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THE PREDICTED PERFORMANCE OF AN UNDERWATER NAVIGATION SYSTEM BASED ON A CORRELATION LOG

P. Atkins and B. V. Smith

Department of Electronics and Electrical Engineering,
University of Birmingham, P.O. Box 363, Birmingham B15 2TT

INTRODUCTION

Acoustic velocity sensors may be successfully used as part of an integrated underwater navigation system. Considerable improvements may be achieved in the the accuracy of the velocity estimate by using bottom referenced systems compared to water-mass referenced systems such as electromagnetic logs.

The traditional acoustic method of obtaining a velocity estimate is by the use of a Doppler log. A suitable compromise must usually be made between good Doppler discrimination and large operational ranges. Correlation logs [1] are a possible alternative to the Doppler log and are in general less affected by secondary disturbances such as temperature variations in the water-mass.

The performance of the correlation log is primarily determined by the transducer geometry. The accuracy of the velocity estimate will also be related to the observation distance allowed to obtain an estimate of the correlation function of the backscattered acoustic field. The large integration distances that are required to obtain velocity estimates make the correlation log an ideal sensor for measuring the elapsed distance of a vessel travelling at a near constant velocity and heading.

ESTIMATION OF THE SPATIAL CORRELATION FUNCTION

Both spatial and temporal correlation logs operate by estimating the position of the spatial cross-correlation function with respect to time. The resolution of this estimate will therefore be determined by the spatial separation and the effective signal-to-noise ratio of the spatial correlation function.

The form of the spatial covariance function will, for most operating conditions, be determined by the geometry of the acoustic projector. For example, the form of the spatial covariance function assuming small slopes for the bottom irregularities and a circular projector may be predicted as [2,3]:

$$\overline{V_1 V_2}^* = W^2 \int_0^1 D^2rt \; x(1-x^2) \; J_o(dkx) \; dx \tag{1}$$

where $Drt = \frac{2J_1(kax)}{(kax)}$, the directivity function of the projector

and W is a constant
x is an integration variable
J_o, J_1 are Bessel functions
k is the wave number
d is the spatial separation of two receivers
a is the projector radius

This function is derived by analytic techniques whereas the measured function is obtained from stochastic signals and will therefore only approach the predicted value as the observation distance approaches infinity. Predicted normalised spatial covariance functions are shown in figure 1 for three circular projectors with diameters of 10mm, 20mm and 30mm, which are operating at a frequency of 150kHz.

Jenkins and Watts [4] show that the covariance of an estimated covariance function may be approximated by :

$$Cov[Cxx(u_1),Cxx(u_2)] \simeq \frac{1}{D}\int_{-\infty}^{\infty} \gamma xx(r)\gamma xx(r+u_2-u_1) + \gamma xx(r+u_2)\gamma xx(r-u_1)\; dr \qquad (2)$$

where Cxx is the estimate of the covariance function
γxx is the theoretical covariance function
u_1, u_2 are spatial lags

This approximation is only valid if the observation distance, D, is large. This assumption will be valid for a practical correlation log where the observation distances are typically of the order of tens of metres. The spatial lag terms u_1 and u_2 are also assumed to be small.

By equating the lag terms u_1 and u_2 the variance of the estimated covariance function may be obtained.

$$Var[Cxx(u)] \simeq \frac{1}{D}\int_{-\infty}^{\infty} \gamma xx^2(r) + \gamma xx(r+u)\gamma xx(r-u) \quad dr \qquad (3)$$

This equation shows that the variance of an ensemble of covariance estimates is inversely proportional to the observation distance, D. The normalised variance of the theoretical function described by equation (1) is shown with respect to the spatial offset from the peak of the function in figure 2. The cases for the three different projector diameters are shown in this figure.

Figure 2 effectively provides an estimate of the normalised noise error power at a particular spatial separation from the peak of the spatial covariance function. However, a correlation log operates by attempting to measure the position of the peak of the estimated function. This estimate of the position of the peak will also depend on the correlation of the noise power across the function. Assuming that the peak of the estimated covariance function lies at a point $u_1 = 0$ then the covariance of the noise power may be obtained by :

$$\mathrm{Cov}[Cxx(0),Cxx(u)] \simeq \frac{1}{D}\int_{-\infty}^{\infty} 2.\gamma xx(r)\gamma xx(r+u) \; dr \qquad (4)$$

This may be plotted with respect to the spatial separation u for varying projector dimensions, as shown in figure 3. It will be noticed that the noise power in the estimate of the spatial covariance function is highly correlated across the likely areas of interest. A comparison of figures 1 and 3 shows that the width of the noise covariance function is greater than the equivalent width of the spatial covariance estimate.

From a knowledge of the variance and the covariance parameters, the uncertainty in the estimate of the position of the peak of the function may be calculated. Assume that the theoretical peak of the spatial covariance function lies at a point u=0. The probability that the measured peak lies at, or beyond, a point, u, may be found by estimating the probability that Cxx(u) is greater than Cxx(0), remembering that Cxx(0) and Cxx(u) are measured functions and are assumed to have normally distributed statistics associated with them. This may be calculated by :

$$p_e(u) = \int_{\infty}^{\infty}\int_{Cxx(u)}^{\infty} p(Cxx(0)).p(Cxx(u)\,|\,Cxx(0)) \;\; dCxx(0)\; dCxx(u)$$

where $p(Cxx(u) >= Cxx(0)) = p_e(u)$

For a normal bivariate distribution this may be shown to be equal to :

$$p_e(u) = \frac{1}{\sqrt{2.\pi.\mathrm{Var}(Cxx(0))}}\int_{-\infty}^{\infty}(1 - \mathrm{erf}(Yo)) \; \exp^{\frac{-(x\,-\,\gamma(0))^2}{2.\mathrm{Var}(Cxx(0))}} \; dx$$

where

$$Yo = \sqrt{\frac{K11.x^2}{2.(K11.K22 - K12^2)}}\left[1 - \frac{K12}{K11} + \frac{K12}{K11}.\,\gamma(0) - \gamma(u)\right]$$

and
$K11 = \mathrm{Var}(Cxx(0))$
$K22 = \mathrm{Var}(Cxx(u))$
$K12 = \mathrm{Cov}(Cxx(0),Cxx(u))$

The results of numerical calculations of $p_e(u)$ are plotted in figure 4 for three projector diameters and observation distances, D, of 1m and 10m. The probability $p[Cxx(u) >= Cxx(0)]$ diminishes very rapidly with the spatial displacement, u. The results can be seen to be highly dependent on the observation distance, D.

The statistics of an ensemble of estimates of the peak of the spatial covariance function are likely to be normally distributed with a mean equal to that of the theoretical position of the peak. The value of u equivalent to one standard deviation may be derived by setting $(1 - p_e(u))$ equal to 0.683, this is justified as the variance of the estimate is highly correlated across the region of interest.

Figure 5 shows the required observation distance, D, plotted with respect to the value of the standard deviation of the estimated position of the peak.

RESOLUTION OF THE VELOCITY ESTIMATE

Both temporal and spatial correlation logs operate by measuring the displacement of the spatial covariance function with respect to time. In the case of the temporal correlation log this displacement is fixed whereas in the case of a spatial correlation log this separation may fall anywhere within a pre-defined range of values.

Assume that the separation of the spatial covariance function is defined as S and that the uncertainty of this estimate is given by ΔS. The accuracy of the system may then be defined as $\Delta S/S$. The spatial uncertainty ΔS is equivalent to one standard deviation associated with the estimate of the peak of the spatial covariance function. In a practical situation the size of the transducer housing will be limited. This places a limit on the separation, S, and on the maximum update rate in terms of the distance travelled for a given accuracy. A typical transducer housing would be of the order of 75mm for a commercial vessel and 300mm for a small, survey-orientated submersible. The maximum transducer separation would typically be of the order of 25mm less than the housing dimensions. Figure 6 shows the expected accuracies with respect to distance travelled of a device with transducer separations of 25mm and 250mm and projector diameters of 10mm and 20mm.

Figure 6 shows the required observation distance to obtain a specified accuracy and shows the importance of using large receiver transducer separations to obtain high accuracies. Practical sonar correlation logs would normally use a pulsed transmitter when ranges in excess of a few metres are required. A sampled version of the acoustic backscattered field is therefore used in obtaining the estimate of the covariance function. The sampling efficiency may be defined as the transmit pulse duration divided by the pulse repetition period. This sampling efficiency would typically be in the range 10% to 20%, resulting in the actual distance travelled by the vessel being between five and ten times the observation distance predicted in figure 6.

LIMITATIONS ON THE RECEIVER TRANSDUCER SEPARATION

A limit is imposed on the maximum value of receiver transducer separation by the velocity and the depth separation (altitude) of the vessel. The spatial covariance function moves at a rate $2\bar{v}$ away from the reference receiver, where $\bar{v}$ is the velocity of the vessel. Equating this to the transducer separation, S, and time:

$$S = 2\bar{v}t$$

Typically, the time t would be of the order of half the transmit pulse period. The transmit pulse period is in turn limited to a value of the order of half the two-way propagation time. i.e.

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$$\text{transmit pulse period} = \frac{z}{c}$$

where z is the depth separation
c is the velocity of propagation

Therefore

$$S <= \frac{v.z}{c}$$

This places a severe limitation on the accuracy of the device at low speeds and in shallow water. Under these conditions, the effective receiver separations must be reduced to compensate. The spatial correlation log has the inherent ability to do this because of its multi-element array structure, such a device will have a very poor accuracy performance under such conditions.

A typical minimum depth with respect to speed trade-off is shown in figure 7. This shows the performance bounds for a number of transducer separations when using a 10mm diameter projector. The predicted accuracy performance for an observation distance of 10m is shown in brackets for comparison. A correlation log will not be able to operate with the defined accuracy in the region enclosed by the ordinates and the performance bound.

OTHER PERFORMANCE LIMITING FACTORS

The above discussion of the predicted performance of a correlation log is based on the spatial characteristics of the acoustic backscattered field. A number of other performance limiting factors must also be considered.

Electrical system noise will affect all the channels used by the correlation log. This will be uncorrelated from channel to channel and will be bandlimited. In a practical system the signal-to-system noise ratio will be large. The time period required for the vessel to travel the required observation distance between velocity updates will ensure that a large number of independent samples are incorporated into the process and hence the effects will be negligible.

Acoustic noise will consist of two components. An uncorrelated noise component will be received, the primary source being flow noise. A partially correlated noise signal will be received from isotropic noise sources and reverberation. Increasing the pulse repetition period may be required to remove the effects of reverberant signals.

The design of the receiver elements may introduce a summation of correlated components caused by the large number of linear paths that may be drawn from one finite sized receiver element to another. This effect may cause significant errors when the vessel is moving with excessive transverse velocities.

The most significant performance limiting factor has been found to be due to returns from the wake of the vessel. In general, the wake of the vessel will be moving at a greater differential velocity with respect to the vessel than that of the sea bed. The wake frequently contains significant levels of aeration and therefore has a large target strength. A simple projector could place a sidelobe pointing to the wake with an attenuation of only 13dB with respect to the main lobe. Significant quantities of information obtained from the wake could thus be processed causing a large error in the velocity estimate with respect to the sea bed.

COMPARISON OF THE PREDICTED AND MEASURED RESULTS

A two axis temporal correlation log has been constructed and field tested [5,6]. The results of a number of track plots obtained by integrating the output of this device with respect to heading are shown in figure 8. The vessel was timed through two theodolites aligned normally to the expected track of the vessel, this provided an estimate of the actual velocity of the vessel. Figure 8 shows two sets of tracks caused by windage and a current acting on the vessel as it steered to a constant heading.

Figure 9 shows an expanded view of the end of the tracks complete with 1% error bounds. The standard deviation-to-mean percentage evaluated over eleven runs was measured as 0.64%. No parameters are available for the likely timing errors in these measurements caused by the manual identification of the position of the vessel. When related to the velocity of the vessel this represents a timing error of about 1.6s, probably limited by the human reaction time of initiating and terminating a run.

The correlation log used for these measurements incorporated a projector of dimension 10mm and a receiver separation of 28.28mm. Velocity estimates were obtained at intervals of approximately 60m. Assuming that a sampling efficiency of 10% was used then the actual observation distance was of the order of 6m. Figure 5 indicates that the predicted error in determining the peak of the spatial covariance function would be of the order of 0.34mm for this observation distance. This corresponds to a value for the predicted accuracy for a single velocity estimate of the order of 1.2%. Fifteen such velocity estimates would be obtained within a single track as shown in figure 8. Assuming a normal distribution of errors, it would be expected that the standard deviation-to-mean ratio of the elapsed distance would be of the order of 0.31%. It can be seen that the predicted and measured performances of the correlation log are of the same order.

CONCLUSIONS

A number of performance limiting criterion have been illustrated for a correlation log. It is the view of the authors that the performance of a correlation log may be accurately predicted from a knowledge of the geometry of the transducer and a value of the observation distance. A high signal-to-noise ratio has been assumed throughout, as this is readily obtained in practice.

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A comparison has been made between the predicted performance and the measured performance of a correlation log. The measured and predicted results were found to agree closely.

ACKNOWLEDGEMENTS

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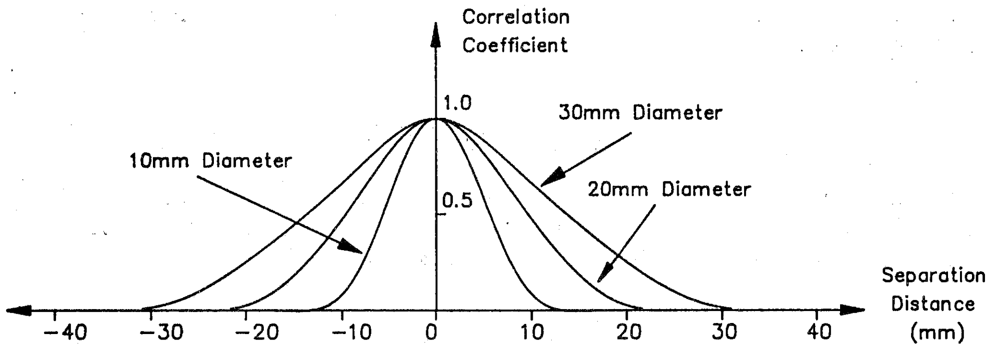

Figure 1 : Predicted Spatial Correlation Functions for Three Circular Projectors

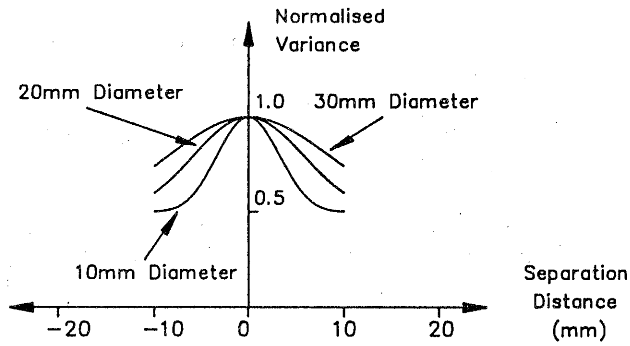

Figure 2 : Predicted Normalised Variance of the Spatial Correlation Function

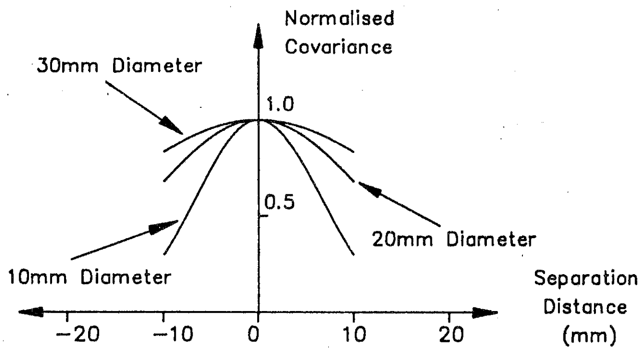

Figure 3 : Predicted Normalised Covariance of the Spatial Correlation Function