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THE PARAMETRIC END-FIRE ARRAY
IN A RANDOM MEDIUM

by

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1. INTRODUCTION

In the parametric method [1] of generating a highly-directive beam of sound underwater the effective difference-frequency radiation volume within the water may be quite large, depending upon the absorption coefficients of the two interacting primary waves. Now in practice, an underwater environment, such as the ocean, has random inhomogeneities and it is thus important to establish how these are likely to effect the difference-frequency volumetric-source distribution within this radiation volume. Of particular interest to the system designer is the relative performance of the parametric transmitter to a conventional source operating in the same random medium.

A theoretical study of these effects has been made [2] based upon the amplitude and phase fluctuations produced on acoustic waves propagating through weakly inhomogeneous media. Some initial results of an experimental investigation have also been reported [3]. The object of this paper is to present some recent theoretical and experimental findings in connection with this subject.

2. THEORETICAL CONSIDERATIONS

The theoretical model studied in [2] has been extended [4] to include the effects of partial coherence of the fluctuations of the primary waves. The following discuscussions are based on these theoretical models.

The coefficient of variation, V_N, at an observation point P, with the geometry as shown in figure 1, is given by

$$V_N^2 = \frac{<|p_3|^2> - <|p_3|>^2}{<|p_3|>^2} = \frac{\iint H_i H_j <B_i B_j> dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j} \qquad (1)$$

where, p_3 is the parametric signal pressure at point P and < > denotes a time average. H_i, H_j are parametric signal differential pressures at P, arising from source distributions in the differential volumes, dv_i and dv_j respectively.

B_i, B_j are amplitude fluctuations at P given by

$B_i = B_{1i} + B_{2i} + B_{3i}$ and $B_j = B_{1j} + B_{2j} + B_{3j}$,

where B_{3i}, B_{3j} are the amplitude fluctuations at P, of the differential parametric signal originating in the differential volumes dv_i, dv_j, respectively and B_{1i}, B_{2i} and B_{1j}, B_{2j} are the amplitude fluctuations of the two primary waves at dv_i and dv_j.

The determination of the terms H_i and H_j, for various beam geometries, has been extensively covered in the literature by Westervelt [1], Berktay [5], Rolleigh [6] and others and hence details will not be given here.

Let us consider the amplitude fluctuation terms B_i and B_j. Making the assumption that $B_{1i,j}$ and $B_{2i,j}$ are not significantly correlated with $B_{3i,j}$, a reasonable assumption since they are produced in different parts of the medium, then equation (1) can be written as

$$V_N^2 = \frac{\iint H_i H_j \left[\langle B_{1i}B_{1j}\rangle + \langle B_{2i}B_{2j}\rangle + 2\langle B_{1i}B_{2j}\rangle + \langle B_{3i}B_{3j}\rangle\right] dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j} \qquad (2)$$

The terms $\langle B_{1i}B_{1j}\rangle$ and $\langle B_{2i}B_{2j}\rangle$ are recognisable as the spatial correlation functions of the amplitude fluctuations at dv_i and dv_j of the primary waves. These may therefore be evaluated using the existing formulae given by Chernov [7], Tatarski [8] and Aiken [9]. The term $\langle B_{1i}B_{2j}\rangle$, which has been assumed to be equal to $\langle B_{1j}B_{2i}\rangle$ in writing equation (2), is a little more complicated and represents the spatial cross-correlation between the amplitude fluctuations of the two primary waves. However, this may be calculated from formulae presented by Eliseevnin [10]. The term $\langle B_{3i}B_{3j}\rangle$ is somewhat different, as it represents the correlation of the amplitude fluctuations at P of two signals, the one arriving from dv_i and the other from dv_j. However, for spherically spreading waves at long ranges, reciprocity may be shown to apply and hence $\langle B_{3i}B_{3j}\rangle$ is equivalent to the spatial correlation function of the amplitude fluctuations at dv_i and dv_j of a single wave transmitted from P. Therefore, $\langle B_{3i}B_{3j}\rangle$ may also be evaluated using existing formulae. Thus equation (2) may be solved numerically for any particular set of long range conditions.

In most applications the two primary waves have frequencies which are very nearly equal, the difference frequency being small, so that the further approximation may be made:-

$$<B_{1i}B_{1j}> \simeq <B_{1i}B_{2j}> \simeq <B_{2i}B_{2j}>$$

Therefore, equation (2) reduces to

$$V_N^2 \simeq \frac{\iint H_i H_j \, (4<B_{1i}B_{1j}> + <B_{3i}B_{3j}>) \, dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j} \qquad (3)$$

Consider that V_N^2 is composed of two components,

$$\frac{4 \iint H_i H_j <B_{1i}B_{1j}> dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j} \quad \text{and} \quad \frac{\iint H_i H_j <B_{3i}B_{3j}> dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j}$$

The first of these includes the term $<B_{1i}B_{1j}>$ which is a function of $\underline{s}_i$ and $\underline{s}_j$ and is therefore independent of the position of P, shown in figure 1 as the vector distance $\underline{R}$. H_i and H_j are dependent on $\underline{R}$ but if the point P is situated outside and beyond the region in which the primary signals interact to produce significant difference-frequency sources, i.e. in the far-field of the effective interaction or radiation volume, then the value of this first component of V_N^2 is largely independent of $\underline{R}$. The second component contains the term $<B_{3i}B_{3j}>$, which is a function of $\underline{r}_i$ and $\underline{r}_j$ and consequently as the value of $\underline{R}$ increases the value of this component as a whole increases. Therefore, for very long ranges, the second component dominates, i.e. for large $\underline{R}$,

$$V_N^2 \simeq \frac{\iint H_i H_j <B_{3i}B_{3j}> dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j} \qquad (4)$$

3. EXPERIMENTAL INVESTIGATION

The experiments were conducted in the laboratory on a 'modelled' parametric-array operating in a water-filled rectangular tank of dimensions 1.8 m x 0.8 m x 0.9 m. An array of heaters covered by a steel mesh were mounted at the bottom of the tank, see figure 2, such that a random thermal microstructure was created in the water by convective mixing. The experimental equipment has been described in detail elsewhere [3], [4].

3.1 Temperature Microstructure

In order to be able to compute the amplitude-fluctuation correlation terms $<B_{1i}B_{1j}>$ etc. and hence make comparisons between experiment and theory, a knowledge of the spatial correlation function of the random refractive-index changes within the water is required. This is obtained from measurements of the spatial correlation function of the temperature microstructure. Figure 3 shows a typical correlation coefficient for the tank.

It is possible to treat temperature fluctuations as passive additives of turbulence [8]. Based on this view, Medwin [11] has proposed a universal model for termperature microstructure in the upper regions of the ocean. By a suitable adaptation of this model, a prediction of the spatial temperature correlation coefficient to be expected in the tank was obtained. This gave excellent agreement with the empirical data of the index correlation function, as shown by figure 3.

3.2 Amplitude Fluctuations of Acoustic Waves

The pulsed carrier acoustic signals used in the experiments had a pulse width of the order of 100 μs and a period between pulses of 4 secs. At the receiver a 10 μs section of the pulse, sampled 35 μs after the arrival of the leading edge, was envelope detected to find its peak amplitude and the value logged onto paper tape. By this method, multipath signals were avoided as well as transients of the leading edge.

The coefficient of amplitude variation, V_L, of 'linear' acoustic signals at 1 MHz and 9 MHz were measured and plotted against range. The results were in good agreement with predictions based on the Medwin model, see figures 4 and 5. The transverse and longitudinal correlation coefficients of amplitude fluctuations of a spherically spreading signal at 1 MHz were measured, see figure 6. Additional measurements were made to determine roughly the shape of the correlation volume. A two-dimensional correlation surface was obtained, shown in figure 7. Similar measurements were attempted for a collimated beam at 9 MHz, but they were only partially successful because of the lateral displacement of the rather narrow beam and the very short correlation distances. A satisfactory result was obtained for the transverse correlation function only.

The coefficient of amplitude variation, V_N, of a parametric signal at 1 MHz, generated by primary signals with frequencies of 9 MHz and 10 MHz respectively, was also measured, see figure 8. It was found to be indistinguishable from that of a linear signal of the same frequency measured on different occasions (figure 4), because of the large scatter of the data which obscured any small differences present. A more decisive comparison was made by measuring the amplitude fluctuations simultaneously and in the same medium. This was achieved by transmitting the parametric and linear signals simultaneously along the same path but in opposite directions. The transmitting transducer of one doubling as the receiver for the other. Linear signals were also transmitted in both directions simultaneously for comparison. From this experiment, it was found that the parametric signal fluctuations were decisively smaller than those of the linear signal at ranges greater than 1 m. The results are shown in figure 9.

4. DISCUSSION OF PARAMETRIC RESULTS

Using the theoretical expressions presented in Section 2 it is possible to make comparisons with the experimental data of figure 9.

In the first instance a comparison can be made using the long-range approximation, equation (4). Assuming that the primary waves are perfectly collimated with a uniform cross-section equal to the surface of the transmitting transducer and applying the modified Medwin model for the tank to the formulae in references 7, 8, 9, estimates of $\langle B_{3i} B_{3j} \rangle$ were obtained. With these estimates applied to equation (4) it was found, for the long-range estimate, that

$$\left(\frac{V_N}{V_L}\right)^2 = 0.86$$

A more detailed calculation may be made using equation (2). In this case empirical values of $<B_{3i}B_{3j}>$, measured at a range of approximately 1.1 m, were used in conjunction with theoretical estimates of $<B_{1i}B_{1j}>$, etc. Values of $(\frac{V_N}{V_L})^2$ were calculated at ranges of 0.8 m, 1.2 m and 1.6 m and are shown plotted in figure 9.

Strictly speaking, the experimental points in figure 9 are the values of $\frac{V_N^2}{V_{Lr}^2}$ and $\frac{V_L^2}{V_{Lr}^2}$, where the subscript r denotes a signal travelling on the the same path but in the reverse direction. Since it has been found that $V_L^2 \simeq V_{Lr}^2$, therefore one can say $\frac{V_N^2}{V_{L}^2{}_r} \simeq \frac{V_N^2}{V_L^2}$.

The rough agreement between these theoretical estimates and the experimental data is reasonable in view of the number of approximations and assumptions made.

At short ranges, that is 0.8 m and less, the measurements of $(\frac{V_N}{V_L})^2$ are less repeatable with a greater scatter. This is probably connected with the lateral movement of the primary signal beam and near-field effects of the parametric array.

The results presented in figure 9 are intriguing because they show that in some situations the parametric signal is likely to be less effected by an inhomogeneous medium than a conventional signal. It is of interest to examine this further.

Consider the long-range approximation to the coefficient of variation given by equation (4). For long ranges and for significant contributions from H_i, H_j in the integrals, the vectors $\underline{s}_i$, $\underline{s}_j$ are negligible compared to $\underline{r}_i$, $\underline{r}_j$, therefore $\underline{r}_i \simeq \underline{r}_j \simeq \underline{R}$ (refer to figure 1.) Hence, $<B_{3i}^2> \simeq <B_{3j}^2> \simeq <B_{3L}^2>$, where $<B_{3L}^2>$ is the mean-square amplitude fluctuation of a spherically spreading conventional signal at the same range and frequency as the parametric signal and by definition.

$$<B_{3L}^2> = V_L^2.$$

Therefore one may write $V_L^2 \simeq <B_{3i}^2> \simeq <B_{3j}^2>$. Also a correlation coefficient M_{3ij} may be defined as

$$M_{3ij} = \frac{<B_{3i}B_{3j}>}{\sqrt{<B_{3i}^2> <B_{3j}^2>}} ,$$

which in this case may also be written as $M_{3ij} \simeq \frac{<B_{3i}B_{3j}>}{V_L^2}$.

Therefore equation (4) becomes:-

$$\left(\frac{V_N}{V_L}\right)^2 \simeq \frac{\iint H_i H_j M_{3ij} \, dv_i \, dv_j}{\iint H_i H_j \, dv_i \, dv_j}$$

This equation demonstrates that the importance of the random medium on the parametric signal will depend strongly upon the amplitude correlation coefficient M_{3ij}. By definition $M_{3ij} < 1$, therefore in general, at long ranges, the parametric signal would not be subject to more fluctuations than a conventional linear signal of the same frequency. In the extreme case where the correlation is very small, i.e. M_{3ij} is only important for small spatial separations of the differential volumes dv_i, dv_j, the parametric signal is likely to be significantly less seriously effected by fluctuations than a conventional signal.

The physical reason for this improvement is that the parametric signal comprises components radiated from different points within the medium. If the signal fluctuations from these points are uncorrelated then we have a situation which in aerial theory, is analogous to a signal to noise improvement achieved by using spatial diversity on transmission.

5. CONCLUSIONS

It has been shown that treating the temperature microstructure as a passive additive of turbulence, along the lines proposed by Medwin, provides a useful and valid description of the random temperative field created in the laboratory tank. The corresponding predictions of amplitude fluctuations on acoustic signals gave agreement within the scatter of the experimental data.

The important conclusion with respect to the parametric signals is that, in general at long ranges, the parametric signal is likely to be less adversely effected by a weakly inhomogeneous medium than a conventional signal.

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Fig.1 Scattering Geometry

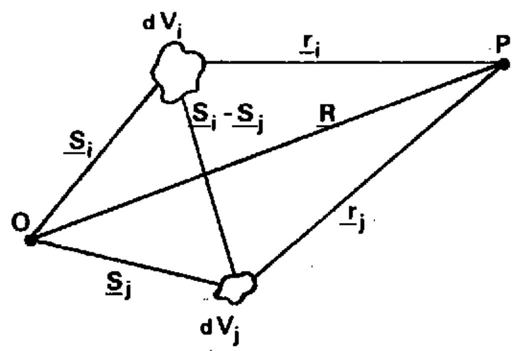


Fig.3 Temperature Spatial Correlation Coefficient

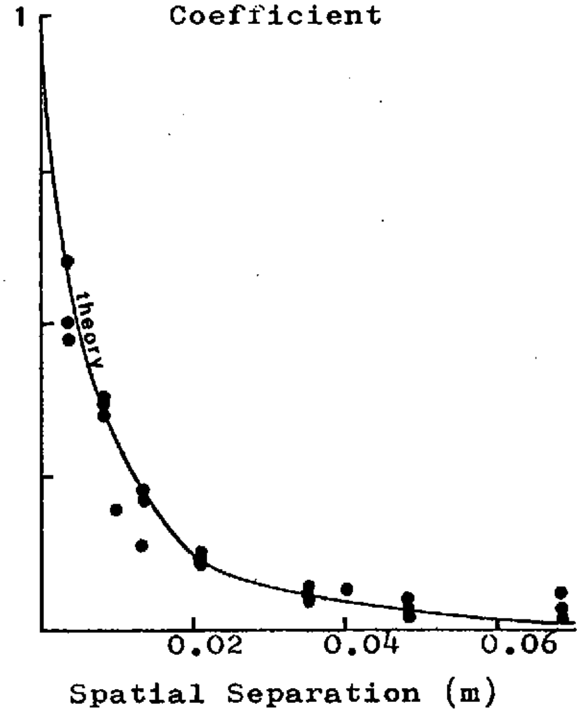


Fig.2 Basic System Block Diagram

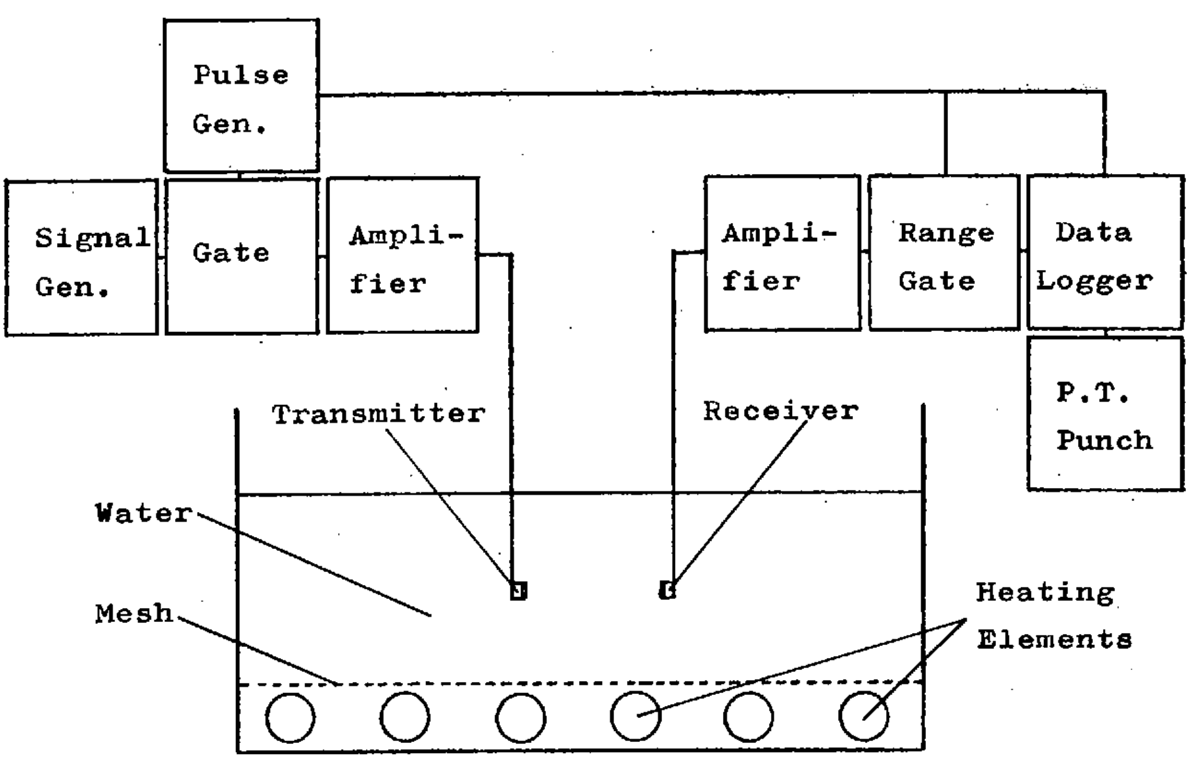


Fig.4 9MHz Collimated Beam

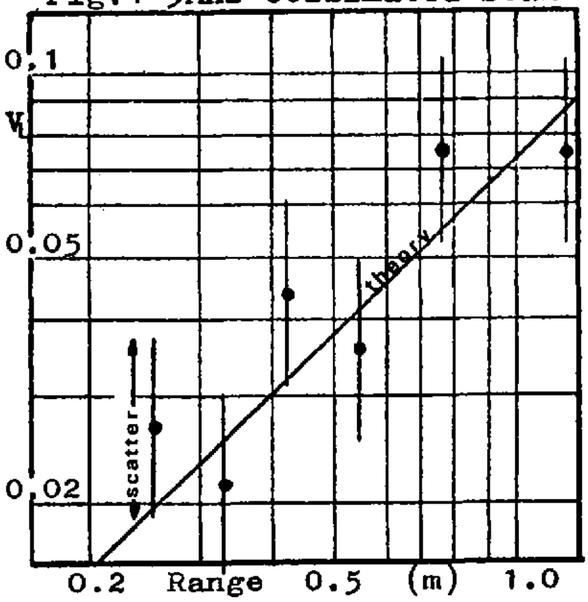


Fig.5 1MHz Spherical Wave

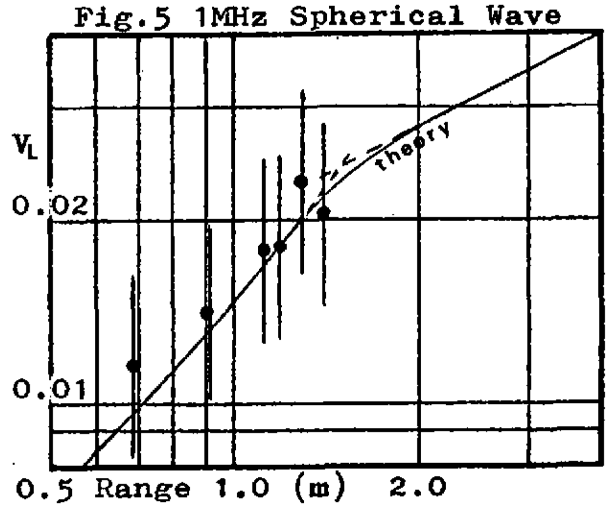


Fig.6 1MHz Spherical Wave
Spatial Correlation

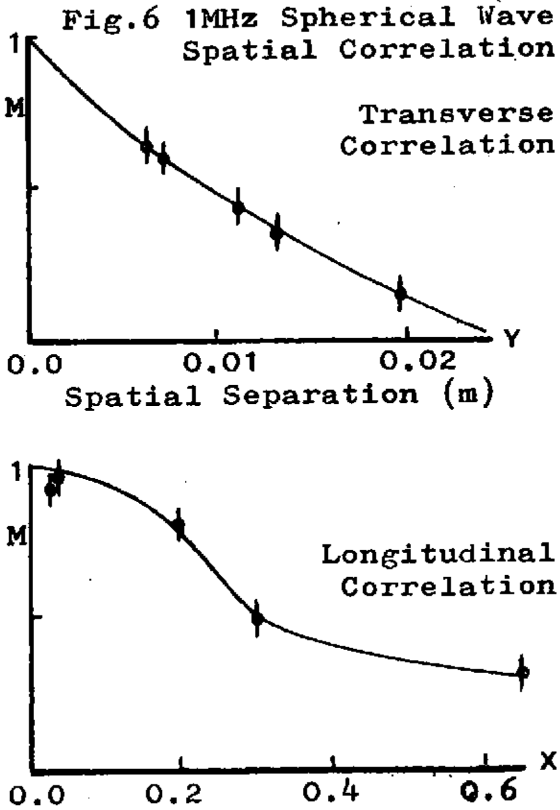


Fig.8 1MHz Parametric Signal

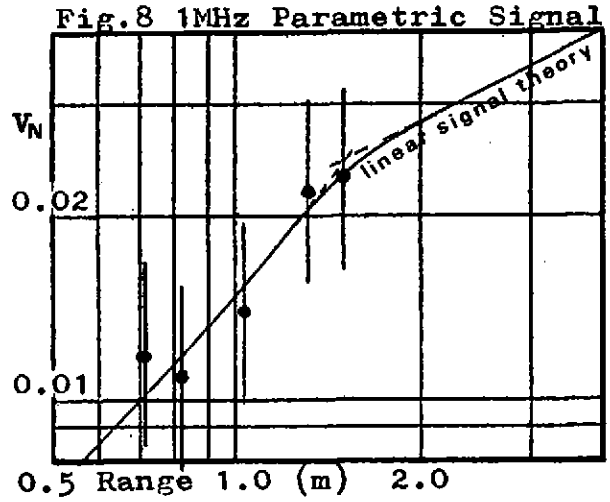


Fig.7 2-D Correlation Surface
1MHz Spherical Wave

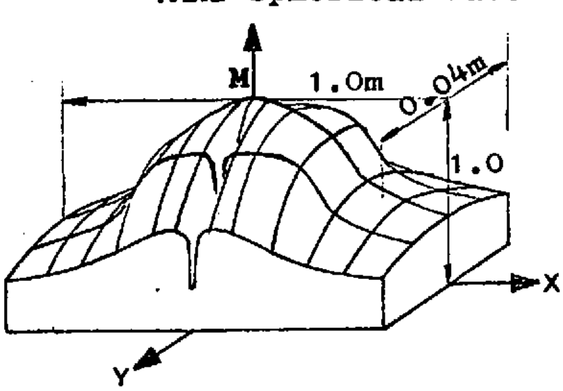
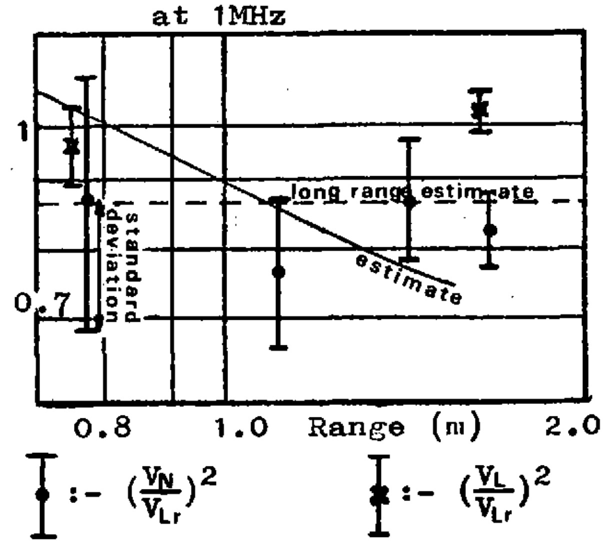


Fig.9 Comparison of V_L and V_N at 1MHz



V_L and V_N are coefficients of amplitude fluctuation of linear and parametric signals, respectively; the subscript $_r$ denotes signal propagation in the reverse direction.