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ONE BIT DIGITAL PROCESSING OF AUDIO SIGNALS

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1. INTRODUCTION

Many analogue-to-digital (A/D) and digital-to-analogue (D/A) converters use an intermediate sigma-delta modulating stage to convert signal inputs and outputs into a simple digital form for high quality conversion. In the case of A/D converters, a multibit representation of the signal is achieved with a decimating filter and, similarly, D/A converters employ interpolators to increase the sampling rate and to remove images of the baseband audio signal that are created by oversampling. The well-documented sigma-delta modulating technique [1] [4], employing Nth order noise shaping, is then used to create a highly quantised two level signal. This one bit signal is a perfectly valid representation because it contains all of the audio band information.

Processing the one bit signal directly offers an alternative approach to audio signal processing and removes the decimating or interpolating requirements in an analogue interface. It also allows a simpler system structure because the interconnections are naturally serial with no implied framing. Also, because the signal is heavily oversampled, the system characteristics can approach those of high quality analogue processors in terms of phase response and distortion effects, while retaining the advantages of digital processing techniques.

This paper aims to describe a second order filter section which directly processes a one bit audio signal. It begins with an account of a filter realisation to achieve this end. The advantages and disadvantages of the system compared with a traditional multibit system are then outlined. In particular the issue of performance, where the advantages of oversampling, the degree of quantisation noise shaping for acceptable signal-to-noise ratios in the audio band, and the effect of finite length coefficient values will be considered. We will show that the processor can be realised without using multibit multipliers. The results of a second order section will be presented and discussed with suggestions for performance improvement. The paper ends with proposals for further work and conclusions drawn from the work completed.

2. FILTER STRUCTURE

This section considers rounding within a digital system and introduces the noise shaping technique. Noise shaping filter realisations are discussed and a complete one bit audio signal processing system is presented.

2.1 Quantisation and Noise Shaping

When an analogue signal is converted to a W bit digital signal, the quantised signal contains 2^W amplitude levels with a difference of Q=(signal amplitude range)/(2^W-1) between adjacent quantisation levels. If the amplitude of a given input is rounded to the nearest level, the maximum quantisation error is Q/2. If the probability density function of quantisation error is

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assumed to be uniform, it is easily shown that the mean square error or average noise power σ^2 is defined as

$$\sigma^2 = \frac{Q^2}{12} = \frac{(V_{max} - V_{min})^2}{12(2^{w-1})^2} \qquad (1)$$

In a digital filter, the multiplication of a W bit signal by an M bit filter coefficient will result in an (W+M) bit output that must be requantised so that the output bit length is the same as the input as shown in Fig. 1. A traditional 16 bit processor will provide a 93 dB output signal to noise ratio as a result of this rounding. A one bit processor on the other hand will produce a one bit output that contains an audio signal that is obscured by noise to an unacceptable level and it is imperative that the quantisation noise is suitably shaped. Noise shaping involves passing the error signal between the multibit input and quantised output through a high pass filter so that most of the quantisation noise energy is moved into frequencies above the audio band. This provides an increased signal-to-noise ratio in the band where the important information exists.

Thus a one bit signal processor must contain two distinct sections, one to filter the audio signal and a second which converts the resulting multibit signal back to one bit one. This section will comprise of a quantiser and a noise shaping filter (Fig. 2). There are many possible ways of arranging these two sections and we will show that the two filters can be combined in such a way as to allow the feedback signal, which comes from after the quantiser, to be one bit.

2.2 Noise Shaping Filter Structures

Tewkesbury and Hallock [1] have suggested a structure where only the error between multibit and quantised signals is fedback and passed through a noise shaping filter. This produces a discrete output Y(z) of

$$Y(z) = X(z) + (1 - H(z))\, Q(z) \qquad (2)$$

where X(z) is the z transform of the input and Q(z) is the z transform of the error signal added by the quantiser.

The input signal is not affected by the noise shaper but in this example all multipliers will have multibit inputs. It should be noted that, as there must be no delay-free loops in the system, the first coefficient of H(z) must be 1. It follows therefor, that the dc. gain of a particular noise shaping filter is not adjustable for a given frequency cut off, giving rise to system stability considerations that will be discussed in a later section. An alternative structure feeds the one bit signal back into the system and contains the noise shaping function in the forward path. This gives:

$$Y(z) = (H(z)/(1+H(z))\, X(z) + (1/(1+H(z))\, Q(z) \qquad (3)$$

One can see that, although the noise is shaped such that $(H_{ns} = (1+H(z))^{-1}$, the input signal X(z) is also filtered. However, this signal filter $(H_a = (1 - H_{ns}))$ will only affect the signal in a frequency region above the audio band, as the noise shaping poles and zeroes will lie outside the audio band, and therefore will not alter the important components of the signal. The above structure also has the advantage of minimising the number of multibit multipliers required for implementation. The structure in Fig. 3 [2] [3] implements 3rd order noise shaping using integrators in place of simple delays. Coefficient multipliers A, B and C have a one bit input and are cost effective in that their output is simply the coefficient value or its negative. Coefficients α and β are required only to achieve a Chebychev type II positioning of filter zeroes and can be

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removed if a Butterworth or Chebychev type I filter will meet specification. This will result in a system that has no multibit multipliers.

2.3 A Complete Processing System

A system has been described that will requantise the resulting multibit signal after the one bit signal has been filtered. An obvious but expensive system to realise a second order filter is place the audio filter and requantiser in series. In this structure the audio filter feedback signal is multibit. However one can arrange for all the inputs to the audio filter coefficient multipliers to be one bit by taking the recursive part of the filter from after the quantiser. This feedback signal not only contains the desired output audio components but also the quantisation noise. This means that the quantisation noise will be shaped by the poles of the audio filter resulting in a noise peak in the middle of the audio band. In truth, because of the non-linear element, the system becomes unstable! This effect can be avoided by adding extra zeroes to the noise shaping transfer function to exactly cancel those added by the audio filter. As the Tewkesbury and Hallock quantising structure is used, noise shaping multipliers are multibit.

A more practical topology combines both the audio and noise shaping filters using a series of integrators and a minimal number of multibit multipliers. In the system shown in Fig. 4 both the noise shaping filter and audio filter unavoidably share poles and what would be a second order audio filter in a multibit PCM processor becomes a $(2+N)^{th}$ order system where N is the order of the noise shaping filter. It is necessary that the audio filter poles in the noise shaping transfer function are cancelled as described above and this is easily achieved by simple manipulation of the coefficients. The system shown requires extra coefficient multipliers in the forward path to provide zeroes in the audio filter to exactly cancel the poles introduced by the noise shaper.

3. THE ADVANTAGES AND DISADVANTAGES OF A ONE BIT PROCESSOR

The benefits and problems involved with a one bit signal processor can be summarised as follows.

Simpler Analogue Interfaces. Interpolation and decimation processes are no longer needed at analogue interfaces. The high sampling rate will reduce the necessity of a stringent pre-sampling filter to remove frequencies greater than half the Nyquist rate in the analogue signal.

Simpler Connection and Synchronisation Hardware. Several system economies result from the single bit word length of each sample. Where many wires and connections would be required for a multibit sample, a single wire will suffice, simplifying design and reducing device size and cost. Similarly, multibit systems require both word and bit synchronisation. As each word is a single bit, word synchronisation hardware becomes redundant.

Simpler Filter Multipliers. As discussed, a second order audio filter is a $(2+N)^{th}$ order system using N^{th} order noise shaping. This increases the number of delays, coefficient multiplications and additions for any given second order section and suggests increased cost in terms of hardware components. But, as both the input and fedback signals contain two levels, +1 and -1, most coefficient multipliers are reduced to switches, passing the coefficient value or its negative directly to the adders. Thus the need for expensive multibit multipliers is reduced.

Indeed, all multibit muiltipliers can be removed from the system. Coefficient multipliers α, β and γ in Fig. 4 are required to move the noise shaping zeroes off the real axis so that a Chebychev type II noise shaping function can be realised. Noise shaping can also be achieved using

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Butterworth or Chebychev type I filters which do not need these coefficients. However sampling rates will need to be increased to move the extra quantisation noise out of the audio band.

It was stated earlier that these coefficient multipliers are also required to exactly cancel the audio filter poles introduced into the noise shaper. The Chebyshev type II zeros will not exactly cancel the poles but will lie at the same angle as the pole providing a similar effect. Also the high signal sampling rate employed by the system moves any audio filter poles very close to the real axis in the z plane, that is, z=1 (approximately). Each integrator produces a noise shaping zero at exactly z=1 and therefor the audio filter poles are essentially cancelled. In fact we discovered that the small discrepancy between unwanted poles and cancelling zeroes actually improves system performance.

Analogue Qualities. The high oversampling rates employed by the system is a factor that can be attributed to the first category. Analogue signals can be considered to be discrete signals with a sampling rate of infinity. If the sampling frequency of a discrete signal is increased, its characteristics move closer to those of an analogue signal and the performance of a digital filter clocking at a higher rate becomes more like that of an analogue filter. Oversampling avoids the need for stringent preliminary filtering and preserves the harmonics in their original positions (although they may be obscured by quantisation noise). Phase response becomes more like that found in high quality analogue systems particularly at high frequencies. Whereas sampling at the Nyquist rate results in a significant difference in the phase and frequency responses of filters or equalisers in the top quarter of the frequency range. The one bit system is also better if one is doing any non-linear processing (e.g. distortion effects) because harmonics produced by these effects are at frequencies above the normally defined audio band. These components are not accounted for when the signal is sampled at the Nyquist rate and will be folded into the audio band. In the case of the one bit system the heavily oversampled nature of the signal means that this will not happen to the same extent.

Signal-to-noise Considerations in relation to Noise Shaping. The signal-to-noise ratio of the output relies on the specification of the noise shaping filter. Adams [2] suggests that, for a particular sampling frequency, type and order of noise shaping filter higher than second, there is a unique filter transfer function that will provide correct comparator gain and system stability. He also describes an iteration procedure to find it. For a given set of system specifications in terms of sampling frequency and noise shaping order, the high pass cut off frequency is already defined. A Chebychev type II filter with suitable dc. gain will have a higher cut off than a Butterworth or Chebychev type I for a given specification. This will move the quantisation noise further out of the audio band and offer improved signal-to-noise ratio. However the cost advantages of avoiding Chebychev type II noise shaping have been described earlier. It should also be noted that the discrepancy between the audio filter poles in the noise shaper, due to the desired filtering function and the integrator poles introduced to cancel them, also produces a highpass filtering function. This has the same cut off frequency as the desired filter and can help to further remove noise from the audio band. An increase in the order of the noise shaping filter also decreases the corner frequency of the filter function allowed for stability. An increase in sampling frequency will allow the corner frequency to move further out of the audio band but will increase the coefficient word length for a given sensitivity specification.

Stability and Sensitivity - their reliance on coefficient word length. As the sampling frequency of the system is large, coefficient words will be longer than those used in a processor using clock rates near the Nyquist frequency. Kaiser (in [5]) has shown that neighbouring poles on the z plane will influence each other's positions depending on the degree of coefficient quantisation. The one bit audio filter has (2+N) poles because the noise shaper introduces N more poles than a multibit second order system. These poles are also close together on the z plane because of the high oversampling rate. This implies that the coefficient sensitivity is significantly affected by coefficient word length. It is easily shown that coefficient lengths do not need to be increased significantly to achieve stability and the sensitivity required of a high quality audio filter. The position of each pole on the complex z plane is governed by the angle, ωT, between the line joining the pole with the origin and the real axis where ω is frequency, T is the time interval between samples, and by r, the distance from the pole to the origin. The system will remain stable if the pole remains inside the unit circle, that is, r remains less than 1. Sensitivity specifications depend on the change in ω. Differentiation of the equation governing pole position will produce an equation that defines the change in pole position with respect to r and ω. and the changes in r and ω.. These variables can be entered according specification and stability limits to give a maximum allowed change in pole position Δ_p. Kaiser's equation will produce a matrix relating the change in pole position to a change in coefficient values in a direct filter realisation. Another matrix can be set up relating the change in these values to a change in actual coefficient values. The coefficient word length required is found by establishing the maximum error in coefficient value due to quantisation and multiplying it by both matrices to find the change in pole position for a particular word length. This can be compared with the maximum change allowed. The word length is increased or decreased to find an acceptable change in pole position. Using this method we have found that a suitable word length for a 50 Hz one bit filter clocking at 1 MHz using the structure shown in Fig. 4 is 32 bits.

4. SYSTEM SIMULATION AND RESULTS

An eighth order low pass filter with a cut off frequency of 8 kHz was designed employing four third order Chebychev type I noise shaping filter sections. It was also implemented using Chebychev type II noise shaping filter sections. The sampling frequency was chosen to be 1MHz (approximately 20 times the Nyquist frequency) resulting in a stable noise shaper with a 33 kHz corner frequency. The coefficient word length required for stability and frequency response accurate to 1 Hz was 20 bits. The processor was simulated using the SPW signal processing software package by COMDISCO with a one bit input signal comprising four sine waves at different frequencies. The spectrum of the one bit outputs are shown in Figs. 5 and 6. The output contains suitably attenuated stopband and acceptable noise attenuation in the audio band but clearly shows the expected rise in noise at higher frequencies. However the type II filter has better noise performance in the stopband.

5. CONCLUSION

A method for realising a filter to process one bit signals has been developed and presented. The report began by detailing quantisation issues and described how a traditional audio filter can be combined with a noise shaping filter and comparator to produce a filtered one bit output for any one bit input. The advantages and disadvantages of this system over multibit signal processors were discussed in relation to cost, where it was seen that many component savings could be made at the price of faster clocking hardware and marginally higher coefficient wordlengths. In this latter section, it was suggested that the magnitude response of the filter could be as good as that of multibit systems and that the phase response was an improvement on systems that clock at the

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Nyquist rate. An essentially multiplierless one bit filter has been simulated successfully, providing unarguable proof of the system's validity. One bit digital filtering shows promise as a technique for achieving high quality system effective audio signal processing.

6. REFERENCES

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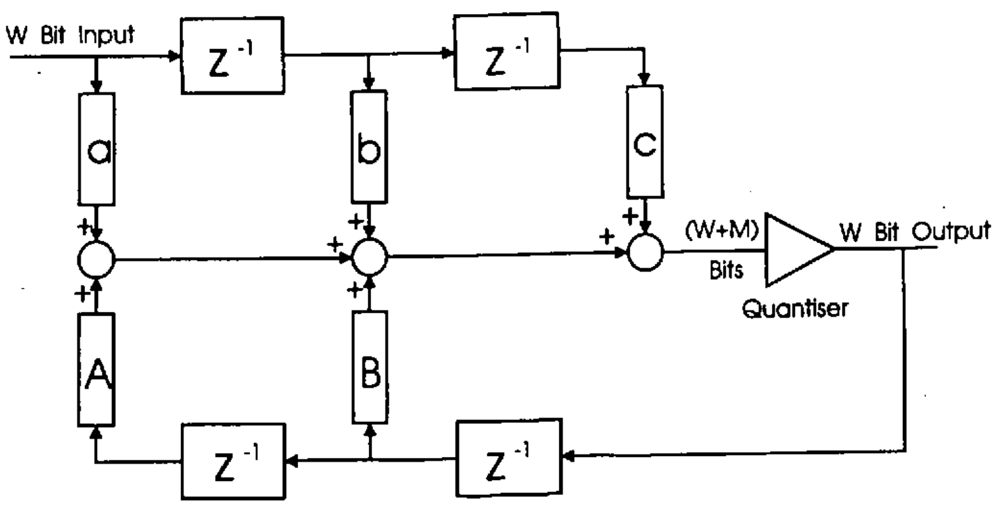

Fig. 1 A Generalised Second Order Section with M Bit Coefficients.

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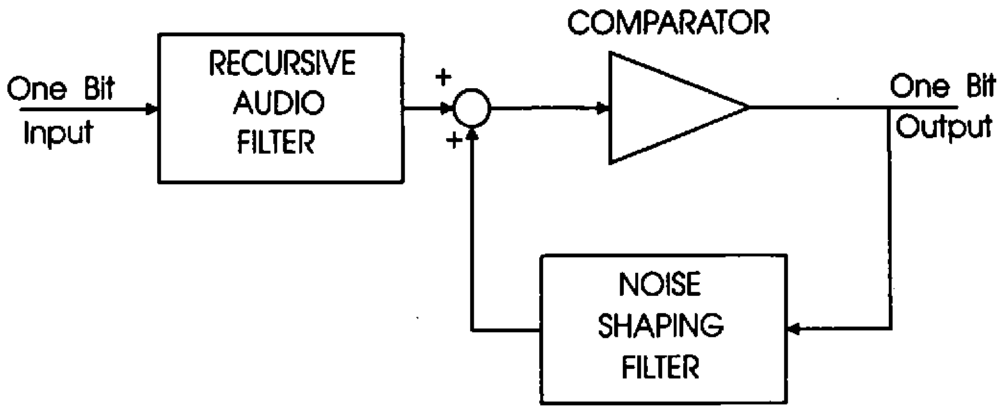

Fig. 2 Generalised Diagram of a One Bit Processor

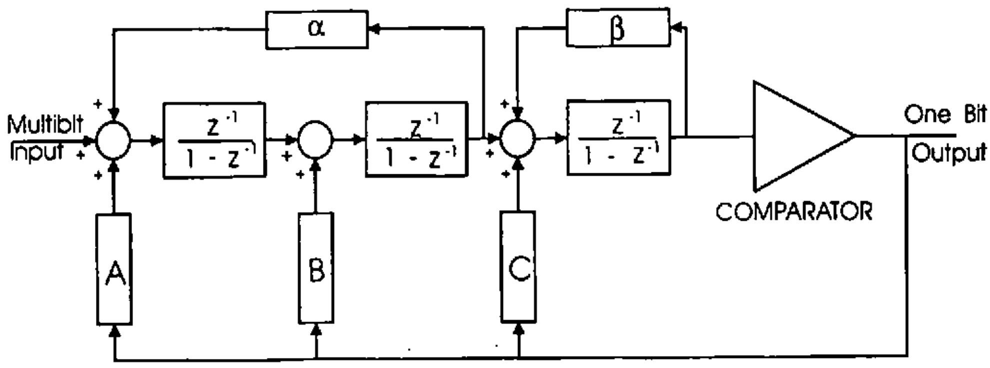

Fig. 3 Quantiser with a Minimum Number of Multipliers.

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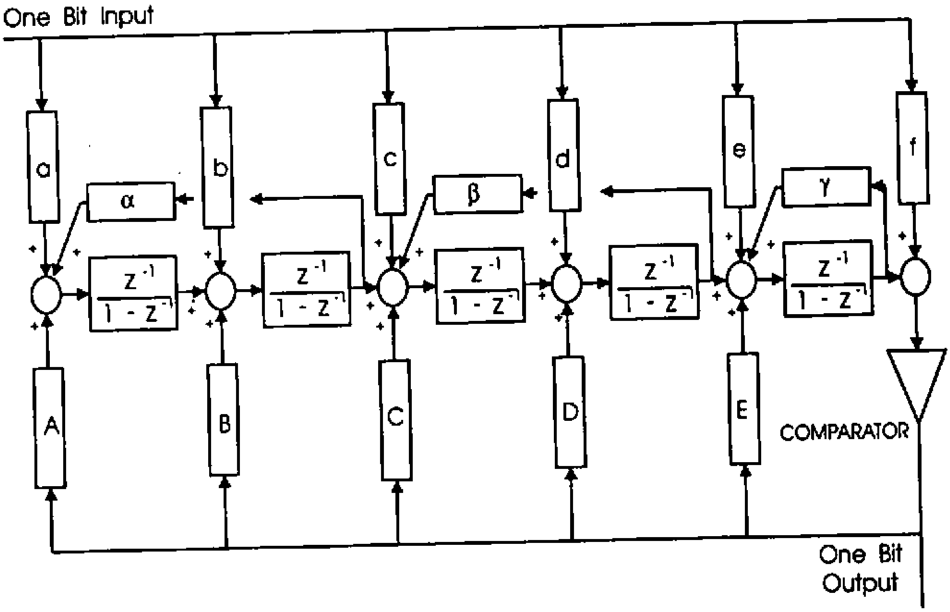

Fig. 4 A Complete One Bit Processor with a Minimum Number of Multibit Multipliers.

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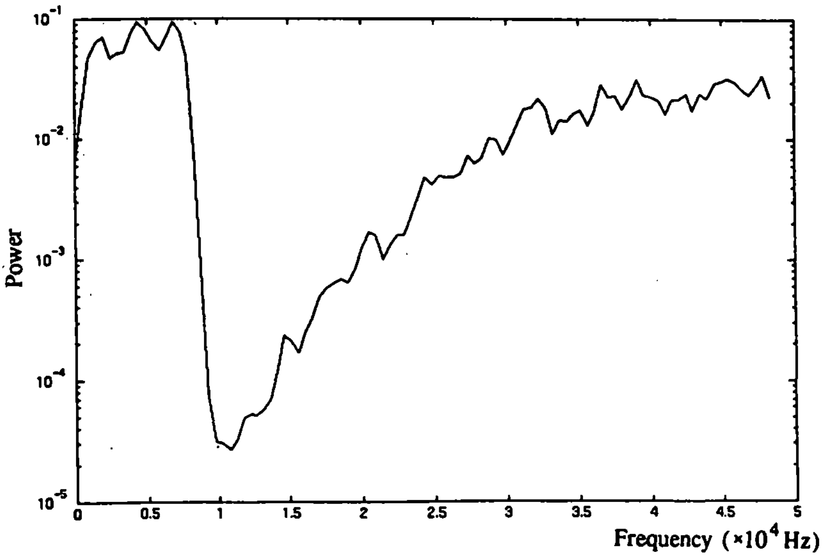

Fig. 5 FFT Plot of the Output of the Simulated System using Chebychev type I noise shaping filter sections

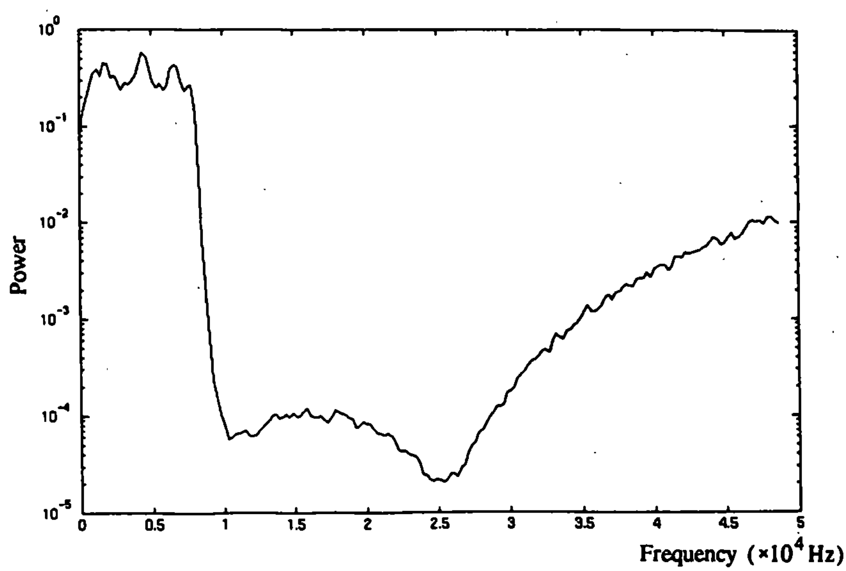

Fig. 6 FFT Plot of the Output of the Simulated System using Chebychev type II noise shaping filter sections.