ACTIVE STRUCTURAL-ACOUSTIC CONTROL OF SOUND TRANSMISSION THROUGH A DOUBLE WALL PANEL BASED ON FREQUENCY WEIGHTED H2– CONTROLLER

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Double wall partition walls and structural panels are being widely used as aerospace and automobile components for their superior sound retardation characteristics. However, it is observed that in the low to mid frequency regime the use of passive sound reduction devices does not effectively reduce the sound transmission through double wall panels. In automobile and aerospace vehicles this frequency regime is often found to be very disturbing to the occupants. The focus of this research work is to develop a suitable active control algorithm based on an frequency weighted optimal H2 control law to alleviate the problem posed in the low and mid-frequency sound transmission through a double-wall panel into an acoustic enclosure. An FE based model is developed separately for the structural panels and the acoustic domain and they are then coupled using Green's Theorem. Based on the coupled modal characteristics of the panels, piezoelectric sensors and actuators are mounted on the structural panels. An objective function is then developed wherein the H2 – norm of the estimated acoustic pressure in the enclosure is minimized by means of active control of the structural panels where the acoustic pressure is used as the performance vector and the sensor voltages proportional to the structural velocities are fed to the controller and subsequently the feedback voltages are supplied to the piezo actuator patches with a limit imposed on the actuation voltages.

Keywords: ASAC, optimal control, double wall

1. Introduction

The study of sound transmission phenomenon through partition wall has been a major area of research through the last 60 years. This area of research has achieved a special significance particularly with the need to have a quieter cavity in room acoustics, in automobile and aircraft interior. The sound transmission through any partition can be broadly classified into two classes – (a) transmission into a free space and (b) transmission into closed space. Although the transmission phenomenon is very similar in both the cases, the treatment of the problem is significantly different. This is due to the fact that in interior acoustics, standing waves are created resulting in the interaction between the acoustic cavity modes and the structural modes that assumes special significance. It has always been the objective of the acousticians to design a proper strategy to alleviate the problem of noise transmission into an enclosure using active and/or passive approaches.

Modern day vehicular structures are built with lightweight structural systems for fuel efficiency which is, however, inherently very poor at reducing the sound transmission from external sources. Hence, it is very important to address the problem of sound transmission and reduction of the same during the design stage itself.In general, optimizing the fuselage sidewalls with respect to their sound transmission capability by applying additional sound proofing materials have resulted in satisfactory sound pressure levels in the cabin. However, it has been reported by several researchers that for broad-band disturbances due to TBL or jet-stream noise, passive control methods cannot provide satisfactory sound level reduction in the low-frequency regime, i.e., below 500 Hz. Passive double wall configurations show good sound attenuation behavior in the mid to high frequency regime above the mass-air-mass resonance. However, at frequencies below the mass-air-mass resonance, the sound transmission ratio is rather high and is controlled by the resonances of the double panel system. Also, the wavelength is large in comparison to the sound-proof layer thickness such that the sound absorption efficiency is rather low [1,2]. It is from this perspective an attempt is made in the present work to develop an active control strategy for sound transmission reduction into an enclosure through a double-wall panel.

Active control strategies for noise reduction inside an enclosure, namely, Active Noise Control, Active Noise and Vibration Control and Active Boundary Control has been reported by several researchers earlier [3,4]. However, the major disadvantage of the aforementioned system is that they are dependent on reference signal and for random or broadband disturbances like TBL or jet-noise, it is practically impossible to obtain the same. Subsequently an alternative strategy has been envisaged by other researchers which aims at reducing the sound transmission into an enclosure by modifying the response of the partition walls using structural actuators and sensors rather than by acoustic actuators and sensors distributed in the enclosure [5,6]. This alternative strategy which is also called the Active Structural Acoustic Control (ASAC) is implemented in the present work for reduction in the transmitted sound inside an enclosure through a double wall partition. A frequency dependent H_2 optimal control strategy is being developed and a brief description of the physical model with the necessary mathematical description is presented in the next section.

2. Problem Definition

The physical model considered in the present study consists of a rectangular acoustic enclosure with one of the walls being flexible and all the other considered as rigid wall. The flexible wall considered is a double-wall panel with an air-gap in between (Fig. 1).

The outer wall, i.e., panel ‘a’ is subjected to a disturbance that is transmitted as sound energy through the air-gap and panel ‘b’into the enclosure. Structural actuators and sensors in the form of piezoelectric patches are mounted on both panel ‘a’ and panel ‘b’ at suitable locations and a few points within the enclosure are identified as observer points and the objective is to reduce the transmitted sound pressure level (SPL) at those locations using a frequency based H_2 control law. The governing differential equations for the structural panels are obtained using the Hamiltonian Principle of variational mechanics and for the acoustic domain wave equation is employed. The governing equations are solved using finite element principles to obtain the free vibration frequencies and mode shapes separately for the structural panels and the acoustic domains. Subsequently, using a modal expansion theory based on Green’s theorem the structural velocity and the acoustic pressure are suitably coupled to obtain the fully coupled structural-acoustic equations for the sound transmission into the acoustic enclosure [7].

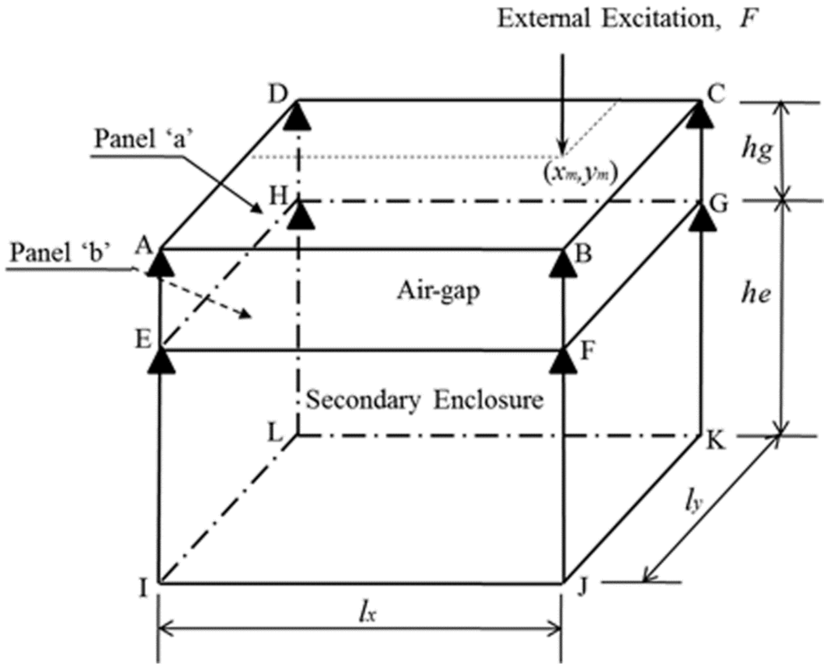

Figure 1: Schematic of the double wall configuration opening into an enclosure

The governing finite element equation can therefore be written as,

$$M\ddot{x}_a + Kx_a = \tilde{F} - P_g \tag{1}$$

$$M\ddot{x}_b + Kx_b = P_g - P_\mathrm{e} \tag{2}$$

where, $\tilde{F}$ is the external disturbance on the primary panel and P_g is the acoustic pressure in the air gap and P_e is the acoustic pressure within the enclosure (Fig. 1).

The acoustic gap pressure P_g is obtained by solving the following wave equation,

$$\nabla^2 P_g - \frac{1}{c_0^2}\frac{\partial^2 P_g}{\partial t^2} = 0 \tag{3}$$

with the following constraints,

$$\frac{\partial P_g}{\partial \mathbf{n}} = \begin{cases} \rho\ddot{w}_a \text{ on panel a} \\ -\rho\ddot{w}_b \text{ on panel b} \\ 0 \text{ on the rigid wall} \end{cases}$$

where, ρ is the equilibrium fluid density and **n** the normal direction directed outwards with w_a and w_b being the normal component of the structural displacements. For the acoustic enclosure a similar procedure follows. Using Green's Theorem,

$$\int_{V_g}\left(P_g\nabla^2\psi_g - \psi_g\nabla^2 P_g\right)\mathrm{d}v = \int_{A_a}\left(P_g\frac{\partial\psi_g}{\partial\mathbf{n}} - \psi_g\frac{\partial P_g}{\partial\mathbf{n}}\right)\mathrm{d}s \tag{4}$$

the governing equations with the introduction of the actuator dynamics can be reduced to the following modal form

$$\ddot{q}_{a,j} = -\omega_{a,j}^2 q_{a,j} - 2\xi_{a,j}\omega_{a,j}\dot{q}_{a,j} - S_a\sum_m L_{m,j}^{ag} p_{g,m} + \psi_{a,j}^T\tilde{F}_m - \psi_{a,j}^T K_{U\varphi}\varphi_a \text{...panel 'a'} \tag{5}$$

$$\ddot{q}_{b,k} = -\omega_{b,k}^2 q_{b,k} - 2\xi_{b,k}\omega_{b,k}\dot{q}_{b,k} + S_b\sum_m L_{m,k}^{bg} p_{g,m} - S_b\sum_n L_{n,k}^{be} p_{e,n} - \psi_{b,j}^T K_{U\varphi}\varphi_b \text{ ...panel 'b'} \tag{6}$$

$$\ddot{p}_{g,m} = \left[\frac{\rho_g c_g^2 S_a}{m_{g,m} V_g} L_{m,j}^{ag}(-\omega^2) \quad \frac{\rho_g c_g^2 S_b}{m_{g,m} V_g} L_{m,k}^{bg}(-\omega^2)\right]\begin{Bmatrix} q_{a,j} \\ q_{b,k} \end{Bmatrix} - \omega_{g,m}^2 q_{g,m} - 2\xi_{g,m}\omega_{g,m}\dot{q}_{g,m} \text{...air-gap} \tag{7}$$

$$\ddot{p}_{e,n} = \left[0 \quad \frac{\rho_e c_e^2 S_b}{m_{e,n} V_e} L_{n,k}^{be}(-\omega^2)\right]\begin{Bmatrix} q_{a,j} \\ q_{b,k} \end{Bmatrix} - \omega_{e,n}^2 q_{e,n} - 2\xi_{e,n}\omega_{e,n}\dot{q}_{e,n} \text{...enclosure} \tag{8}$$

where, $L_{k,i}^{ag}$ and the like terms represent the coupling coefficient between the various modal vectors of the structural and the acoustic domain. The terms, q_a, q_b, p_g and p_e are the modal velocity of panel 'a' and 'b' and modal pressures for the acoustic gap and the secondary enclosure, respectively.

In the present model it is assumed that both panel 'a' and panel 'b' are controllable. Hence piezoelectric sensor patches are mounted on both the panels. The governing sensor equations can then be written as,

$$\varphi_a^s = R_f K_s^T \psi_{a,j} \dot{q}_{aj} \text{…panel 'a'} \tag{9}$$

$$\varphi_a^s = R_f K_s^T \psi_{a,j} \dot{q}_{aj} \text{…panel 'b'} \tag{10}$$

Assuming the plant to be strictly proper with no feed through from the exogenous input w(external disturbance) to the regulated output z (SPL within the enclosure) the State – Space equations can be written as,

$$\dot{x} = Ax + B_{mw}w + B_{mu}u \tag{11}$$

$$z = C_z x + D_{zu}u \tag{12}$$

$$y = C_y x + D_{yw}w \tag{13}$$

where, x defines the state of the system represented by,

$$x = [q_a \quad q_b \quad p_g \quad p_e \quad \dot{q}_a \quad \dot{q}_b \quad \dot{p}_g \quad \dot{p}_e]^T \tag{14}$$

y defines the sensor output and u is the control input.

The focus is therefore to find an admissible controller, K, which minimizes the quadratic norm (H_2 norm) of the transfer matrix from w to z with an upper bound on the input control voltage. In the present model a structural velocity feedback is implemented with an assessment of the observed state. However, as the actual state,x is not available from the measurements, the concept of the estimated state, $\hat{x}$ is introduced which is obtained from a part of the controller. Hence, the estimated state, $\hat{x}$ and the controller output, u can be formulated similar to an LQG model as,

$$\dot{\hat{x}}(t) = (A + B_u K_c)\hat{x}(t) + K_e\left[y(t) - C_y \hat{x}(t)\right] \tag{15}$$

$$u(t) = K_c \hat{x}(t) \tag{16}$$

where, K_c and K_e are the controller gain and the filter (estimator) gain, respectively obtained by considering the process and measurement noises to be uncorrelated and having a constant power spectral density.

$$K_c = -(D_{zu}^T D_{zu})^{-1} B_u^T \bar{S} \tag{17}$$

$$K_e = \bar{P} C_y^T \left(D_{yw} D_{yw}^T\right)^{-1} \tag{18}$$

The terms $\bar{\mathbf{S}}$ and $\bar{\mathbf{P}}$ are solutions of the controller algebraic Riccati equationand the estimator algebraic Riccati equation, respectively. It can be shown that the state matrix A and the estimator matrix Cz are frequency dependant and hence the solution to the Ricatti equations and finally the state-space equations needs to be solved at each frequency step. In the present ASAC strategy, the objective is to minimize the averaged quadratic sound pressure level in the frequency range of interest for these selected observer locations due to an external excitation subjected to an applied control voltage to the actuator patch defined by $|\boldsymbol{\varphi}^{\mathbf{a}}| \leq 100$ Volts. Therefore, the control objective may similarly be stated as,

Minimize $\langle\overline{P^2}\rangle$
subjected to,

$$|\varphi^a| \leq 100 \text{ Volts: voltage limitation.}$$

$$\langle\overline{P^2}\rangle = \frac{1}{2nz} P_{obs}.P_{obs}^* \tag{19}$$

where, P_{obs} is the vector of sound pressure at the observation points (number of observation points = nz). In the next section a numerical case study is performed on a physical system as described earlier and the efficiency of the developed controller model is presented.

3. Results and Discussion

The physical dimension of the configuration considered in the following examples is as follows:
Panel 'a' – 0.5 m x 0.35 m x 0.002 m; Panel 'b' - 0.5m x0.35m x0.003m
Air gap depth (h_g) – 0.33 m; Enclosure depth (h_a) – 0.55 m
Both the structural panels are made of Aluminum and the fluid considered within the air-gap and the enclosure is air. The material properties assumed are as follows –
Aluminum: Young's modulus (E) - 70 GPa, Poisson's ratio (ν) – 0.3
Air:Velocity of sound – 343.3 m/s, Density (ρ) – 1.2 kg/m^3
The external panel, i.e., panel 'a' is subjected to an oblique pressure loading expressed in the form as given,
$P(x, y, t) = 2P_0 \exp(j\omega t - jk_0 z\cos\lambda - jk_0 x\sin\lambda\cos\theta - jk_0 y\sin\lambda\sin\theta)$
where, P_0 is the amplitude of the incident pressure which is 1.0 Pa in the present work. λ, θ are the elevation angle and the azimuth angle (Fig. 2) with k_0 as the wave number.

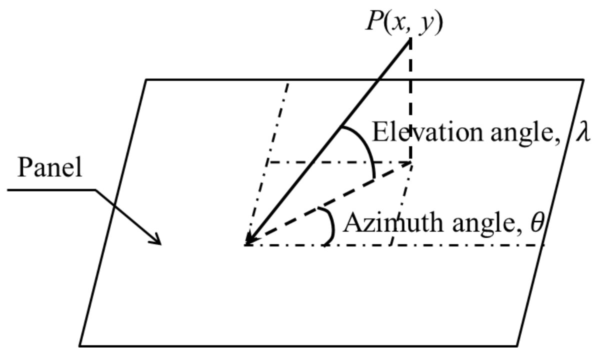

Figure 2: Illustration of the incident acoustic oblique plane wave

As has been reported in the earlier literature, to obtain the maximum transmitted energy through a flat panel, the azimuth angle and the elevation angle is assigned as 60^0 and 30^0, respectively. After performing an optimization study to locate the actuator and sensor positions on panel 'a' and panel 'b', a set of patch positions for the panels are identified and are shown in Fig. 3.

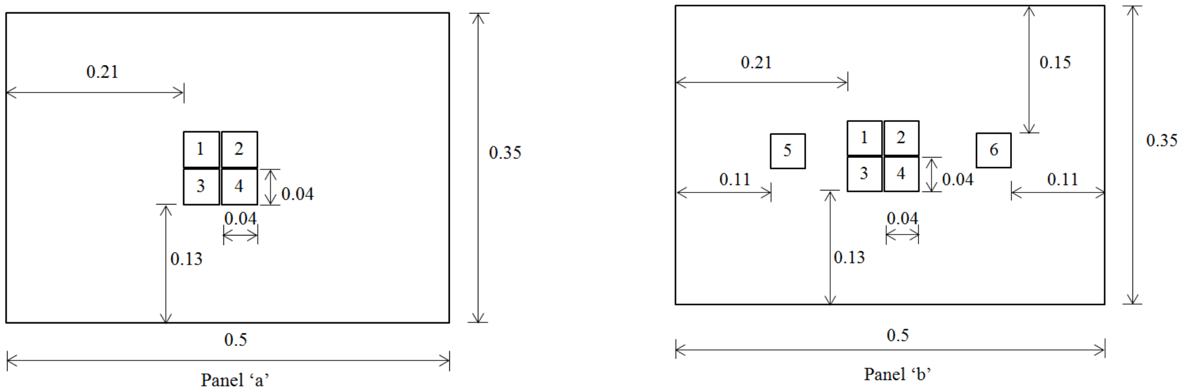

Figure 3. Optimized location of Sensor – Actuator Patches for the Panels

The numerical analysis on the developed model is performed and the average sound pressure level is calculated following Eq. 19. The averaged uncontrolled SPL and the controlled SPL obtained are presented in Fig. 4. It is important to mention in this context that in the frequency range of interest, i.e., between 0 – 200 Hz, all the piezo patches are not utilized at a time. The sequence in which the piezo patches are utilized are as follows -

1 – 80 Hz, Panel 'a' patch 1,2,3,4
81 – 106 Hz, Panel 'b' patch 1,2,3,4
107 – 200 Hz, Panel 'b' patch 5,6

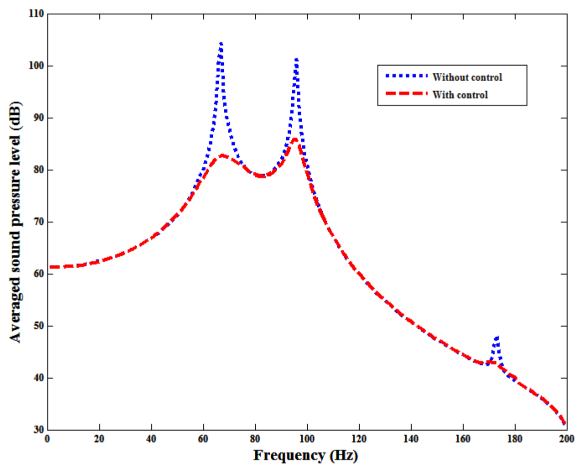

Figure 4 Uncontrolled and controlled averaged SPL in the enclosure

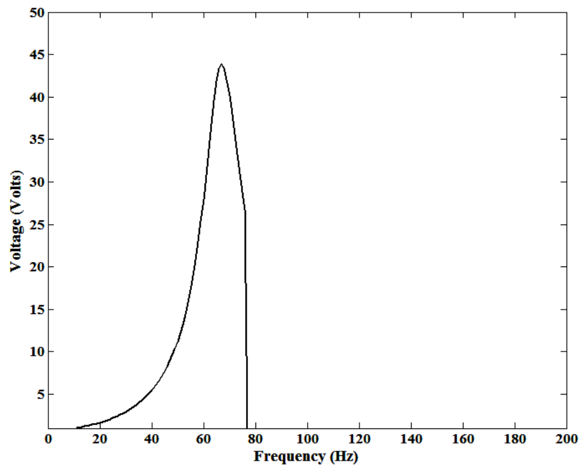

Figure 5 A typical voltage pattern for patch 1 (panel 'a') corresponding to coupled mode 1

A typical optimal voltage pattern obtained for one such piezo patch is shown in Fig. 5. It is noted that the the developed H_2 optimal controller with velocity feedback based on ASAC strategy appreciably could reduce the average SPL within the enclosure.

Conclusion

A state-space model of the acoustic transmission mechanism through a double wall panel based on modal expansion theorem is successfully implemented in the present work. An optimal frequency weighted H_2 controller based on ASAC strategy is subsequently developed for attenuation of the transmitted sound into an enclosure. A significant reduction in the SPL could be achieved with a distributed sensor-actuator network mounted on the structural panels. The need for piezo patches to be mounted on both the panels of the double wall is highlighted and the obtained results are discussed.

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