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ADDITIONAL PAPER RECEIVED TOO LATE FOR INCLUSION IN MAIN PROCEEDINGS.

"INTERACTION BETWEEN SOUND WAVES PROPAGATING IN THE SAME DIRECTION.

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1. Introduction.

In a previous work (Tjötta 1967) we considered the mutual non-linear interaction between two sound waves travelling in the same direction. The basic equations of motion for a homogeneous thermoviscous fluid were approximated by linearizing the diffusion terms due to viscosity and heatconduction. Approximated solutions were obtained by applying the method of successive approximations up to a third order, the solution of the linearized equation being taken as a first order approximation. Especial attention was directed to a study of the possibility of parametric amplification. To a third order approximation it was concluded that amplification of a small signal wave with frequency ω_2 due to interaction with a stronger pump wave with frequency $\omega_1 > \omega_2$ is possible if and only if the generated wave with sum frequency $\omega_1 + \omega_2$ (in the second order approximation) is blocked. However, an upconverter type of parametric amplification was found to be possible, i.e. it is possible to obtain combined frequency waves with amplitudes that exceed the one of the primary ω_2 wave for moderate and higher volume of ω_1/ω_2. The effect was small - of order one only. In a more recent work by Berktey and Al - Temimi (1970) this theoretical result seem to have bee verified experimentally (larger than one only at high intensity where higher harmonics in the pump wave become significant).

We now present the results of an analysis of this interaction problem on the basis of the Burgers' equation. The exact solution of this equation is expanded in a series in the parameters, $R_1 = M_1/S_1$ and $R_2 = M_2/S_2$, where M_1 and M_2 are the Mach numbers, S_1 and S_2 the Stokes numbers of the two interacting waves (defined in section 2). The series converge for all finite R_1 and R_2 (The solution of the Burgers' equation is not valid for $R_1 = \infty$ or $R_2 = \infty$). The three first terms in the expansion are given explicitly and the range of validity of calculations based on the method of successive approximation is considered. We also add some comments on the interaction problem between two collimated beams, and the theory developed is used to interpret some recent experimental observations by Berktay and Al-Temimi, and by Hobæk and Vestrheim (this meeting).

2. Basic equations.

We assume one-dimensional motion and start with the Burgers' equation (see, for example Blackstock 1964)

$$\frac{\partial v}{\partial x} - Bv \frac{\partial v}{\partial \tau} = D \frac{\partial^2 v}{\partial \tau^2} , \qquad (1)$$

where v is the particle velocity, x is the distance, $\tau = t - \frac{x}{c_o}$, t is the time, c_o is the speed of an isentropic and infinitesimal sound wave at the local values of the temperature and density. Further B and D are two constants defined by $B = \frac{\beta}{c_o^2}$ and $D = \frac{\delta}{2c_o^3}$, when

$\delta = \frac{\kappa + \frac{4}{3}\mu}{\rho} + \frac{\gamma - 1}{\gamma}K$ is the sound diffusivity expressed in terms of viscosity and heat conduction, $\beta = \frac{1}{2}(\gamma + 1)$ for perfect gases and $\beta = \frac{1}{2}(\gamma_1 + 1)$ for arbitrary fluids, where γ is the ratio of specific heats and γ_1 is a constant related to the equation of state of the fluid.

For a monochromatic wave this equation describes the motion correctly up to order M and S relative to the largest term retained. Terms of relative order M^2, MS and S^2 are neglected. Here M and S denote the Mach number and the modified Stokes number respectively, i.e.,

$$M = \frac{v}{c_o} \quad , \quad S = \frac{\delta\omega}{c_o^2}$$

where ω is the anuglar frequency of the wave. To this order of approximation the excess velocity u - which normally enters in the Burgers' equation - is related to v by the expression $u = \frac{1}{2}(\gamma_1 + 1)v$.

The boundary conditions are:

$$v = 0 \quad \text{for} \quad t \leq 0 \; , \quad x = 0$$

$$v = v_o = v_{o1}\cos\omega_1 t + v_{o2}\cos\omega_2 t \quad \text{for} \quad t > 0 \; , \quad x = 0. \qquad (2)$$

We have here linearized the boundary conditions. A more correct condition would be

$$v\Big(x(t), t\Big) = \frac{dx(t)}{dt} \quad \text{for} \quad t > 0 \; ,$$

where $x(t) = \frac{v_{o1}}{\omega_1}\sin\omega_1 t + \frac{v_{o2}}{\omega_2}\sin\omega_2 t$, as the source will oscillate with finite amplitude. However, Eq.(2) is a proper approximation in our case (cf. Lauvstad, Naze and Tjötta 1964).

3. Solution.

An exact solution of Eq.(1) is well-known and can be expressed in the following form (cf. Blackstock 1964)

$$v = \frac{2D}{B}\frac{\partial}{\partial\tau}\ln\,\Theta\,, \tag{3}$$

with

$$\Theta = \frac{1}{2\sqrt{\pi Dx}}\int_{-\infty}^{\infty}\Theta_o(\lambda)\,e^{-\frac{(\lambda-\tau)^2}{4Dx}}\,d\lambda\,,$$

$$\Theta_o(\lambda) = e^{\frac{B}{2D}\int_o^{\lambda}v_o(t)dt}.$$

Substituting $\lambda-\tau = -2\sqrt{Dx}\,q$, and inserting the boundary conditions (2), we find:

$$\Theta = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}e^{-q^2+\beta R_1\sin[\omega_1(\tau-2\sqrt{Bx}\,q)]+R_2\sin[\omega_2(\tau-2\sqrt{Bx}\,q)}dq + T(t), \tag{4}$$

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Equation 4 should read

$$\theta = \sqrt{\frac{1}{\pi}} \int_{-\infty}^{\infty} \exp\Big\{-q^2+\beta R_1 \sin[\omega_1(\tau-2\sqrt{D\chi}\ q)]$$

$$+\beta R_2 \sin[\omega_2(\tau-2\sqrt{D\chi}\ q)]\Big\} dq+T(t)$$

p.9, second line following Equation 8: "of Eq. 7" should read "of Eq. 6."

p.10, eighth line should read "neglect the reaction on the ω_2 wave from the generated waves"

p.12, fourth line "Fig. (2)" should read "Fig. (12)."

where $R_i = \frac{v_{oi} c_o}{\delta \omega_i}$ $(i = 1,2)$ are the Reynolds numbers, and $T(t)$ denote transient terms. We have $T(t) \to 0$ for $t \to \infty$ (x fixed).

We now develop the integrand in (4) in a double power series in R_1 and R_2, and integrate term by term (which is proved to be allowed for finite R_1 and R_2 and t large enough so that the transient terms are negligible).

Further, putting

$$v = a_o + a_{11}R_1 + a_{12}R_2 + (a_{21}R_1^2 + a_{22}R_2^2 + a_{23}R_1R_2)$$

$$+ (a_{31}R_1^3 + a_{32}R_2^3 + a_{33}R_1^2R_2 + a_{34}R_1R_2^2) + O(R^4),$$

we obtain after some calculations

Order R:

$$v_1 = v_{o1}e^{-\alpha_1 x}\cos\omega_1\tau + v_{o2}e^{-\alpha_2 x}\cos\omega_2\tau \qquad (5)$$

where $\alpha_i = B\omega_i^2$ is the absorption $(i = 1,2)$.

Order R^2:

$$v_2 = -\beta R_1 v_{o1} e^{-3\alpha_1 x}\sinh(\alpha_1 x)\sin 2\omega_1\tau \qquad (6)$$

$$- \beta R_2 v_{o2} e^{-3\alpha_2 x}\sinh(\alpha_2 x)\sin 2\omega_2\tau$$

$$+ \frac{\beta\delta}{2c_o} R_1R_2(\omega_1-\omega_2)e^{-(\alpha_1+\alpha_2)x}\left(1-e^{2\sqrt{\alpha_1\alpha_2}\,x}\right)\sin\left[(\omega_1-\omega_2)\tau\right]$$

$$+ \frac{\beta\delta}{2c_o} R_1 R_2 (\omega_1+\omega_2) e^{-(\alpha_1+\alpha_2)x}\left(e^{-2\sqrt{\alpha_1\alpha_2}x} - 1\right)\sin\left[(\omega_1+\omega_2)\tau\right]$$

<u>Order R^3</u>:

$$v_3 = -\frac{\beta^2}{2} R_1{}^2 v_{o1} e^{-3\alpha_1 x}\left[\sinh(\alpha_1 x)\right]^2 \cos\omega_1\tau$$

$$+ \frac{\beta^2}{2} R_1{}^2 v_{o2} e^{-(2\alpha_1+\alpha_2)x}\left[1 - \cosh(2\sqrt{\alpha_1\alpha_2}x)\right]\cos\omega_2\tau$$

$$- \frac{\beta^2}{2} R_1{}^2 v_{o1} e^{-3\alpha_1 x}\left[2 - 3e^{-2\alpha_1 x} + e^{-6\alpha_1 x}\right]\cos 3\omega_1\tau$$

$$+ \frac{\beta^2}{8} R_1{}^2 v_{o2}\left(2\,\frac{\omega_1}{\omega_2} - 1\right) e^{-(2\alpha_1+\alpha_2)x}\left[e^{2\sqrt{\alpha_1\alpha_2}x} - 1\right]$$

$$\times \left[e^{-2\alpha_1 x}\left(e^{2\sqrt{\alpha_1\alpha_2}x} + 1\right) - 2\right] \times \cos\left[(2\omega_1-\omega_2)\tau\right]$$

$$- \frac{\beta^2}{8} R_1{}^2 v_{o2}\left(2\,\frac{\omega_1}{\omega_2} + 1\right) e^{-(2\alpha_1+\alpha_2)x}\left[e^{-2\sqrt{\alpha_1\alpha_2}x} - 1\right]$$

$$\times \left[e^{-2\alpha_1 x}\left(e^{-2\sqrt{\alpha_1\alpha_2}x} + 1\right) - 2\right] \times \cos\left[(2\omega_1+\omega_2)\tau\right]$$

$$+ \left[\omega_1 \Leftrightarrow \omega_2\,,\quad R_1 \Leftrightarrow R_2\,,\quad v_{o1} \Leftrightarrow v_{o2}\,,\quad \alpha_1 \Leftrightarrow \alpha_2\right] \quad (7)$$

Here the square brackets denote a series of terms obtained from the one given explicitly in Eq.(7) by interchanging ω_1 and ω_2, R_1 and R_2, v_{o1} and v_{o2}, α_1 and α_2.

The series expansion obtained in this way converge and agree term by term with the result that is obtained if we take the solution of the linearized equations as a first approximation and carry through the successive approximations. Thus, for $M_1 >> M_2$ we have only one significant $\cos\omega_2\tau$ term in Eq.(7):

$$\frac{\beta^2}{2} R_1^2 v_{o2}\, e^{-(3\alpha_1+\alpha_2)x}\left[1 - \cosh(2\sqrt{\alpha_1\alpha_2}x)\right]\cos\omega_2\tau$$

which agrees with the result obtained previously by the method of successive approximations (Tjøtta 1967). The amplitude here is always negative, which proves that amplification of the ω_2 wave is not possible in this model.

Putting $v_{o2} = 0$, we have:

$$v_1 = v_{o1}\, e^{-\alpha_1 x}\cos\omega_1\tau$$

$$v_2 = -\frac{1}{2}\beta R_1 v_{o1}\, e^{-2\alpha_1 x}\left(1 - e^{-2\alpha_1 x}\right)\sin 2\omega_1\tau$$

$$v_3 = -\frac{\beta^2}{8} R_1^2 v_{o1}\, e^{-\alpha_1 x}\left\{\left(1 - e^{-2\alpha_1 x}\right)^2\cos\omega_1\tau + e^{-2\alpha_1 x}\left(2 - 3e^{-2\alpha_1 x} + e^{-6\alpha_1 x}\right)\cos 3\omega_1\tau\right\},$$

which for v_2 and v_3 agree with formulas found in well-known solutions for the second and the third harmonic in a sound wave of finite amplitude (cf. for example, Kech and Beyer 1960, Blackstock 1965).

Further, we find $v_2 = O(x)$ and $v_3 = O(x^2)$ for small x , and

$$v = v_1 + v_2 + v_3 \to v_{01}\cos\omega_1 t + v_{02}\cos\omega_2 t \quad \text{for} \quad x \to 0$$

in accordance with presumed linearized boundary condition for $x = 0$.

4. Interpretation of the solution.

Let $\omega \circ \Omega$ denote nonlinear interaction between waves with frequencies ω and Ω in a first order (quasi-linear) approximation. We may then interprete the different terms in v_3:

Term	Interaction
$\cos\omega_1\tau$	$\omega_1 \circ 2\omega_1$
$\cos\omega_2\tau$	$\omega_1 \circ (\omega_1-\omega_2)$ and $\omega_1 \circ (\omega_1+\omega_2)$
$\cos 3\omega_1\tau$	$\omega_1 \circ 2\omega_1$
$\cos[(2\omega_1 - \omega_2)\tau]$	$\omega_1 \circ (\omega_1-\omega_2)$ and $\omega_2 \circ 2\omega_1$
$\cos[(2\omega_1 + \omega_2)\tau]$	$\omega_1 \circ (\omega_1+\omega_2)$ and $\omega_2 \circ 2\omega_1$

A similar table is obtained for the terms in the square brackets in Eq.(7).

In v_2 the following standard interactions are represented: $\omega_1 \circ \omega_1$, $\omega_2 \circ \omega_2$, $\omega_1 \circ \omega_2$. In the literature studies of such non-linear effects as end-fire arrays and up-converter type of parametric amplifications are based on the quasi-linear approximation of the interaction between two waves, $\omega_1 \circ \omega_2$. More precise information of the range of validity of this approach can now be obtained by calculating the terms in v_3 . This is discussed in the next section.

5. Up-converter parametric amplification.

Introducing

$$G_\pm \overset{def}{=} \frac{v_\pm}{v_{o2}\, e^{-\alpha_2 x}}$$

we find

$$G_\pm = \frac{\omega_2 \pm \omega_1}{\omega_2} \frac{\beta R_1}{2} e^{-\alpha_1 x}\left(e^{\pm 2\sqrt{\alpha_1 \alpha_2} x} - 1\right) \tag{8}$$

Here v_+ and v_- denote the velocity amplitudes of sum and difference frequency wave, respectively (cf. Eq.(7)).

For $\omega_1 >> \omega_2$, we have

$$G_\pm = - \beta R_1 \alpha_1 x\, e^{-\alpha_1 x} , \tag{9}$$

and thus

$$\mathrm{Max}|G_\pm| = \beta R_1 e^{-1} \quad \text{for} \quad x = \frac{1}{\alpha_1} .$$

Berktay (1965), Berktay and Al-Temimi (1970) have put much emphasis on the possibility of obtaining $|G_\pm| > 1$, as this could form the basis of design for up-converter parametric amplifiers. Experimentally, they have thus far observed $|Q_{\pm}| = O(1)$.

A measure of the reaction from the generated waves in the second approximation on the primary waves is found by studying the third approximation, or the term of order R^3 in our expansion.

We now assume $M_1 >> M_2$. A condition which enables us to neglect the reaction on the ω_2 wave the generated waves with sum and difference frequencies, have then been given previously, Tjøtta (1967). The results here, based on Burgers' equation, agree with his results. We find:

$$I_{1c} = \frac{\beta^2}{2} R_1^2 e^{-2\alpha_1 x}\left[1 - \cosh(2\sqrt{\alpha_1\alpha_2}x)\right] \qquad (10)$$

for the amplitude of the second term (dominant $\cos\omega_2\tau$ term for $M_1 >> M_2$) in Eq.(7), when measured in terms of the amplitude of the primary wave with frequency ω_2 (i.e. $v_{o2}\, e^{-\alpha_2 x}$). For $\omega_1 >> \omega_2$ this becomes

$$I_{1c} = -\beta^2 R_1^2\left(\frac{\omega_2}{\omega_1}\right)^2 (\alpha_1 x)^2\, e^{-2\alpha_1 x} \qquad (11)$$

wich leads to

$$\mathrm{Max}|I_{1c}| = \beta^2 R_1^2 \left(\frac{\omega_2}{\omega_1}\right)^2 e^{-2} \qquad (12)$$

Similarly, the amplitude of the first term (dominant $\cos\omega_1\tau$ term for $M_1 >> M_2$) in Eq.(7) is given by

$$I_{1h} = -\frac{1}{2}\beta^2 R_1{}^2 e^{-2\alpha_1 x}\left[\sinh(\alpha_1 x)\right]^2 \qquad (13)$$

when measured in terms of the amplitude of the primary wave with frequency ω_1 (i.e., $v_{o1} e^{-\alpha_1 x}$). Thus I_{1h} decreases from zero at $x = 0$ to an asymptotic value $-\beta^2 R_1{}^2/8$ for x large. We have

$$\mathrm{Max}|I_{1h}| \approx \frac{\beta^2 R_1{}^2}{8} \quad \text{for} \quad \alpha_1 x \geq 2 \, . \qquad (14)$$

In order to have a gain in the waves with sum and difference frequencies, i.e. $|G_\pm| > 1$, it is necessary that $\beta R_1 > e$. The convergence of our expansion is then slow, and the two first terms are not sufficient to predict the solution accurately enough. For $\omega_1 \gg \omega_2$ we may still have $\mathrm{Max}|I_{1c}| \ll 1$ and the ω_2-wave is not modified by third approximation (term of order R^3). However, the ω_1-wave is modified at some distance $(x > 1/\alpha_1)$ from the source due to $\omega_1 \circ 2\omega_1$ interaction and higher harmonic effects have to be accounted for, (when $\beta R_1 = e$, $I_{1h} = -0.7$ for $\alpha_1 x = 1$, $\mathrm{Max}|I_{1h}| = 0.9$). Further the combined frequencies $\omega_1 \pm \omega_2$ are now close to ω_1, making a separation between them difficult.

In their recent experiments Berktay and Al-Temimi (1970) worked with an intensity of about 0,16 W/com^2 for the pump wave (ω_1-wave) in order to avoid exstra attenuation effects due to higher intensity. This corresponds to $M = 3{,}2\cdot 10^{-5}$ in water, and with $\beta = 3.5$ this leads to $\beta R_1 \approx 2$ and $\beta R_1 \approx 1$ at the frequencies of $f_1 = 2.79$ MHz and $f_1 = 5.9$ MHz, resp., used in the experiment. Here $\mathrm{Max}|G_\pm| < 1$ in qualitative agreement with the observations. Further, our theory predicts $G_\pm$ independent of ω_2 for $\omega_1 \gg \omega_2$, which agrees with observations (Fig.(2) ref. [2]). They also observe $|G_\pm| > 1$, but then with higher intensities where effects of third order and higher in the pump wave become important.

6. Interaction between beams.

So far we have only considered infinite plane waves. For the case of two plane beams propagating in the same direction, we have on the axis of symmetry the following expression for the pressure of the difference frequency wave (to order R^2):

$$P_- = \frac{2\pi}{i\chi} A\, Q(x) \ , \quad x > 0 \tag{15}$$

$$Q(x) = \int_0^\infty dz\, e^{i(\chi_1-\chi_2^*)z} \left\{ e^{i\chi\sqrt{(x-z)^2+a^2}} - e^{i\chi\sqrt{(x+z)^2+a^2}} \right\} + \mathcal{J}(x) \tag{16}$$

$$\mathcal{J}(x) = \int_0^\infty dz\, e^{i(\chi_1-\chi_2^*)z} \left\{ e^{i\chi(x+z)} - e^{i\chi|x-z|} \right\}$$

$$= \frac{2\chi}{\alpha_1+\alpha_2-\alpha} \, \frac{e^{i(\chi_1-\chi_2^*)x} - e^{i\chi x}}{2k+i(\alpha_1+\alpha_2-\alpha)} \tag{17}$$

Here are: $\chi = k+i\alpha$, $\chi_1 = k_1+i\alpha$, $\chi_2 = k_2+i\alpha_2$, $k = k_1-k_2 > 0$, a the beam radius, and

$$A = -\beta\rho_o k^2 \psi_{1o}\psi_{2o}^* .$$

The primary wavws are: $\psi_1 = \psi_{1o}\, e^{i(\chi_1 x-\omega_1 t)}$, $\psi_2 = \psi_{2o}\, e^{i(\chi_2 x-\omega_2 t)}$
$\mathfrak{J}(x)$ corresponds to the interaction between two infinite plane waves. It is well-known that $\mathrm{Max}|\mathfrak{J}|$ is obtained at a distance

$$x_M = \frac{\ell n \dfrac{\alpha_1+\alpha_2}{\alpha}}{\alpha_1+\alpha_2-\alpha}$$

from the sound-source. For the case of two interacting beams, the generated difference frequency wave is observed to have maximum amplitude much nearer the sound than predicted by the infinite plane-wave theory, (cf. Hobæk (1967)).

The integral in Eq.(16) is now evaluated numerically, and some results are shown in Figs. 1-3.

For the case $\omega_1 \gg \omega_2$ (cf. Fig.1) we find $|Q|$ fluctuating about $|\mathfrak{J}|$. The generated difference frequency wave obtains its maximum amplitude near $x = \frac{1}{\alpha_1}$, which is the position for maximum of $|\mathfrak{J}|$, and is not propagated for out of the interaction region as is the case for $\omega_1 \approx \omega_2$.

Berktay (1967), Berktay and Al-Timimi (1970) have previously pointed out and emphasized the difference between the model of infinite plane waves and the one with plane beams, when analyzing the possibility of an upconverter type of amplification. The results here do not support this conclusion, in that $|Q|$ and $|\mathfrak{I}|$ lead to approximately the same values of the factor $G_{\pm}$ for $\omega_1 \gg \omega_2$.

When $\omega_1 \approx \omega_2$ (cf. Figs. 2,3) we find for the highest value of ka a fluctuation of $|Q|$ about $|\mathfrak{I}|$, similar to the case with $\omega_1 \gg \omega_2$. For the lower values of ka (cf. Fig.2) there are such fluctuations only near the source, and the position of maximum is moved toward the source for decreasing ka. This is in qualitative agreement with experimental observations by Hobæk (1967) and by Hobæk and Vestrheim (1971).(The intensities are kept high in the first work, and direct comparison is not justified). In the last work also these authors have computed this effect for low values of ka and have made some quantitative comparisons with their recent experimental observations. They neglect, however, the effect due to backscattering. We have computed $Q(x)$ when neglecting in Eqs.(16) and (17) the terms due to back-scattering (second term in (16), first term in (17), and integrating from $z = 0$ up to $z = x$ only). The effect from these terms is small, and in our examples it is significant only very near the source (up to about 5%). Here, of cource, also the approximations introduced by linearizing the boundary conditions may lead to errors in the results. In a discussion of the fine-structure of the fluctuation in this region, it may be necessary to take into account the effect due to back-scattering. Near-field effects in the primary waves of frequencies ω_1 and ω_2

may also lead to a fluctuation and thus fine-structure in the generated waves, but these are not taken into account in the present theory. Neither are the transient effects (from $T(t)$ in Eq.(4)) taken into account. For $\omega_1 \approx \omega_2$ we find, for the case of infinite plane waves, that I_{1c} decreases from zero at $x = 0$ to the asymptotic value

$$I_{1c} \approx -\frac{\beta^2 R_1^{\,2}}{4} \quad \text{for large } \alpha_1 x \quad (\alpha_1 x \geq 1 \text{ in practice})$$

and I_{1h} is as before.

In this case Hobæk and Vestrheim (1971) observe an increase in the intensity of the difference frequency wave with increasing intensity of the primary waves (having about the same intensity) for $\beta R \geq 1$, indicating effects of higher order than the second. This is in qualitative agreement with the present theory as $\mathrm{Max}|I_{1c}| = \frac{1}{4}$ and $\mathrm{Max}|I_{1h}| = \frac{1}{8}$ for $\beta R \approx 1$ and the third order term in our expansion becomes important (Note, however, that their experiment is for two beams and not for infinite plane waves as here).

References.

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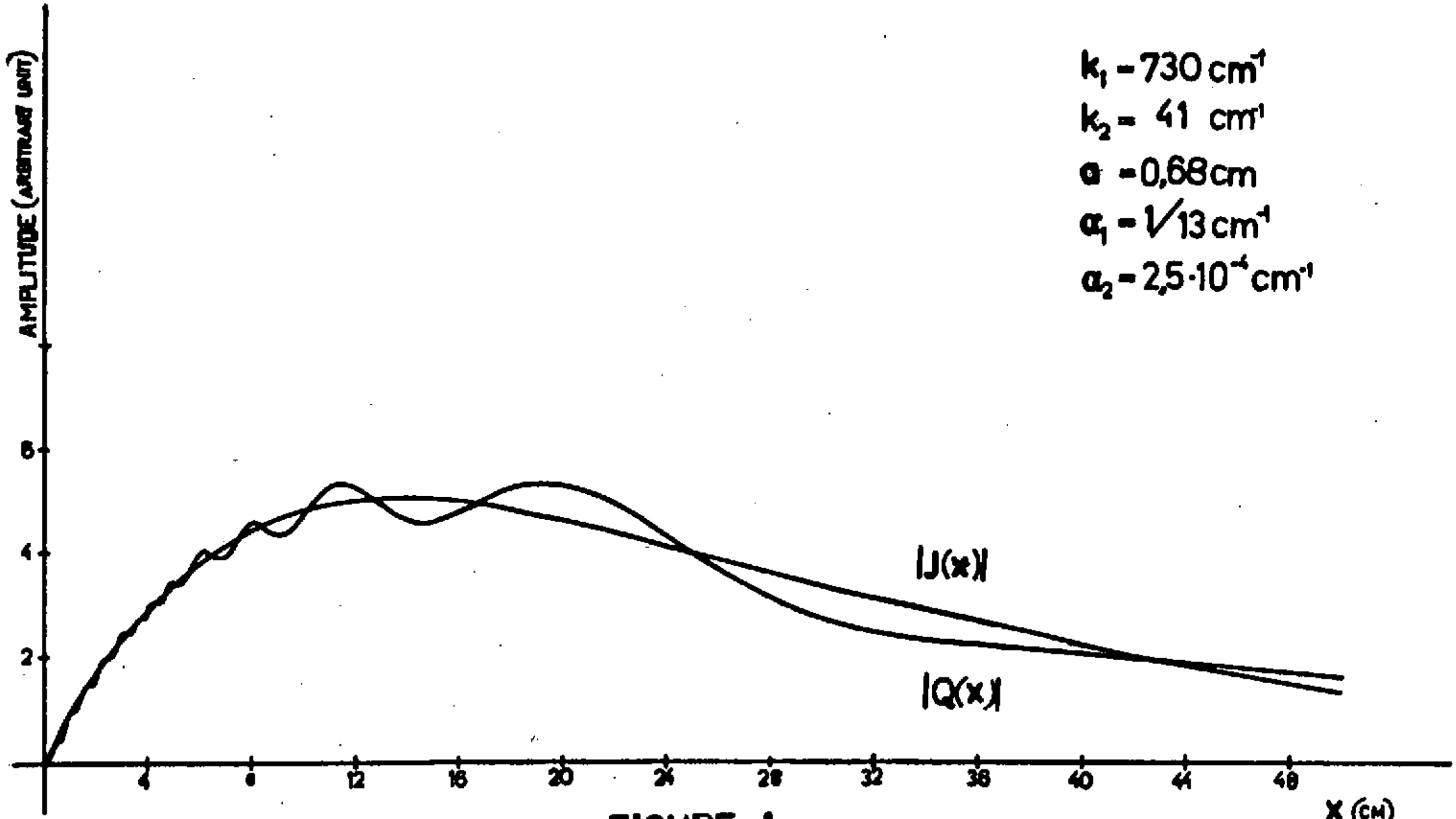
Figure Captions.

Figures 1 - 3.

Variation of the difference-frequency pressure level with the distance from the sound source.

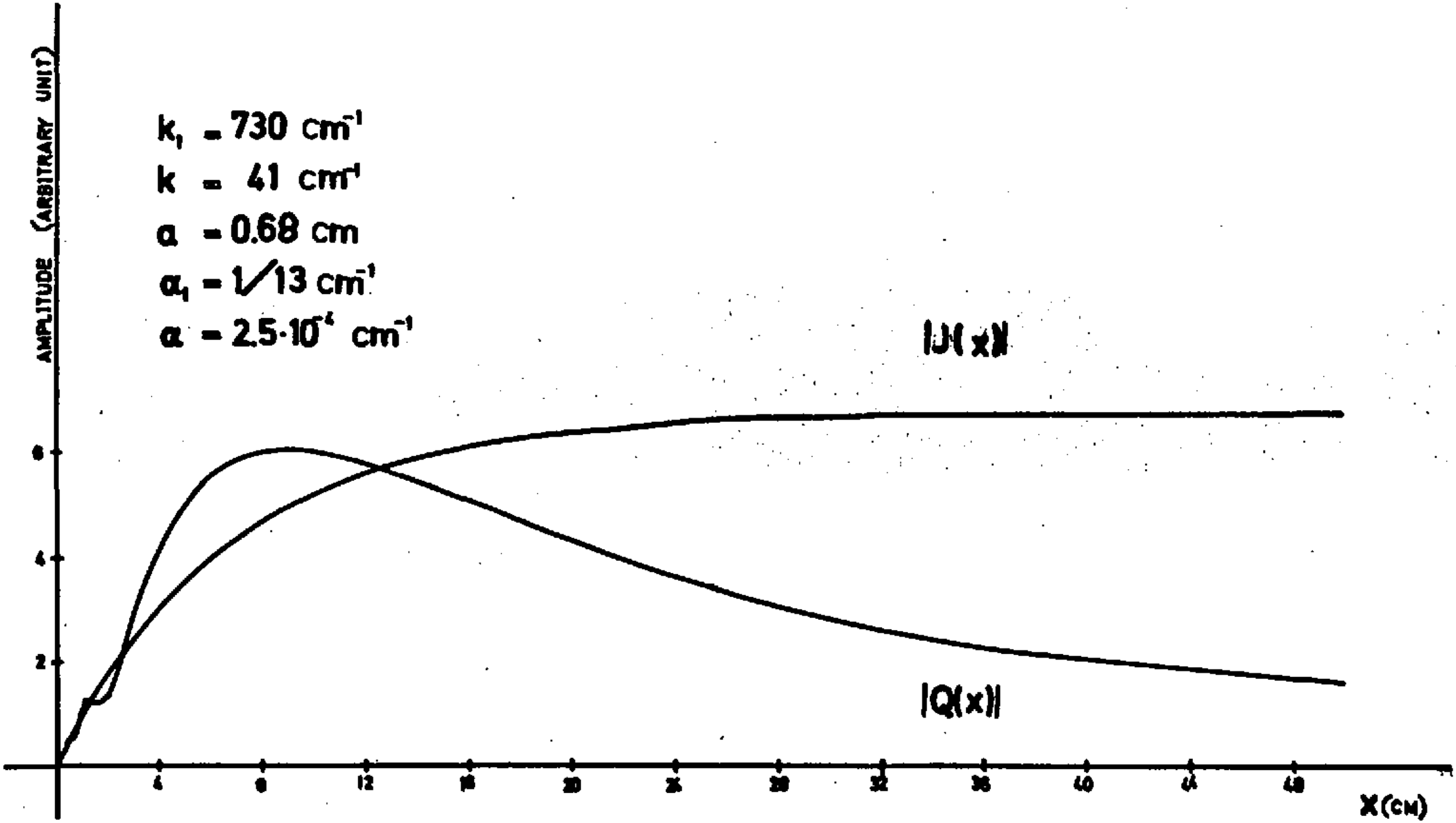
$|\mathfrak{J}(x)|$: Infinite plane waves.

$|Q(x)|$: Plane beams.



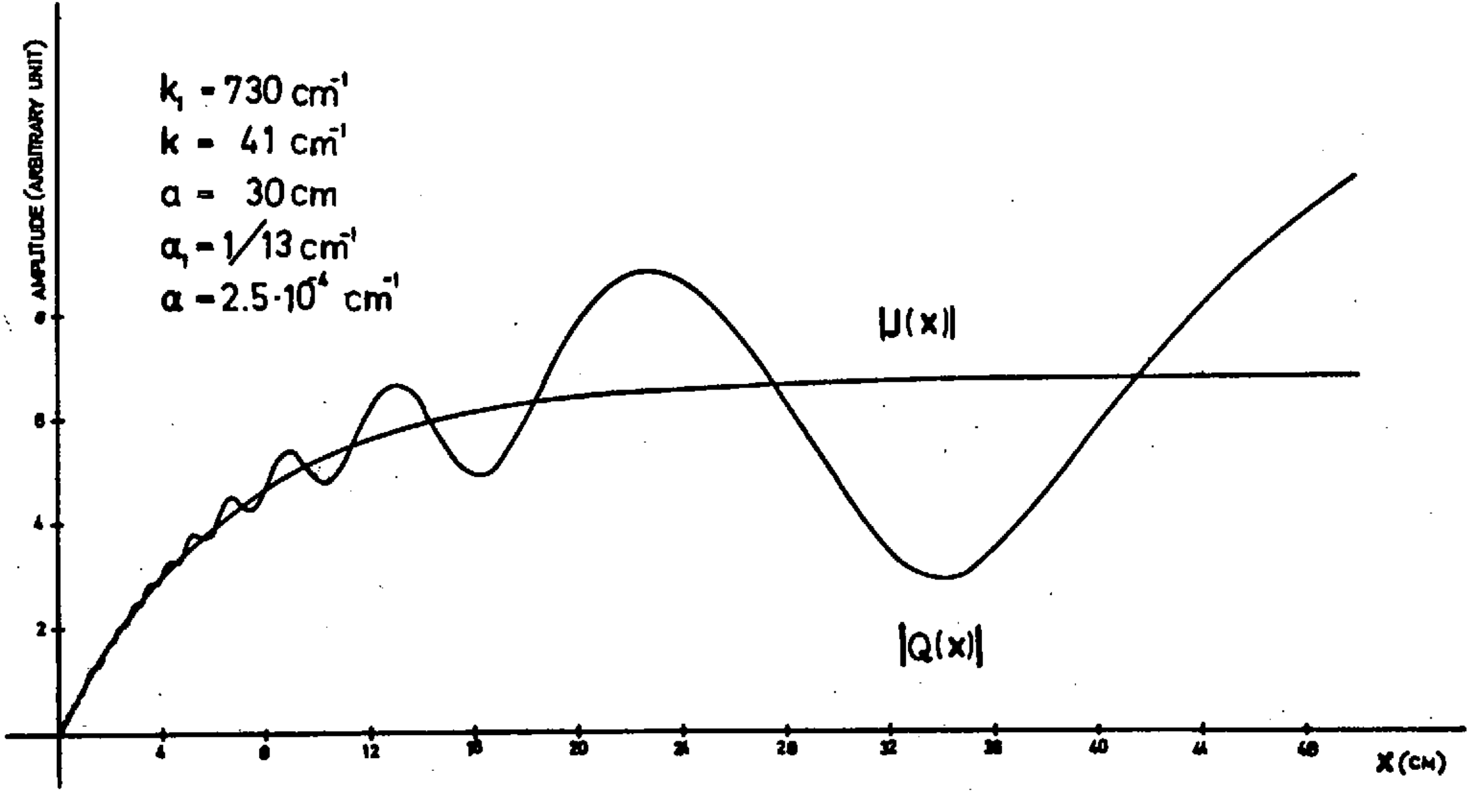
AMPLITUDE (ARBITRARY UNIT)
k1 = 730 cm-1
k2 = 41 cm-1
a = 0,68 cm
α1 = 1/13 cm-1
α2 = 2,5·10-4 cm-1
|J(x)|
|Q(x)|
6
4
2
4
8
12
16
20
24
28
32
36
40
44
48
X (CM)

FIGURE 1.



AMPLITUDE (ARBITRARY UNIT)
k1 = 730 cm-1
k = 41 cm-1
a = 0.68 cm
α1 = 1/13 cm-1
α = 2.5·10-4 cm-1
|J(x)|
|Q(x)|
X (CM)

FIGURE 2.



AMPLITUDE (ARBITRARY UNIT)
k_1 = 730 cm^{-1}
k = 41 cm^{-1}
a = 30 cm
α_1 = 1/13 cm^{-1}
α = 2.5·10^{-4} cm^{-1}
|U(x)|
|Q(x)|
2
4
6
8
4
8
12
16
20
24
28
32
36
40
44
48
X (CM)

FIGURE 3.