Proceedings of The Institute of Acoustics

THEORETICAL RESPONSES OF MICROPHONE CAPSULE ARRAYS

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INTRODUCTION

Stereo recording requires the use of two or more microphones. In British practice, the main signal usually comes from a centrally positioned pair of near-coincident microphones. Two particularly common techniques are the use of a pair of L and R directional microphones, eg. cardiods at 120° or figure-of-eights at 90°, as pioneered by A.D. Blumlein; and the use of an omni-directional microphone giving a mono signal in conjunction with a figure-of-eight microphone giving a left-right difference signal (MS). More recently the soundfield microphone has been developed, consisting of four or more microphone capsules in a spherical array. This microphone is capable of providing a truely coincident stereo signal, but can also do much more, being able to capture the three-dimensional information needed for full ambisonic surround-sound reproduction. Experience suggests that the technology has reached the point where further advance requires greater understanding of the theoretical responses of microphone capsules both singly and in arrays; particularly if complex microphones with large numbers of capsules are to be developed.

The basic capsule is considered to be a capacitive displacement device consisting of a thin metallic, or metallized, circular diaphragm supported next to a metal grid to form a parallel plate capacitor. Changes in the position of the diaphragm result in changes in the capacitance between the diaphragm and the grid. If a constant charge is held on the capsule by applying a DC polarizing voltage through a high resistance, then these changes in capacitance give a fluctuating output voltage across the capsule. Alternatively, the capsule can be connected as the capacitor in an LC oscillator circuit, so that the changes in its capacitance alter the frequency of oscillation.

The various relationships between the diaphragm displacement, the capsule capacitance, and the final electrical output are non-linear. In practice, however, the ranges over which these quantities vary is so small that the non-linear terms can be ignored, except at high sound-levels, and the final electrical output can be considered to be proportional to the diaphragm displacement. Hence throughout this work the output of the simulated microphone array is given as the overall (volumetric) displacement of its diaphragms.

MODELLING THE BASIC CAPSULE

Capsule resonance

The capsule diaphragm can be modelled by a simple spring-mass system, having a single resonant mode. The frequency response of such a system, giving the displacement due to an applied force, is given by :

$$Y(f) = \frac{M}{(f_r^2 - f^2) + j2ff_rF_d} \qquad (1)$$

where f is the frequency (in Hz), f_r is the resonant frequency, F_d is the damping factor (the ratio between the applied damping and that required to give a critically damped response), and M is a constant.

At low frequencies the diaphragm will move with the applied force, the movement being controlled by its compliance; the capsule is 'compliance controlled'. From (1), the low-frequency response is independent of the frequency and inversely proportional to f_r^2. This is in agreement with Lamb [1], who shows that the fundamental resonant frequency of a circular diaphragm is proportional to the square root of the tension across it, while its displacement is inversely proportional to the tension; ie. a slacker diaphragm has a lower resonant frequency and gives a greater output.

At high frequencies, the inertia of the diaphragm will dominate the movement, and the capsule is then 'mass controlled'; the response falling with f^2.

The response at and around the resonant frequency is determined by the damping factor : if $F_d < 1$ the response is under-damped, rising to a peak before falling off; if $F_d = 1$ the response is critically damped, with a double break frequency at f_r; and if $F_d > 1$ the response is over-damped and has two break frequencies, above and below f_r. In the region between the two break frequencies of an over-damped diaphragm the capsule is 'resistance controlled', and $|Y(f)|$ falls with f.

Capsule sensitivity

As the diaphragm is clamped around its circumference, its freedom of movement, and so its sensitivity will vary across its surface; in addition to the variation with frequency given above.

Under a constant pressure the diaphragm will become spherical, with the greatest displacement at its centre and zero displacement around its circumference. Hence, for small displacements, the sensitivity variation with position can be given by the quadratic function :

$$S = h.(1 - r^2/a^2) \qquad (2)$$

where h is a constant, a is the radius of the diaphragm, and r is the radial distance from its centre.

This equation is in agreement with that obtained by Lamb [1] for the displacement across a diaphragm subjected to a uniform force at low frequencies.

Sound pressure variation across an angled capsule

At the higher audio frequencies the size of the diaphragm will be a significant proportion of the wavelength of the sound wave. Hence, when the plane of the diaphragm is at an angle to the wavefront (assumed plane), there will be a significant variation in the sound pressure across its surface.

In a rectangular xyz co-ordinate system, let the diaphragm lie in the yz-plane with its centre at the origin, and let the sound wave approach along an axis x' normal to the z-axis but at an angle B from the x-axis; ie. the wavefront is at an angle B to the diaphragm about the z-axis. If the pressure at the origin due to the sound wave is :

$$P_o = p.\cos(2\pi ft + A) \qquad (3)$$

where t is time (in seconds), and p and A are constants; then, as the axes have been chosen to make the sound pressure independent of z, the pressure variation across the surface of the diaphragm is given by :

$$P = p.\cos(2\pi ft + A + 2\pi fy.\sin(B)/c) \qquad (4)$$

where c is the speed of sound in air.

General response for a single capsule

The total output from a single capsule will be given by :

$$I = \int_s H.Y.S.ds \tag{5}$$

where H is the pressure applied to the diaphragm, and s represents its surface. Since Y varies only with f, and the axes have been chosen to make H independent of z, (5) becomes :

$$I = Y.\int H.(\int S.dz).dy \tag{6}$$

Assuming that the metal grid forming the other half of the parallel plate capacitor covers the whole of the diaphragm, so that the electrical output is dependent on the displacement over the whole diaphragm, y varies between ±a while, for a given y, z varies between $\pm\sqrt{\{a^2 - y^2\}}$. The required integral in z is thus :

$$\int S.dz = \frac{4ah}{3}.(1 - y^2/a^2)^{3/2} \tag{7}$$

SINGLE CAPSULES

Response of a single closed-back capsule

A closed-back capsule has a small enclosed cavity on one side of the diaphragm. The air pressure in this cavity is maintained at the ambient surrounding pressure through a small opening in the enclosure, but does not vary with the incident sound waves. The pressure applied to the diaphragm is thus the pressure variation in the air due to the sound waves, with H = P as given in (4); this type of capsule more commonly being referred to as 'pressure-responding'. Combining (4), (6) and (7), performing the required integration in y to give the total output from the capsule, and dividing by the applied sound wave as given in (3); the (pressure) response of a single closed-back capsule can be given by :

$$Y.\frac{hc^2}{\pi f^2.\sin^2(B)}.\sum_{k=0}^{\infty}\frac{(-1)^k}{k!.(2+k)!}.(\pi af.\sin(B)/c)^{2(k+1)} \tag{8}$$

To simulate the theoretical response of a single closed-back capsule a programme was written which allowed the user to enter the required parameter values, and then plotted either the frequency or polar response from computed modulus values of the response obtained using (1) and (8).

Since, from (4), the magnitude of the pressure variation applied to the diaphragm of a closed-backed capsule is independent of its frequency, to obtain a flat frequency response the microphone needs to have a mechanical response which is 'flat' (ie. constant) over its operating frequency range. Hence the capsule diaphragm must be compliance controlled, having its resonant frequency above the required frequency range and being approximately critically damped.

The simulation programme was initially run for a capsule having a diaphragm 18 mm in diameter with a critically damped resonance at 20 kHz. When the capsule faces the sound source (B = 0) the pressure will be constant over the surface of the diaphragm, and the response will be determined solely by the resonant characteristics. Hence the frontal frequency-response is flat at low frequencies with the expected double break-point at f_r and a 40 dB per decade fall-off at high frequencies.

When the capsule is at an angle to the sound waves ($B \neq 0$), the variation in sound pressure across the diaphragm will cause a reduction in the response; this reduction increases with both f and B (and a), dropouts occurring when there is complete cancellation across the diaphragm. This was confirmed by the simulation programme, the reduction in response being found to be significant at high frequencies; for B = 45° the response was 3 dB down (on the front response) at approximately 16 kHz with the first dropout at 43 kHz, while for B = 90° the respective frequencies were 13 and 29 kHz.

This behaviour was further investigated by plotting a set of polar responses. At 1 kHz the polar is the familiar omni-directional circle, this frequency being low enough for there to be no noticeable reduction in the side response. However, as the frequency is increased, the polar response first narrows, becomes pinched at the sides, and then becomes four-lobed once the frequency is high enough to cause a dropout at certain angles; with the side lobes having opposite sense to those at the front and back. At still higher frequencies more side lobes appear, of alternating sign, but with the overall shape becoming steadily narrower.

Increasing the diameter of the diaphragm was found to increase the low-frequency output, which can be shown to be proportional to the area of the diaphragm, but at the cost of lowering the frequency at which the response starts to fall off due to the pressure variation when $B \neq 0$. Hence there has to be a compromise between the output level of the capsule and its high-audio-frequency response.

From the discussion of the mechanical model of the diaphragm it can be seen that a similar compromise has to be made in setting f_r. It was found, though, that the situation could be improved by making the diaphragm slightly under-damped; reducing F_d slightly boosted the response around f_r, flattening out and generally improving the high-audio-frequency response.

It should be noted that at the higher frequencies, where the size of the capsule is significant, diffraction effects will significantly curb and modify the response to the rear of the capsule, while slightly boosting it in front [2][3].

Response of a single open-back capsule

In an open-back capsule both sides of the diaphragm are open to the surrounding sound field. If the capsule has two grids, one on either side of the diaphragm, and is mechanically symmetrical front to back, it should have identical front and back responses. The grids must have holes in them to allow the sound pressure variations to reach the diaphragm, the dimensions of these holes to a large extent determining the mechanical damping applied to the diaphragm.

If the grids are equi-distant from the diaphragm, the pressure at the diaphragm can be considered to be the pressure difference between the front and back of the capsule; the transmission delays from the faces of the capsule to the diaphragm will be equal and so can be ignored. Hence, whereas a single closed-back capsule responding only to the pressure at a point in the sound field has been shown to be non-directional (except at high frequencies), the response of a single open-back capsule will be directional, the pressure difference between its faces varying with its orientation in the sound field. Alternatively the diaphragm can be considered to follow that component of the movement of the air around it normal to its diaphragm. This type of capsule is thus commonly referred to as 'pressure-difference-responding' or 'velocity-responding'.

Using the same co-ordinate system as for (4), if the capsule has thickness q, the sound pressure across each face will be :

$$P = p.\cos(2\pi ft + A + 2\pi fy.\sin(B)/c \pm \pi fq.\cos(B)/c) \quad (9)$$

The pressure applied to the diaphragm, H, is thus the difference between the two terms of (9) and, using this in place of (4), it turns out that the response of a single open-back capsule is equal to the response of a single closed-back capsule multiplied by the term :

$$-2j.\sin(\pi fq.\cos(B)/c) \quad (10)$$

To simulate the theoretical reponse of a single open-back capsule, the same programme was used as for a single closed-back capsule, the responses now being multiplied by the modulus of the term in (10).

From (10); for reasonable values of q, the low-frequency response will be proportional to fq.cos(B), while at high frequencies there will be a further set of dropouts corresponding to cancellation between the two faces of the capsule. Therefore the displacement of the diaphragm now needs to fall with frequency over most of the operating range; the capsule diaphragm must be resistance controlled, being well over-damped with f_r and F_d chosen to give the required operating frequency range.

On running the simulation programme for a capsule 3 mm thick and having a diaphragm 18 mm in diameter, with the break frequencies of its resonant characteristic set at 20 Hz and 20 kHz; the front response was found to be fairly flat between the break frequencies as required, while falling off as these were approached and passed. The low-frequency fall-off went towards a 20 dB per decade slope as would be expected; the high-frequency response had an additional reduction as f approached the first front-back cancellation dropout at 110 kHz, the term in (10) no longer being proportional to f.

Frequency responses at $B = 30^\circ$ and 60°, and polar plots, showed that the low-frequency response was indeed proportional to cos(B); polars at 1 kHz and 20 Hz gave the familiar figure-of-eight response, the two lobes having opposite sense, with the 20 Hz polar somewhat smaller, due to the specified resonant characteristic, but otherwise identical to that at 1 kHz. Again from (10); as B is increased to 90°, so the front-back cancellation dropout frequencies will rise towards infinity, the thickness of the capsule being reduced in the direction of propagation of the sound wave (x'). Hence the high-frequency responses were dominated by the same reduction effect, due to the pressure variation across the diaphragm, as for the single closed-back capsule; with an additional reduction due to the front-back cancellation effect which disappeared as B approached 90°.

The variation in response with diaphragm size will be the same as for the single closed-back capsule; a compromise being required between the output level from the capsule and the integrity of its high-audio-frequency response. For the single open-back capsule the same compromise was found to be necessary in setting the (effective) thickness; the output level being proportional to q, while the front-back cancellation dropout frequencies varied with 1/q.

It was also found that, whereas for the single closed-back capsule the resonant frequency of the diaphragm dictates a further compromise between output and high-frequency response; it is not f_r as such, but the higher break frequency of the over-damped resonant characteristic that controls this for the open-back capsule. The lower break frequency can in fact be set as low as the available damping will allow, to give a flat low-frequency response within the required frequency range.

In terms of overall output and integrity of frequency and directional response, the plots obtained seemed to show that there is very little to choose between the single closed-back and open-back capsules. As for the closed-back capsule, there will be diffraction effects around the open-back capsule at the higher frequencies where the capsule is large relative to the wavelength of the sound; and at higher frequencies still there will be additional effects due to diffraction around the holes in the grids. However, as it is sensitive on both front and back faces, the alteration in the overall directional behaviour of the open-back capsule will not be as severe as for the closed-back capsule.

TWO CAPSULES PLACED BACK TO BACK

The array

Using the same axes as above, the microphone will now be considered to have two capsules placed back-to-back along the x-axis, equi-distant from the origin with their diaphragms separated by a distance u. The total output of the array can be found as either the sum of, or the difference between, the outputs of the two capsules. The response of the array can then be given as this total output divided by the input to the array; this being the pressure variation at the origin, P_o , as given in (3).

When the response is obtained from the sum of the outputs of the two capsules, this will be called the 'summation response', while that obtained from the difference between the outputs will be termed the 'difference response'. Hence, if the two capsules were facing in the same direction, then the summation response would basically be an augmented version of the response for a single capsule. However, the capsules here are facing in opposite directions and, while this will not affect the responses obtained with the non-directional closed-back capsules, the responses obtained with open-back capsules will be interchanged; turning an open-back capsule back-to-front being equivalent to inverting its output. Thus it will be the difference response of a back-to-back pair of open-back capsules which will be similar to the response for a single capsule, not the summation response.

Two closed-back capsules

The output from each capsule will be given by the response of a single closed-back capsule multiplied by the 'input' pressure, this being :

$$p.\cos(2\pi ft + A \pm \pi fu.\cos(B)/c) \qquad (11)$$

It follows that for a back-to-back pair of closed-back (pressure-responding) capsules, the summation and difference responses are given by the expression (8) (the response of a sigle closed-back capsule) multiplied respectively by :

$$2.\cos(\pi fu.\cos(B)/c) \qquad (12)$$

$$-2j.\sin(\pi fu.\cos(B)/c) \qquad (13)$$

The simulation programme was again extended to incorporate these factors.

Summation response. For low enough frequencies the cosine term in (12) will be approximately equal to unity, and the summation response of a back-to-back pair of closed-back capsules was found to be simply twice that of a single closed-back capsule, as would be expected. At high frequencies there will be another set of dropouts corresponding to cancellation between the two capsules. These dropouts behaved in a similar way to those due to front-back cancellation for a single open-back capsule, as would be expected on comparing (12) and (10); the dropout frequencies being inversely proportional to u (in place of q), and rising to infinity as B was increased to 90°.

However, due to the cosine in (12) where (10) has a sine, the output level of this array was found to be independent of u, while the extra dropouts were at relatively lower frequencies than for the single open-back capsule. For u = 10 mm (B = 0) the first front-back cancellation dropout was at 16.5 kHz, while for u = 5 mm it was at 33.0 kHz, the overall response level being the same in both cases. For u = 3 mm the first dropout in the front response would be at 55 kHz, while for the single open-back capsule of the same thickness (q = 3 mm), the first dropout was at 110 kHz.

This type of array, if short enough, could thus provide a small, high output, pressure-responding microphone with a good high-frequency response. There may well be additional advantages, with regards to diffraction effects, in having a symmetrical structure of this type, either with the capsules immediately back-to-back, or perhaps even sharing a common central electrode.

Difference response. Comparing (13) with (10), it can be seen that the difference response of a back-to-back pair of closed-back capsules is in fact identical to the response of a single open-back capsule; the thickness, q, of the single capsule being replaced by the separation, u, of the two capsules in the array. This is hardly surprising, both microphones measuring the pressure difference between two parallel planes.

The use of two pressure-responding capsules in this way was proposed by A.D. Blumlein in 1931 [4]. However, he suggested that their separation, u, should be of the same order as the width of the human head, to prevent any differences in the individual responses of the capsules from dominating the overall response at low frequencies, where the pressure difference between the capsules would be small. With more modern capsules this problem could still arise for small capsule separations, a compromise having to be made between the low and high-frequency response integrity. A pressure-difference or velocity-responding microphone is thus probably best constructed from a single open-back capsule.

Two open-back capsules

The responses for this array will be the same as for the previous array with two closed-back capsules, but with the terms in (12) and (13) multiplied by the expression for a single open-back capsule; this in turn being the response of a single closed-back capsule multiplied by (10). However, as explained above, the responses obtained using (12) and (13) are now interchanged; (12) giving the difference response, and (13) the summation response.

Difference response. The term in (12) acts in the same way for the difference response of this array as it did for the summation response of the previous array using closed-back capsules; at low frequencies the response was simply twice that of a single open-back capsule, while at high frequencies the same extra set of cancellation dropouts was added.

However, whereas the closed-back capsule array may have an advantage as regards diffraction effects, it would seem likely that, for this array, the necessary inclusion of a narrow gap between the capsules would aggravate the diffraction, and indeed may cause the array to bahave more as if it consisted of two faces and a connecting ring rather than four faces.

Summation response. Applying the term in (13) to the previous array using closed-back capsules gave a first-order pressure-difference response as would normally be obtained from a single open-back capsule. Hence, applying this term to this array of open-back capsules should now gave a second-order response. From (13) and (10); the low-frequency response is now proportional to $f^2 qu.\cos^2(B)$, and the high-frequency response has two sets of similar but independent front-back cancellation dropouts, due to q and u. With u = 10 mm (B = 0 and u > q), the lowest dropout is at 33.0 kHz.

To compensate for the frequency relationship, the microphone needs a mechanical response which falls with the square of frequency over its operating range. Hence the diaphragms must now be mass controlled with the resonant frequency below the operating range and approximately critically damped. Following on from the results for a single open-back capsule, it was now found that the response for this array is in fact completely independent of the resonant characteristic, provided f_r is low enough.

Because of the $\cos^2(B)$ term, the low-frequency polars now gave a two lobe response similar to that for a single open-back capsule, but which was narrower with both lobes of the same sense. More directional, higher order, responses can be obtained using arrays with greater numbers of capsules on a common axis, but the dimensional loses soon become significant [5].

It should also be noted that it is probably not possible to build a suitable capsule with a low-frequency resonance which is under-damped; beyond a certain point the air damping will always dominate the resonant characteristic. Hence, for an array of this type, it will probably be necessary to use external electronics to flatten the frequency response.

CONCLUDING REMARKS

The simple mechanical system used to model the diaphragm uses lumped parameters giving a single resonant mode. In reality the mass and compliance of the diaphragm are distributed over its surface allowing it to resonate at many other higher frequencies, setting up patterns of circular and radial nodal lines [1][6]. However, recent work [7] has shown that the simple model used is quite good enough provided the damping is approximately critical or greater. The more complex modes of vibration also mean that (2) is only really true below the fundamental resonance ($f < f_r$); but while the details of the effect of the pressure variation across the surfaces of the capsules, on the high-frequency responses may be inaccurate, the general result of a reduction in response with dropouts at certain frequencies still holds.

Within these reservations, the model has shown how the responses of single capsules and of back-to-back pairs of capsules, vary with the dimensions of the microphone, and the resonant characteristic of its diaphragms. It is now intended to use this model to simulate more complex microphones, consisting of four or more capsules in a spherical array, and starting with the four-capsule tetrahedral array as used in the first-order soundfield microphone.

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The effects of diffraction around these capsules are complicated and have not been included in this model. However, it is considered that there may be advantages in constructing future microphones with the capsules in the surface of a closed sphere; the equations for the diffraction around a sphere have been solved [6] and so the effects could be calculated and compensated for. Hence, it is also intended to include such effects in the model.

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