PARABOLIC APPROXIMATION FOR SOUND IN MOVING FLUID: APPLICATION TO OCEAN CURRENTS EVALUATION AND SOURCE LOCALIZATION

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1. INTRODUCTION

Parabolic approximation is a widely used approach to acoustic fields modeling in range dependent environments. For the first time sound field properties in waveguide with currents were analyzed using this approximation in [1,2]. Later extensive studies of acoustic effects of oceanic currents were made in [3-5]. In]2-5] under rather restrictive assumptions a set of parabolic equations (PEs) is derived and these PEs are solved, when existing computer codes, elaborated for PE in motionless medium with constant density, are applicable.
To study influence of currents in the ocean on sound propagation underwater, we use another PE. Both sound speed c and flow velocity $\vec{u}$ as well as medium density are supposed to be time independent and can depend on all three space coordinates. When currents are absent, the parabolic equation reduces to standard PE proposed by F.D.Tappert, and for $\vec{u} \neq 0$ also contains as its special cases a number of PEs, proposed previously in [2-5].
An implicit finite-difference scheme is proposed to solve the PE. Unconditional stability of the scheme is proved. Numerical solution of the PE is used to demonstrate strong influence of flow presence as well as $\vec{u}$ and c range dependence on transmission loss of low frequency sound propagating through the Gulf Stream.
Two inverse problems are addressed in framework of the parabolic approximation under realistic environmental conditions. Amplitude and phase of continuous wave signal radiated by a point source and received by a vertical array is used as initial data in numerical experiments. First , source localization in moving medium by inverting direction of wave propagation is investigated. This approach is based on flow reversion theorem proved, which generalizes the reciprocity principle, valid in motionless fluid, on acoustic fields in moving medium. Second, results of reciprocal transmission experiment are inverted to estimate horizontal components of flow velocity dependence on vertical coordinate.

2.PARABOLIC WAVE EQUATION FOR SOUND IN MOVING MEDIA

Departing from an exact set of differential equations of linear acoustics of moving steady-state medium, it was show in [6], that complex envelope

$$\Psi(r,\varphi,z) = r^{1/2}p(\vec{r},t)\exp(i\omega t - ik_0 r) \tag{1}$$

of a harmonic pressure field p obeys approximately the 2-D narrow-angle PE

$$2ik_0\frac{\partial\Psi}{\partial r} + \frac{\partial^2\Psi}{\partial z^2} - \frac{\partial\Psi}{\partial z}\,\frac{\partial}{\partial z}\ln(\rho\beta^2) + \left[k^2\beta^2 - k_0^2 - ik_0\frac{\partial}{\partial r}\ln(\rho\beta^2)\right]\Psi = 0, \tag{2}$$

Here $k = \omega/c(\vec{r})$ is a wave number, $k_0 =$ const, $\rho(\vec{r})$ is medium's density, $c(\vec{r})$ and $\vec{u}(\vec{r})$ are sound and flow velocities, $\beta \equiv 1 - k_0 u_r/\omega$, u_r is a radial component of $\vec{u}$. It is assumed that z-axis of cylindrical coordinates (r,φ,z) is vertical and comes through a sound source. PE(2) is valid provided $\varepsilon \ll 1$, $m \ll 1$, $k_0 r(m^2 + M\varepsilon + \varepsilon^2) \ll 1$ [6]. Here $M \simeq u / c$ is the flow's Mach number, $m \simeq u_\varphi/c$ is the Mach number defined with respect to transverse component of $\vec{u}$, $\varepsilon \simeq |1 - \xi_n/k_0|$ is a relative spread of mode propagation constants ξ_n. In ray terms, ε is of the order of maximum grazing angle squared. In the ocean $\varepsilon \gg M$ ($m \lesssim M \lesssim 10^{-3}$, $\varepsilon \simeq 10^{-2} \div 10^{-1}$) and therefore presence of currents doesn't limit range of parabolic approximation applicability.

We prefer to use PE(2) instead of the set of PEs, proposed in [2 - 5], as (2)
(i) is valid in moving media under more general conditions than all the PEs [2 - 5] together;
(ii) leads to exact energy conservation and reciprocity relations;
(iii) may be easily solved by IFD method.

Difference equation

$$2ik_0\left(F_j^{n+1} - F_j^n\right) + \frac{\Delta r}{2(\Delta z)^2}\hat{L}^n\left(F_j^{n+1} + F_j^n\right) + \frac{\Delta r}{2}C_j^n\left(F_j^{n+1} + F_j^n\right) = 0 \qquad (3)$$

approximates PE(2) to within $O((\Delta z)^2) + O(\Delta r)$. Operator $\hat{L}$ is defined to be

$$L^n(a_j) = A_j^n A_{j+1}^n B_{j+1/2}^n a_{j+1} + (A_j^n)^2 a_j [B_{j+1/2}^n + B_{j-1/2}^n] + A_j^n B_{j-1/2}^n A_{j-1}^n a_{j-1}. \qquad (4)$$

In (3) , (4) $A_j^n \equiv A(n\Delta r, \varphi, j\Delta z)$ etc, n and j are integers, $F(r,\varphi,z) = \Psi / A$, $A = (\rho\beta^2)^{-1/2}$, $B = A^{-2}$, $C = k^2\beta^2 - k_0^2$. To calculate starting field, i.e. initial conditions for solution of (3), we use normal modes or ray theory.

The PE(2) and corresponding difference equation (3) have physically attractive properties of rigorous validity of the reciprocity principle in media at rest and of the flow reversion theorem, as applied to solutions of (2) and (3). To prove this statement for Eq.(3), note that within parabolic approximation waves, which propagate to diminishing r values in a medium with flow velocity $-\vec{u}(\vec{r})$, and unchanged dependencies $c(\vec{r})$ and $\rho(\vec{r})$, obey equation

$$-2ik_0\left(\tilde{F}_j^{n+1} - \tilde{F}_j^n\right) + \frac{\Delta r}{2(\Delta z)^2}\hat{L}^n\left(\tilde{F}_j^{n+1} + \tilde{F}_j^n\right) + \frac{\Delta r}{2}C_j^n\left(\tilde{F}_j^{n+1} + \tilde{F}_j^n\right) = 0 \qquad (5)$$

similar to (3). Here and below tilde indicate quantities corresponding to the medium with reversed flow. Multiply (5) by $w_j^n \equiv F_j^{n+1} + F_j^n$ and then subtract product of (3) and $\tilde{w}_j^n$. Summing the result over j one obtain after some transformations

$$\sum_j F_j^{n+1}\tilde{F}_j^{n+1} = \sum_j F_j^n \tilde{F}_j^n , \qquad (6)$$

at arbitrary n. Relation (6) expresses the flow reversion theorem for the difference equation. Of course, the theorem is also valid for PE(2) [6 - 8] and has clear physical meaning: acoustic field in moving medium is invariant with respect to source and receiver permutation provided direction of the flow is simultaneously reversed throughout the medium. When $\vec{u} \equiv 0$, the theorem reduces to the reciprocity principle.

Quite analogously, using complex conjugate of Eq.(3) instead of Eq.(5) one can prove the identity

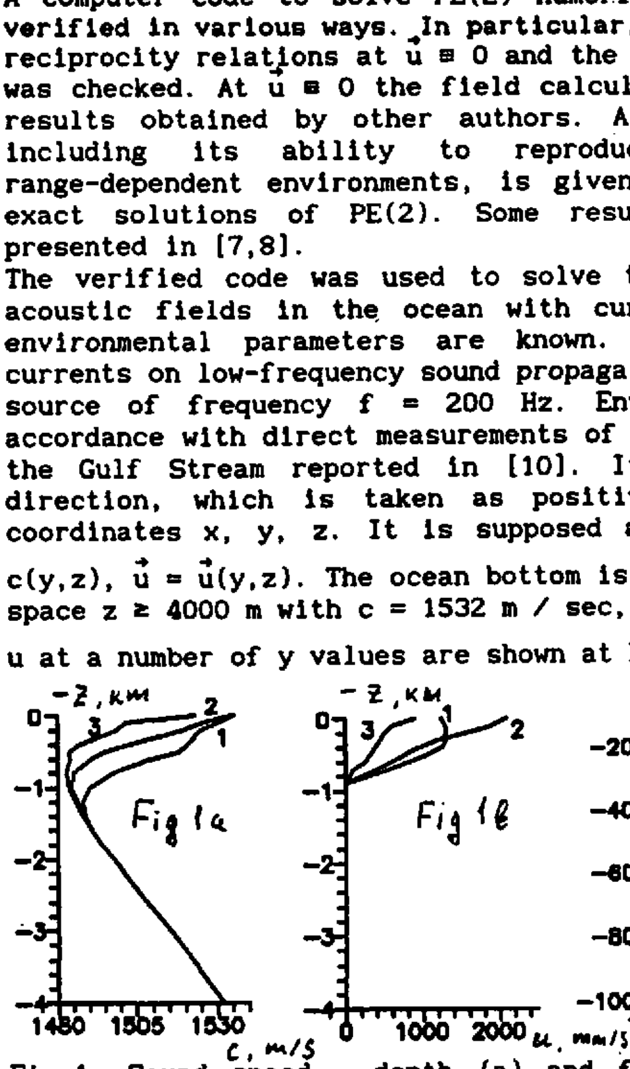
$$T^{(n)} - T^{(n+1)} = \frac{\Delta r}{4k_0} \sum_j (\beta^2)^j_n \mathrm{Im}[(k^2)^j_n] |w^n_j|^2, \quad T^{(n)} \equiv \sum_j |F^n_j|^2. \qquad (7)$$

If there is no absorption, $\mathrm{Im}\, k^2 \equiv 0$ and $T^{(n)}$ doesn't depend on n. This fact may be interpreted as acoustic energy flux conservation at wave transmission through a flow. In general, $T^{(n+1)} \le T^{(n)}$. This property ensures unconditional stability of difference scheme [9, p.324].

3. DIRECT PROBLEM

A computer code to solve PE(2) numerically by IFD method is developed and verified in various ways. In particular, fulfillment by numerical solutions of reciprocity relations at $\vec{u} \equiv 0$ and the flow reversion theorem in moving media was checked. At $\vec{u} \equiv 0$ the field calculated using our code was compared with results obtained by other authors. A severe check of the computer code, including its ability to reproduce correctly acoustic effects of range-dependent environments, is given by comparing numerical results with exact solutions of PE(2). Some results of the code's verification are presented in [7,8].

The verified code was used to solve the direct problem, i.e. to calculate acoustic fields in the ocean with currents, when sound source as well as environmental parameters are known. To illustrate influence of oceanic currents on low-frequency sound propagation consider acoustic field of a point source of frequency f = 200 Hz. Environmental parameters are chosen in accordance with direct measurements of water temperature and flow velocity of the Gulf Stream reported in [10]. It is assumed that $\vec{u}(\vec{r})$ has constant direction, which is taken as positive direction of x-axis of Cartesian coordinates x, y, z. It is supposed also, that $\rho \equiv \rho_0$, $\rho_0 =$ const and $c \equiv c(y,z)$, $\vec{u} = \vec{u}(y,z)$. The ocean bottom is taken to be a homogeneous liquid half-space $z \ge 4000$ m with c = 1532 m / sec, $\rho = 1.53\rho_0$. Vertical profiles of c and u at a number of y values are shown at Fig.1.



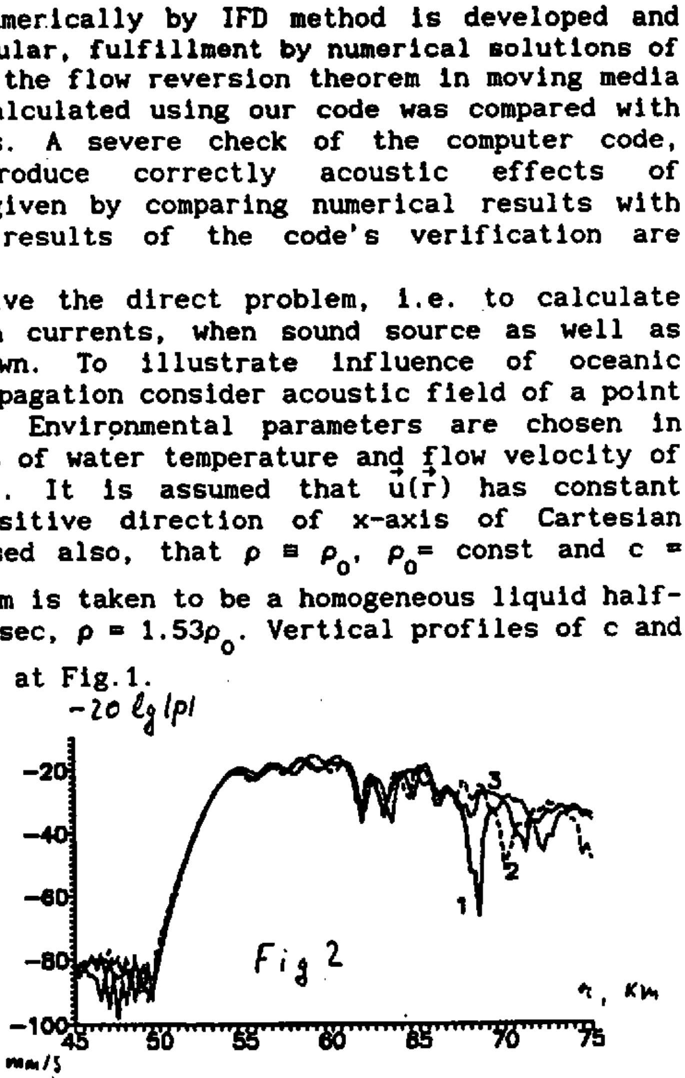


Fig.1. Sound speed - depth (a) and flow velocity - depth (b) profiles at

selected distances from the Gulf Stream axis: y = 68 km (1), y = 45 km (2), y = 22 km (3).
Fig.2. Transmission losses versus range for a receiver and a source at a depth of z_0= 200 m, and a frequency of 200 Hz. 1: the field in range-dependent moving medium, 2: the field in medium at rest, 3: the field in medium with range-dependent flow velocity corresponding to $\vec{u}$ at original medium at y = 22 km.

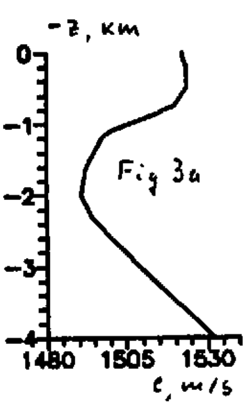
Let coordinate of the sound source be $x_0 = 0$, $y_0 = 68$ km, $z_0 = 200$ m. Transmission losses along horizontal line $y - y_0 = 3^{-1/2}(x - x_0)$, $z = z_0$, making an angle 30^0 with the Gulf Stream axis, are shown at Fig.2. To our knowledge, this is the first case, when sound field in the ocean with both range- and depth - dependent current is calculated using the parabolic approximation. Interference maxima shift, caused by the current, is evident at Fig.2. The flow changes transmission loss at a given point up to 40 dB. Comparison of curves 1 and 3 at Fig.2 shows that assumption of flow velocity to be range-independent leads to great errors, which are close to errors due to complete disregard of the medium's motion.

4. SOURCE LOCALIZATION

Due to significant influence of ocean currents on acoustic fields one should take medium's motion into account not only in simulating sound propagation but also in solving inverse problems. It will be shown below that solving inverse problems for moving media possesses some qualitative differences from solving similar problems when $\vec{u} \equiv 0$.
In Sects.4,5 for simplicity the ocean is assumed to be layered medium with $c(z)$ and $\vec{u}(z)$, which are shown at Fig.3 and are typical to the Gulf Stream. Field of a given sound source in such media is analyzed in [7].
If amplitude and phase of an acoustic field of an unknown source are measured by some array at narrow frequency band (or at several bands), and parameters of the medium at rest are given, it is the backpropagation method [11] which is convenient way to estimate position of the source within parabolic approximation. The reciprocity principle is the key stone of this method. Currents violate acoustic reciprocity. Nevertheless, the flow reversion theorem (Sect.2) makes it possible to extend the backpropagation method to moving media, both layered and range-dependent. Let a point source generates an acoustic pressure $P(z) = |P|\exp(i \arg P)$ at some section of a waveguide. According to the flow reversion theorem, for acoustic field be focused at position of the source, the measured field $P(z)$ should be propagated backwards in the medium with reversed direction of flow. This implies solving PE with P as starting field.
According to Fig.4a, the amplitude of the PE solution so obtained has a pronounced absolute maximum. Although backpropagation is accomplished in two dimensions, i.e. omitting factor $r^{1/2}$ in (1), position of this maximum coincides exactly with true position of the source (x_0= 72 km, z_0= 250 m). If

a flow velocity profile different from - $\vec{u}(z)$ is used to specify medium in which P(z) is propagated, then the field focusing isn't so pronounced. For instance, at Fig.4b, obtained without reversion of the flow direction, the absolute maximum and its difference from other maxima are much less then at Fig.4a. Besides, position of the absolute maximum (x = 69.5 km, z = 90 m) is far from true position of the source.

Currents induce acoustic anisotropy of layered media in horizontal plane. That is acoustic field of undirected source depends on azimuthal angle φ of cylindrical coordinates in addition to range and depth. If it is known that a point sound source is situated somewhere in the waveguide, strong focusing of backpropagated field may be used to localize the source not only with respect to range and depth but to estimate also the azimuthal direction from the vertical array to the source. This is illustrated by Fig.5, where $\varphi = 45^{\circ}$ corresponds to propagation of P(z) exactly backwards. In the case considered the vertical array makes it possible to determine angular position of the source to within 10^{0}- 15^{0}.



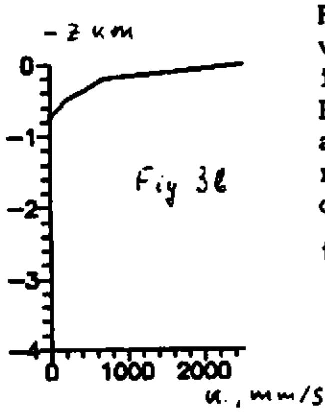
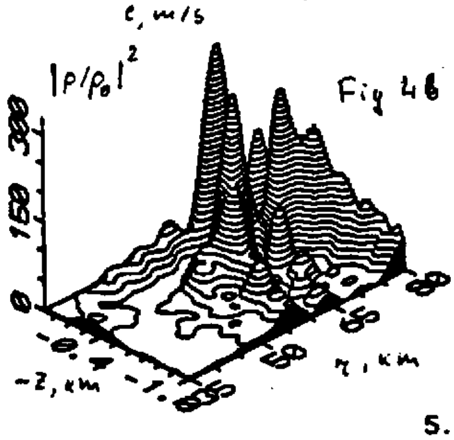
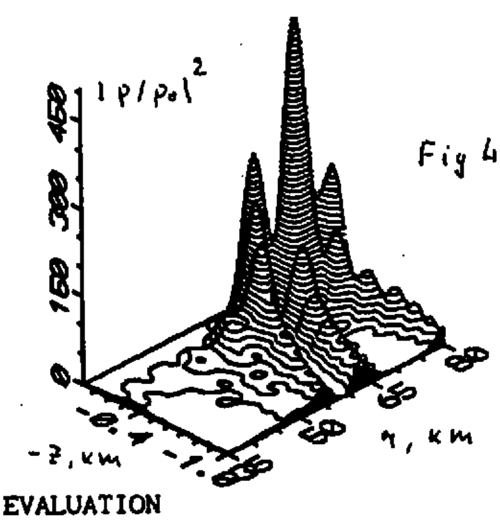


Fig.3. Vertical dependencies of flow velocity (a) and sound speed (b) assumed in the inverse problems.

Fig.4. Intensity $|p|^{2}$ of backpropagated acoustic field in the media with reversed (a) and initial (b) flow direction. Normalization constant p_0 is the same for both figures.





5. CURRENTS EVALUATION

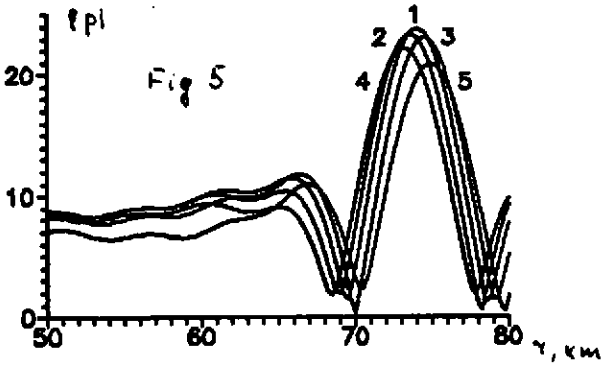
Let positions of a vertical array and a point source as well as amplitude u(z) of current velocity be known, but direction of $\vec{u}$ is unknown. Then the flow direction may be found by calculating backpropagated field, as in Sect.4, for various flow directions and by comparing position of inverted sources to true source position. In principle in this way other unknown parameters of $\vec{u}(z)$ may also be estimated. However, small inaccuracies in c(z)or in positions of the source and the array lead to inadmissible errors in such estimates.

One can get rid of this disadvantage by using reciprocal transmissions. Let close sound source and receiver be placed at some point $\vec{r}_1$ and another identical source-receiver pair is placed at point $\vec{r}_2$. Consider a difference $F = p_1(\vec{r}_2) - p_2(\vec{r}_1)$ of acoustic pressure at point $\vec{r}_2$ due to source at $\vec{r}_1$ and at point $\vec{r}_1$ due to source at $\vec{r}_2$. According to the reciprocity principle $F \equiv 0$ if there is no flow. Let F be known as a function of depth of one source-receiver pair with the second being fixed.
F(z) may be used to determine some parameters of currents. For this purpose we introduce the quantity

$$I = L^{-1} \int dz \, [|F_e - F_t| \, / \, (|F_e| + |F_t|)]^2. \tag{8}$$

Here F_e is experimentally measured F(z) and F_t is F calculated for some trial $\vec{u}(z)$. Integral is taken over interval $(z_0, z_0 + L)$ of depth for which $F_e(z)$ is given. In numerical experiments we simulate F_e by solving PE for $\vec{u}(z)$ and c(z) profiles shown at Fig.3 and some specified φ. F_t is calculated for various φ values. It is of great importance that the use of the flow reversion theorem allows us to find $F_t(z)$ far all z by solving PE only twice.

Quantity $0 \le I \le 1$ by definition. I = 0 when direction of trial flow coincides with the true direction. The latter (or, equivalently, magnitude of $\vec{u}$) is determined rather accurately in this way (see Fig. 6). It is straightforward to extend this procedure to include determination of several parameters of $\vec{u}$.



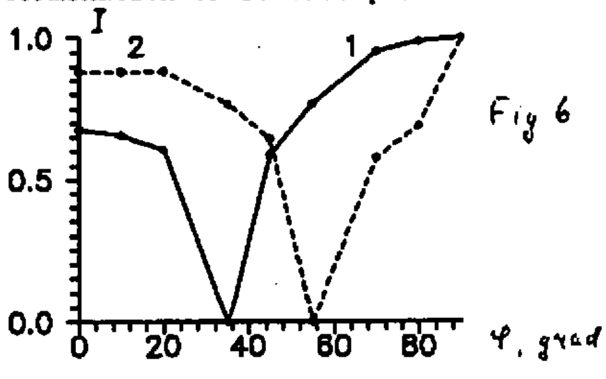


Fig.5. To determining the source position in horizontal plane. Intensity of backpropagated field at source depth versus range is shown for selected azimuthal directions of propagation: $\varphi = 45^\circ(1)$, $\varphi = 35^\circ(2)$, $\varphi = 55^\circ(3)$, $\varphi = 25^\circ$ (4), $\varphi = 65^\circ(5)$.

Fig.6. Dependence of quantity I (8) on direction of trial current. "Measured" data are simulated by calculating field of the point source in direction making an azimuthal angle $\varphi = 35^\circ$ (1) and $\varphi = 55^\circ$ (2) with Ox - axis.

6. SUMMARY AND CONCLUSIONS

We have considered the parabolic approximation as a means of modeling acoustic fields in ocean with currents and determining some properties of sound source and of the ocean. It is shown that under conditions typical to the Gulf Stream not only currents themselves but also range dependence of their velocity have dramatic influence on transmission loss of low-frequency sound. In more detail solution of direct problems in considered in [7,8]. Here we have emphasized application of the parabolic approximation to solve inverse problems. It is shown that a vertical acoustic array possesses directivity in horizontal plane when currents are present. Hence, localization of sound source in three dimensions using vertical array is possible. A simple approach to ocean currents tomography via reciprocal transmissions of continuous waves is proposed. This approach seems to be especially advantageous when rays can't be resolved. Further studies are necessary to reveal what amount of environmental information may be collected using this approach under realistic conditions.

7. REFERENCES

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