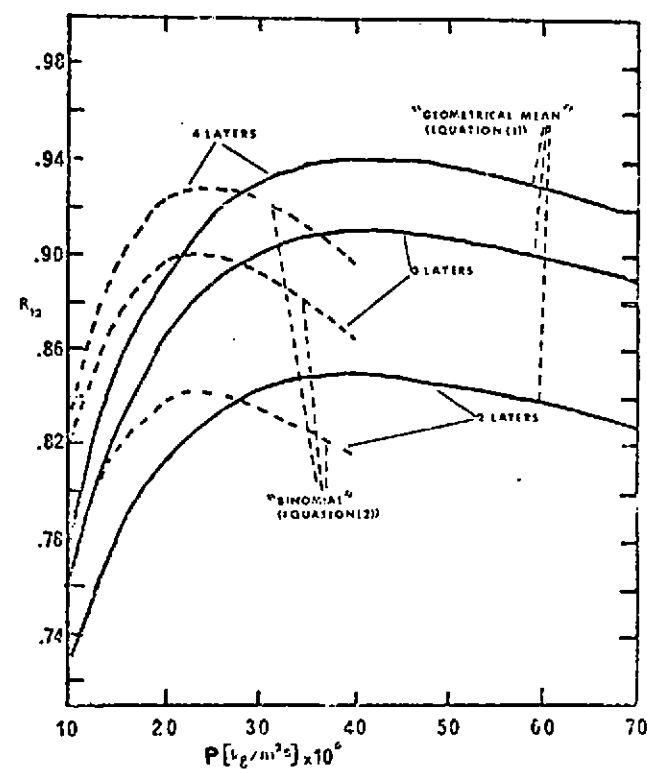
- [1] Kossoff, G., IEEE Trans. Sonics & Ultrasonics SU-13, 20, (1966).
- [2] Mason, W.P., "Electromechanical Transducers and Wave Filters"  
2nd Edition, Van Nostrand, (1948).
- [3] Collin, R.E., Proc. IRE, 43, 179, (1955).



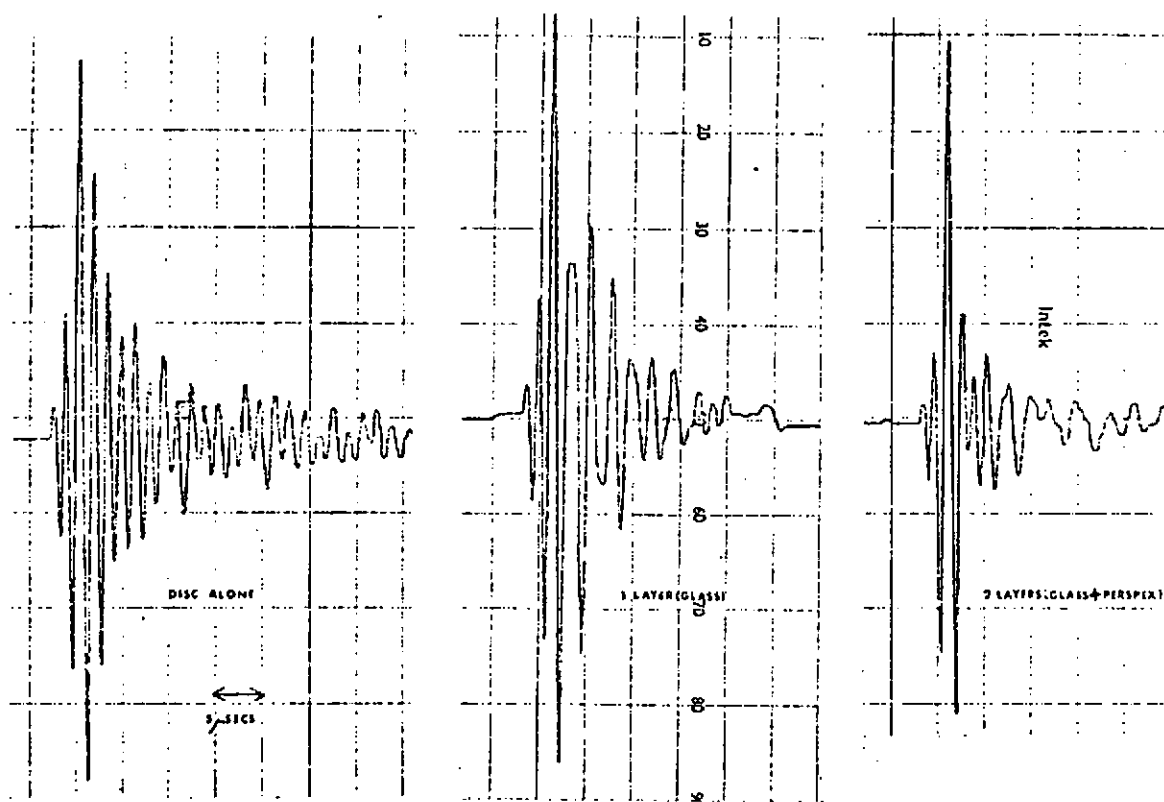
Figure(1) Equivalent circuit representation of the transducer.  $V$  is the input voltage and  $F$  is the output force.



FIG(2) DETAILED RESULTS OBTAINED UNDER METHOD A TOGETHER WITH THE BEST RESULTS OBTAINED UNDER METHODS B & C



FIG(3) DETAILED RESULTS OBTAINED UNDER METHOD C



Figure(4a) Acoustic responses of the transducer at various stages of its construction to a voltage impulse excitation

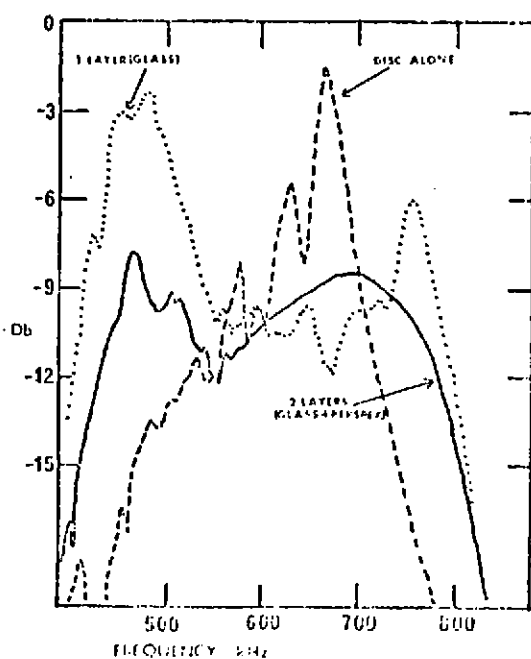


FIGURE 4b THE POWER SPECTRUM OF THE RESPONSES SHOWN IN FIG. 4a

$R_{12}$	$P_1$	$P_2$	Comments
0.82	5.3	18.4	Method C, $P = 65$
	5.4	19.4	Method C, $P = 70$
	3.3	15.5	Method A, $\alpha_p = 0$
	4.6	10.9	Method A, $\alpha_p = 0.5$
0.84	2.9	10.5	Method C, $P = 20$
	3.1	12.3	" $P = 25$
	4.1	11.0	" $P = 30$
	5.0	16.6	" $P = 55$
	5.1	17.5	Method C, $P = 60$
	3.5	14.6	Method A, $\alpha_p = 0.2$
0.85	4.3	12.3	Method C, $P = 35$
	4.5	13.4	" $P = 40$
	4.7	14.5	" $P = 45$
	4.8	15.5	" $P = 50$
	3.7	13.5	Method A, $\alpha_p = 0.1$
	4.1	12.2	Method A, $\alpha_p = 0.4$
	3.5	11.1	Method B

Table 1. Combinations of  $P_1$  and  $P_2$  which produce an  $R_{12}$  value between 0.82 and 0.85: units are  $10^6 \text{ kg/m}^2\text{s}$