RECONSTRUCTION OF VERTICAL DISTRIBUTION OF SOUND AND HORIZONTAL CURRENT VELOCITIES BY NONPERTURBATIVE INVERSION OF ACOUSTIC TRAVEL TIMES

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1. INTRODUCTION

Acoustic tomography is a very promising tool to study dynamics of the ocean [1-5]. Being proposed by Munk and Wunsch [1], the program of global ocean monitoring includes acoustic remote sensing of currents as its important part. Determining sound speed c and current velocity $\vec{u}$ by means of ocean acoustic tomography implies solution of an inverse problem. This paper treats a kinematic inverse problem (KIP), where acoustic travel times T for various source-receiver separations are considered as input data.

A number of approaches to solve KIP in motionless media were studied in detail by many authors (see [6,7] and bibliography therein), but explicit exact solutions were found only for layered media. These solutions are based on Abels's integral transformation. When a flow is present, fluid is acoustically anisotropic. Hence, solution of KIP becomes much more complicated. If currents are slow, the inverse problem may be addressed via linear inversion [3,4]. However, linearization with respect to $\vec{u}$ isn't usually permissible, when sound propagates through powerful currents such as Gulf Stream. Another approach, to which this paper is devoted, consists in generalization of some results, obtained earlier using Abels's transformation for media at rest, on moving media.

Present research is a part of theoretical study of possibilities of acoustic monitoring of powerful oceanic currents from moving ships. It should be noted that errors Δr in positioning of source and receiver give rise to significant changes $\Delta_1 T \cong \Delta r/c$ in travel times. For typical $\Delta r/r$ and $|\vec{u}|$ values, $\Delta_1 T$ is greater than or comparable to the change $\Delta_2 T \cong ur/c^2$ in travel times due to flow. Nevertheless, one can measure the currents velocity by tomographical techniques provided travel times in opposite directions are measured almost simultaneously. In this case of reciprocal transmission the positioning errors result only in relative error $O(\Delta r/r)$ in estimate of $\vec{u}$.

We shall consider KIP for layered media, in which sound velocity c and flow velocity $\vec{u}=(u_1,u_2,0)$ are functions of a vertical Cartesian coordinate z. Dependence $T(\vec{r})$ of acoustic pulses travel times on source-receiver separation is taken as input data. For simplicity it is supposed that both source and receiver are at the same depth $z=z_0$. Then T appears to be function of two variables : $T=T(x,y)$. However, our goal consists in determining $c(z)$ and $\vec{u}(z)$ departing from values of T (for propagation in opposite directions) collected along two nonparallel sections $\vec{r}=r\vec{e}_{1,2}$, where $r=|\vec{r}|$, $\vec{e}_{1,2}$=const.

The problem of inversion of $T(x,y)$ is well-posed, if in the given medium there are no horizons which are not turning horizons of any ray [8]. The solution of the problem in closed form is known for a particular case only, when one of the Cartesian components of $\vec{u}$ is constant [8]. However, even in this case function of two variables T(x,y) is used as input data. In the ocean experimental determination of such a function is quite unrealistic. To simplify solution of the problem it is expedient to utilize smallness of flow velocity $\vec{u}$ and variations of sound speed Δc compared to mean sound speed. In the ocean typical values of flow's Mach number $M=|\vec{u}|/c\le 10^{-3}$ and $\Delta c/c\le 0.03$. Taking into account smallness of these parameters makes it possible to find explicit solution of KIP in moving media in terms of T values measured at two nonparallel straight lines instead of the whole plane $z=z_0$. The approximate solution of KIP is obtained in Sect.2. Its accuracy and stability are studied in Sect.3 using simulated data. Results are summarized in Sect.4.

2. ANALYTIC SOLUTION OF THE INVERSE PROBLEM

Let sound source is situated at coordinates' origin. Hence $z_0=0$. Denote $c_0=c(0)$, $\vec{u}\equiv(u_{10},u_{20},0)=\vec{u}(0)$, $\vec{\nu}=(\vec{\nu}_\perp,\nu_3)$, $\nu_3=(\nu_\perp\cos\psi,\ \nu_\perp\sin\psi,0)$, $\nu_3=\pm[(c_0-\vec{u}\vec{\nu}_\perp)^2c^{-2}-\nu_\perp^2]^{1/2}=\pm\nu\cos\chi$. $\vec{\nu}$ is a unit normal to wave front, ν_3 is a vertical component of $\vec{\nu}$, χ is a grazing angle. $\vec{\nu}_\perp$=const at rays in layered media. If ray reaches a turning horizon $z=z_0$, where $\nu_3=0$, and then returns to plane z=0, we have [9,§16.1]

$$\vec{r}=(x,y,0)=2\int_0^{z_r}dz[(c_0-\vec{u}\vec{\nu}_\perp)\vec{u}+c^2\vec{\nu}_\perp](c^2|\nu_3|)^{-1},\quad T=2\int_0^{z_r}dz(c_0-\vec{u}\vec{\nu}_\perp)(c^2|\nu_3|)^{-1}, \qquad (1)$$

To find a parameter $\vec{\nu}_\perp$ for a ray, which comes to a point, consider function $\tau=c_0T-\vec{\nu}_\perp\vec{r}$. It follows from (1) that $d\tau/d\vec{\nu}_\perp=-\vec{r}$. By differentiating τ with respect to $\vec{\nu}_\perp$, one finds $\vec{\nu}_\perp=c_0dT/d\vec{r}$. This equation makes it possible to express $\vec{r}$, T and τ as function of $\vec{\nu}_\perp$. In addition, it is straightforward to find c_0 and $\vec{u}_0$ by considering $T(\vec{r})$ at $\vec{r}\to 0$. Therefore, we assume c_0, $\vec{u}_0$, $T(\chi,\psi)$, $\vec{r}(\chi,\psi)$ and $\tau(\chi,\psi)$ to be known in what follows.
Suppose $\vec{u}=(u(z),0,0)$. Then $\vec{r}=(x,0,0)$ for rays with $\psi=0$ and $\psi=\pi$. By measuring travel times along line y=0 one obtains two dependencies $T^{(\pm)}(\chi)$, which give rise to $\tau^{(+)}\equiv\tau(\chi,0)$ and $\tau^{(-)}\equiv\tau(\chi,\pi)$. (Here and below superscripts + and - refer to quantities relevant to rays with $\psi=0$ and $\psi=\pi$ respectively.) According to (1)

$$\varphi^{(+)}\equiv\tau^{(+)}(\chi)(c_0\cos\chi+u_0\cos\chi)/2c_0=\int_0^{z_r}dz[(c_0-(u-u_0)\cos\chi)^2c^{-2}-\cos^2\chi]^{1/2}. \qquad (2)$$

This is an integral equation with respect to functions u(z) and c(z). In general moving medium the equation cannot be reduced to the Abel integral equation [6] due to complicated dependence of the integrand on $\cos\chi$. Note that $\chi^2\cong\Delta c/c_0\ll 1$. Values of $\cos\chi$ are very close for all refracted rays and $(u-u_0)\cos\chi$ may be substituted approximately by some function $(u-u_0)g(z)$

independent on χ. Let us take $g(z)=(c_0/c)^{\beta}$, β=const. Omitting terms $O(M^2)$ in radicand, we obtain then the Abel integral equation with solution

$$z(\eta^{(+)})=\frac{2}{\pi}\frac{d}{d\eta^{(+)}}\int_0^{\eta^{(+)}}\varphi^{(+)}(a)[\eta^{(+)}-a]^{-1/2}da \qquad (3)$$

Here $a=\sin^2\chi$, $\eta^{(+)}=1-(c_0/c)^2+2(u-u_0)c_0^{-1}(c_0/c)^{2+\beta}$. Velocities c and u are expressed in terms of solutions $z(\eta^{(\pm)})$ of two Abel's equations as follows :

$$c=c_0(1-0.5\eta^{(+)}-0.5\eta^{(-)})^{-1/2},\quad u=u_0+0.25c_0(\eta^{(+)}-\eta^{(-)})(c/c_0)^{2+\beta}. \qquad (4)$$

Approximate solution (3), (4) is obtained under conditions $M\ll1$ and $\chi^2\ll1$. Under these very conditions the effective sound speed profile (ESSP) approximation becomes applicable. In framework of this approximation expressions for T and $\vec{r}$ at a ray coincide with those in motionless media with sound speed $c(z)+\tilde{u}(z)$, where $\tilde{u}$ is the projection of $\vec{u}$ on the vertical plane, which contains both source and receiver. Solution of KIP within ESSP approximation follows immediately from well-known solution of the inverse problem for medium at rest :

$$z(\xi^{(\pm)})=\frac{1}{\pi}\frac{d}{d\xi^{(\pm)}}\int_0^{\xi^{(\pm)}}\tau^{(\pm)}(a)[\xi^{(\pm)}-a]^{-1/2}da, \qquad (5)$$

$$c=0.5[E^{(+)}+E^{(-)}],\quad u=0.5[E^{(+)}-E^{(-)}],\quad E^{(\pm)}=(c_0\pm u_0)(1-\xi^{(\pm)})^{-1/2}. \qquad (6)$$

It is of importance, that estimates of c(z) and u(z), which are valid at higher grazing angles, than (3),(4) and (5),(6), can be obtained by using solutions obtained above as initial approximations in iterative solution of the exact integral equations. Derivation of one such wide-angle approximation for c and u is presented in [10].

Now consider currents with arbitrarily changing direction of horizontal flow, i.e. with $\vec{u}=(u_1(z),u_2(z),0)$. From (1) and inequality $M\ll1$ it follows that $\psi=O(u_2/c)$ for all the rays coming to observation points at the line $z=z_0$, $y=0$. Hence, up to terms $O(M^2)$ $T(\chi,0)$ isn't affected by the transverse component of the flow. Therefore in the case at hand Eqs.(4),(6) give c(z) and projection of $u_1(z)$ of $\vec{u}$. By measuring $T^{(\pm)}$ along two nonparallel straight line one can find both components $u_1(z)$ and $u_2(z)$ of the flow velocity.

Note one more way of determining $u_2(z)$. Let $T^{(\pm)}$ is measured along line $y=0$ and input data include also dependence $\psi(x,0)$. Then omitting terms $O(M^2)$ in (1) one obtains from condition $y=0$ an integral equation of Abel's type with respect to $u_2(z)$. Solving of the latter equation is straightforward.

3. PROPERTIES OF THE APPROXIMATE SOLUTIONS OF THE INVERSE PROBLEM

To begin with, consider the simplest particular case of fluid with linear profiles $c=c_0(1+Bz)$, $u=c_0ABz$ of flow and sound speeds. It is assumed $Bz\ll1$ and $A\ll1$.The results are set in Tab.1, where in the first column the approach used for inversion is indicated. The terms of the order of $O(B^kz^k),k\geq2$ are methodical errors, and using their explicit expression one can evaluate accuracy of each method. One can see that the least error in reconstruction of profile of $\vec{u}$ corresponds to wide-angle approximation and method (3),(4) under optimal value 5/4 of parameter β. Relative errors in reconstruction of u(z) are equal $O(u/c_0)$ and $O((\Delta c/c_0)^2)$ respectively. Inversions by Eqs. (5), (6) and Eqs. (3), (4) (with $\beta\neq5/4$) have similar accuracy.

Approach	Results of inversion	
	c/c_0	u/c_0
Eqs.(3),(4)	$1+Bz-\frac{7}{4}A^2B^2z^2$	$ABz+(\beta-\frac{5}{4})AB^2z^2+O(AB^3z^3)$
Eqs.(5),(6)	$1+Bz-\frac{1}{4}A^2B^2z^2$	$ABz-\frac{1}{4}AB^2z^2$
Wide-angle approximation	$1+Bz-\frac{7}{4}A^2B^2z^2$	$ABz+\frac{5}{4}A^2B^2z^2$

Tab.1 Results of analytic solution of KIP by various approximate methods.

Evaluation of the results of inversion demands use of a micro computer, when original profiles of c and $\vec{u}$ are more complicated. Accuracy and stability of three methods of nonperturbative inversion, considered in Sect.2, were studied by solving KIP with simulated input data. Typical results of numerical experiments for three various environments are illustrated by Figs. 1-4. In these examples $u_2\equiv0$ and values of $T^{(\pm)}(x,0)$ calculated at N discrete points are used to simulate measured data.

(i) Consider inverse problem for a model case, in which flow's Mach number and grazing angles are large compared to their values in the ocean, but small compared to 1. Original c(z) and u(z) profiles and results of travel times inversion are shown at Fig.1. Accuracy of the results of different inversion methods are in good agreement with the accuracy estimates derived above for medium with linear c(z) and u(z) profiles. The wide-angle approximation gives the best results. Difference between original c(z) and all its estimates is too small to be seen at the figure.

The above inversion formulas involve continuous data. It is of great importance for practical implementation, what amount of input information is really needed to find c(z) and u(z). This question is addressed at Fig.2 via wide-angle approximation. In spite of complex shape of original profiles it turns out, that satisfactory accuracy of the inversion is preserved even when the travel times along only 15 rays (in each direction) are given.

(ii) Consider c and u profiles (Fig.3) typical to Gulf Stream as the next example. Source and receiver are at SOFAR axis at depth z_0=1100 m. c(z) and u(z) are assumed to be known at $z>z_0$. The problem consists in determining c(z) and u(z) above the waveguide's axis. In the medium at hand the travel times difference $T^{(+)}(x,0)-T^{(-)}(x,0)$ due to current mounts to 15 ms per ray cycle. Equations (3), (4) with β=5/4 are used to solve the inverse problem.

The root mean square and maximum errors of inversion of sound speed and flow velocity turn out to be $\sigma_u=4\cdot10^{-3}$ m/s, $|\delta u|_{max}=1.2\cdot10^{-2}$ m/s and $\sigma_c=1.3\cdot10^{-2}$ m/s, $|\delta c|_{max}=7.8\cdot10^{-2}$ m/s.

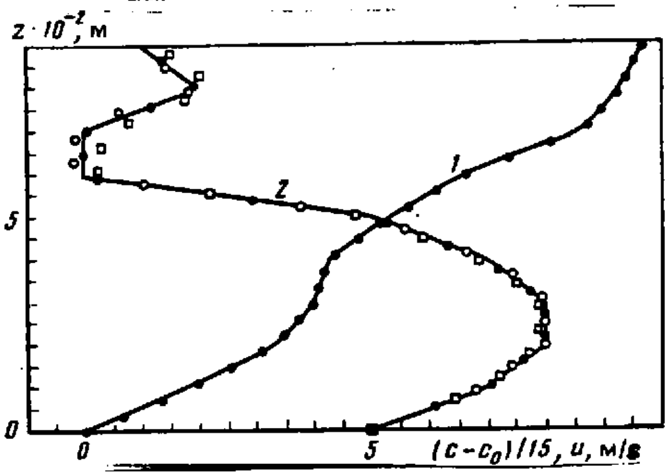

Fig.1 Numerical experiment on travel times inversion. Original profiles c(z) (1) and u(z) (2) are shown by solid lines. Results of c(z) inversion is shown by dark circles. Estimates of u(z) obtained using Eqs. (3),(4) with $\beta=0$ and $\beta=5/4$ as well as using wide-angle approximation are shown respectively by squares, light and dark circles. Total number of rays N=100, $c_0=1500$ m/s.

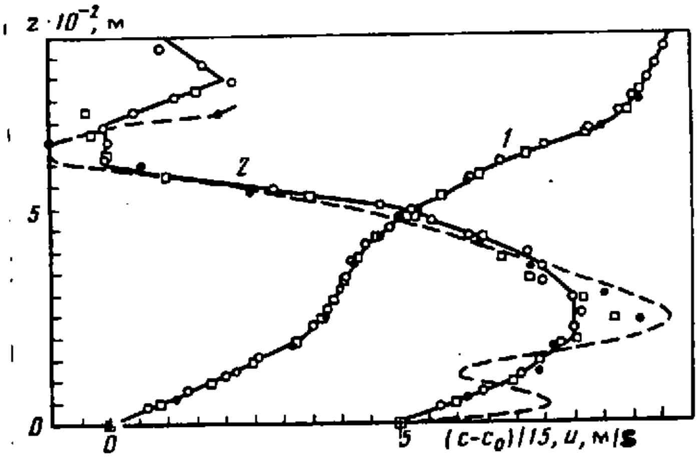

Fig.2 Influence of number of rays used on accuracy of inversion of c(z) (1) and u(z) (2). Original profiles are shown by solid lines. Light circles, dark circles and dashed line stand for results of inversion with N=50, N=20, N=15 and N=10 respectively.

Let the source and receiver move with respect to the ocean bottom with velocity equal to flow velocity at z=0, i.e. at the surface of the ocean. This is to model ships' drift, which is usually present in oceanic experiments. Under these conditions solution of KIP using (3), (4) with $\beta=5/4$, N=100 has errors similar to that in the previous problem. Namely, $\sigma_u=4\cdot10^{-3}$ m/s, $|\delta u|_{max}=1.5\cdot10^{-2}$ m/s, $\sigma_c=1.2\cdot10^{-2}$ m/s, $|\delta c|_{max}=7.1\cdot10^{-2}$ m/s. Hence, accuracy of the inversion isn't affected by the drift.

(iii) Experimentally measured travel times unavoidably have some errors. To simulate these errors exact input values T(x,0) were substituted by $T(x,0)+\varepsilon_A\theta_j+\alpha\varepsilon_s x_j|x_j|$. Here ε_A and ε_s are amplitudes of random and systematic errors, θ_j is a stochastic quantity with $|\theta_j|\le1$, $\alpha=(\max_j x_j^2)^{-1}$ is a normalizing constant. To model relative clock drift at two ships, the systematic error has different signs for rays propagating in opposite directions. Fig.4 illustrates solution of the inverse problem within wide-angle approximation for a medium with linear dependencies u(z) and c(z)=1500 +0.02z, where c is expressed in m/s and z in meters. In three cases, when $\varepsilon_A=3$ ms, $\varepsilon_s=0$; $\varepsilon_A=2$ ms, $\varepsilon_s=1$ ms; $\varepsilon_A=0$, $\varepsilon_s=3$ ms, errors in inversed c(z) are respectively $\sigma_c=4.8\cdot10^{-2}$ m/s, $|\delta c|_{max}=11\cdot10^{-2}$ m/s; $\sigma_c=5.6\cdot10^{-2}$ m/s, $|\delta c|_{max}=1.5\cdot10^{-1}$ m/s and $\sigma_c=2.7\cdot10^{-2}$ m/s, $|\delta c|_{max}=1.3\cdot10^{-1}$ m/s. Note that errors up to 3 ms in the travel times can't prevent from satisfactory inversion in the case considered.

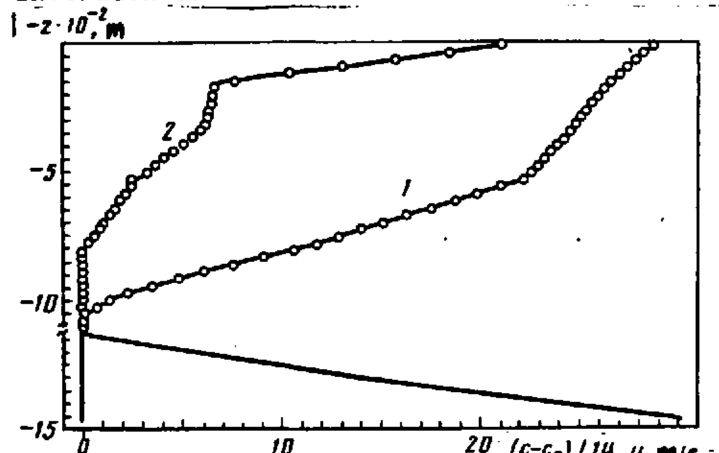

Fig.3 Solution of KIP under conditions typical to Gulf Stream. Original and estimated profiles of c(z) (1) and u(z) (2) are shown by solid lines and by light circles, respectively.

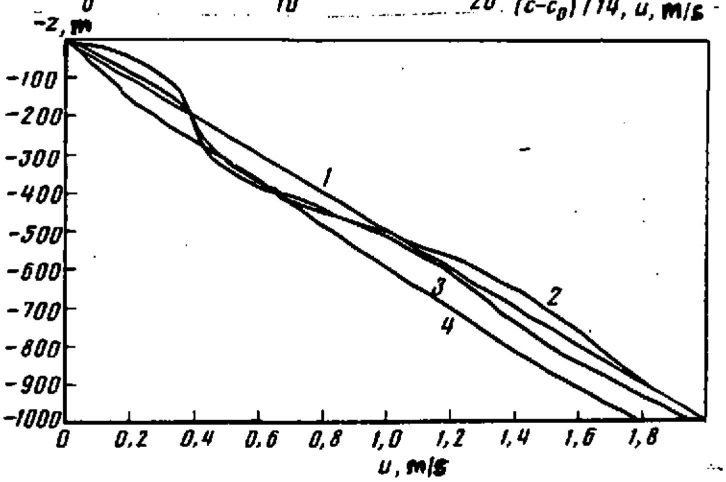

Fig.4 Effect of random and systematic errors in travel times on estimates of u(z). Original profile (1) is compared with results of inversion, when $\varepsilon_A=3, \varepsilon_s=0$ ms(2), $\varepsilon_A=2$ ms, $\varepsilon_s=1$ ms (3) and $\varepsilon_A=0$, $\varepsilon_s=3$ ms (4). Total number of rays N=100.

4. CONCLUSION

Several approaches are developed to solve the kinematic inverse problem of ray acoustics in moving layered media. These approaches are extensions of some known methods [6,7] designed for media at rest. The equations obtained are appliable for all possible values of current velocity and grazing angles of refracted rays in the ocean. Analogous approaches to companion problem, in which modes can be resolved instead of rays, are developed in [11]. Some additional tomographic schemes and corresponding modifications of the inversion procedures are considered in [10].

Two main shortcomings should be noted. (i) Extension of our methods on range-dependent environments isn't straightforward. (ii) It is difficult to use any a priori knowledge of medium's properties to improve the inversion.

The main advantage of our methods is its nonperturbative nature. That is, any reference profiles of c(z) and $\vec{u}(z)$ are not used during the inversion. Numerical implementation of the solution of the inverse problem doesn't require significant computer time and memory. Although integrals over continuous variables enter the inversion formulas, in fact the solution may be found using discrete and rather sparse input data. In practice, it is expedient to use the nonperturbative methods of inversion to obtain some initial estimate of c and $\vec{u}$ vertical dependence, which can be refined, including determining c and $\vec{u}$ variability in horizontal plane, via standard linear inversion.

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