Proceedings of The Institute of Acoustics

ADAPTIVE CONTROL OF PERIODIC DISTURBANCES IN RESONANT SYSTEMS

P. Darlington and S.J. Elliott

Institute of Sound and Vibration Research
University of Southampton, Southampton SO9 5NH

ABSTRACT

The effects of the dynamics of an acoustic system upon the conventional LMS noise canceller are introduced by considering the control of a 2nd order network. Simulating the control of sinusoidal disturbance shows that a stability limit exists which is explained in terms of an analytical model of the adaptive system.

INTRODUCTION

"Adaptive Noise Cancelling" is a technique for the removal of additive noise from a signal. Early applications and the theoretical background of the technique were developed within the electronic engineering discipline [1]. This signal conditioning technique can be easily recast into a noise control problem in which the adaptive canceller attempts to control the behaviour of some system under control in a "Model/Reference" sense. The analysis of the Widrow Hoff LMS noise canceller has been widely reported in the context of electrical cancellation but, unfortunately, many potential applications of adaptive noise cancelling involve the control of complex dynamic systems. The performance of conventional adaptive cancelling techniques in attempting to control such systems is the subject of this paper.

ADAPTIVE CONTROL OF ACOUSTIC SYSTEMS

When adaptive techniques are applied to the control of distributed parameter systems, the duration of their impulse response becomes important. This impulse response has two important components. Firstly, as a result of transmission times between transducers, there are pure delay factors to be included in the description of the controlled system. These almost inevitably lead to a non-minimum phase structure. Secondly, if the distributed parameter system is bounded, it will display resonant/antiresonant characteristics which will influence the behaviour of the controller. Both these effects radically influence the convergence and stability criteria for an adaptive cancelling system. The behaviour of an adaptive canceller, controlling a second order system is reported, both by simulation and theoretical analysis, below. This yields general results which can be readily extended to predict the performance of noise cancellers controlling complex acoustic systems.

THE ADAPTIVE CANCELLING SYSTEM WITH A FILTER IN THE CONTROL LOOP

The concept of an adaptive canceller controlling the response of a resonant system can best be modelled in the conventional structural notation of Widrow [1] by including the system as a filter in the "control loop" [2] of the canceller, see fig. 1. In this position it is intuitively seen that:

ADAPTIVE CONTROL OF PERIODIC DISTURBANCES IN RESONANT SYSTEMS

1) The system under control, C, may influence the behaviour of the cancelling system by introducing memory into the "error" path

2) Signals uncorrelated with the reference input are not influenced by the cancelling system.

The signal present at the summing junction in the absence of the adaptive filter is conventionally referred to as the "primary" signal and in this case is composed of the "desired" input and the "signal" input which are perfectly correlated and uncorrelated respectively with the reference input.

The system of fig. 1 serves as the basis for an experimental computer simulation which is reported below.

ADAPTIVE CONTROL OF A 2ND ORDER NETWORK - SIMULATION

The system of fig 1 was implemented as a computer simulation in which all signals could be examined and recorded. The filter chosen was a second order resonator with natural frequency $f_o = 1/4T$, where T is the sampling period and damping ratio $\zeta = .2$. The parameters were obtained by the impulse invariant method, such that the transfer function was:

$$H(z) = \frac{1.46z^{-1}}{1-0.047z^{-1}+0.533z^{-2}} \quad (1)$$

The adaptive filter was a standard LMS controlled 11th order FIR device; the unusual choice of N=11 is discussed later. The adaption parameter was chosen as $\mu = .02$, and the filter was excited by the following reference and desired inputs and initial conditions:

$$x(k) = \cos(2\pi f_r k/T) \qquad d(k) = U(k)\frac{x(k-2)}{2}$$

where U(k) is the unit step fn.

No 'signal' input was used, and the adaptive filter weights were initialised to zero.

The reference normalized frequency, f_r, was an experimental variable.

The observed error response of the simulated system to the above inputs is shown below, for various reference normalized frequencies as figs. 2. The system is seen to be stable and convergent, successfully cancelling the periodic noise applied to the system under control up to a reference frequency of $\sim$ 0.3. At higher frequencies, the system is unstable.

The results of the computer simulation show that the presence of a simple linear filter in the control loop of an adaptive cancelling system can radically influence it's performance; this effect can be fully understood and predicted by the following analysis.

THE ADAPTIVE CANCELLING SYSTEM WITH A FILTER IN THE CONTROL LOOP - THEORETICAL ANALYSIS

In the seminal paper by Glover [3] it was demonstrated that an adaptive noise canceller with a periodic reference signal implemented a fixed linear comb filter for certain reference frequencies. This approach is extended in this section to include a fixed linear filter in the control loop.

The response of the adaptive filter can be written as [4]

$$y(k) = 2\mu \sum_{i=0}^{k-1} e(i) X_k^T X_{k-i} \qquad (2)$$

$$\text{where } X_k = \begin{matrix} x(k) \\ x(k-1) \\ \cdot \\ \cdot \\ x(k-N+1) \end{matrix}$$

and substituting for e(i) in fig. 1:

$$y(k) = 2\mu \sum_{i=0}^{k-1} \sum_{j=0}^{Q-1} c_j (d_{i-j} - y_{i-j}) X_k^T X_{k-i} \qquad (3)$$

where c_j is the jth element of the Qth order impulse response of the error filter. For certain reference frequencies [3,5] the reference autocovariance estimates become time invariant. At these frequencies, Z transforming 3 and rearranging gives a fixed linear transfer function between the observed error and the desired input:

$$\frac{E(z)}{D(z)} = \frac{1-2z^{-1}\cos\omega_0 T+z^{-2}}{1-2z^{-1}\cos\omega_0 T+z^{-2}+\mu N(z^{-1}\cos\omega_0 T-z^{-2})C(z)} \qquad (4)$$

exactly describing the behaviour of the adaptive system in the presence of the filter

ADAPTIVE CONTROL OF A 2ND ORDER NETWORK - THEORETICAL ANALYSIS

Given eqn 4 it is possible to predict the behaviour of the system simulated above. With the 11th order adaptive filter used in the simulation the reference autocovariance is not exactly time invariant at those frequencies used in the simulation, however, the theory used to develop eqn. 4 remains approximately true [3].

The convergence rate of the adaptive system with an error filter will be controlled only by the denominator of eqn. 4; the zeros effect the frequency response but not stability. Thus finding the roots of the denominator of eqn. 4 will describe the convergence rate of the system.

Substituting the parameters of eqn. 1 for C(z) in eqn. 4 and solving for the denominator roots numerically gives the following results. The pole locations for a normalised reference frequency of $f_r/f_o = 1$ are shown plotted as Fig 3. The pole pair outside the unit circle near the imaginary axis dominate the behaviour of the system.

The modulus of the dominant pole pair is shown plotted against reference frequency as fig. 4. The system is seen to be stable up to reference frequencies of $\sim$ .3, after which the dominant poles migrate outside the unit circle causing instability. The time constant of the instability is shortest at f_r= .5 - .6 after which it lengthens slightly. This effect is clearly seen in the results of the computer simulation (figs 2).

Note that the extreme frequencies of fig 4 (shown dotted) are coarser approximations than the centre of the plot, as the reference autocovariance becomes more time-variant at these frequencies [5].

The prediction of the performance of the noise cancelling system based on eqn 4 is seen to follow closely the results obtained from the computer simulation. To further examine the effects of the error filter several other filter designs were investigated having the same natural frequency, 1/4T, but with damping ratios ζ = .1, .2, .3, .4, .5 (see fig 5). Substituting the appropriate C(z)'s in eqn 4 and solving for the poles gives the required prediction of stability and convergence. The moduli of the dominant pole pairs are shown as fig. 6.

An increase in damping ratio is seen from fig 6 to generally reduce the radial position of the dominant pole pair, tending to make the system "more stable". This effect is most clearly seen at the natural frequency of the error filter, presumably due to the increased damping controlling the magnitude of the resonant effect.

The frequency at which the system becomes unstable appears to increase with increasing damping towards a limiting value. This is an effect caused by the phase shift across the error filter; the asymptote corresponds to approximately 70° of phase lag. This phase condition can be found from examining the effects of a pure delay element in the error path, using a similar analysis to that discussed above (i.e. substituting $C(z)=z^{-1}$ in eqn 4). For the case of a highly damped error filter, the stability criteria is governed by this phase asymptote; the filter will become unstable when the error filter produces 70° of phase lag. In more lightly damped situations the onset of instability is governed by a combination of the phase shift across the filter and the effects of the resonance.

For any damping ratio, given that the 70° phase condition is obeyed, it is possible to reduce the adaption parameter μ to such a value as to ensure stability; this effect of constraining the adaption of the filter so as to remove the influence of the dynamics of the system under control is then analogous to the "quasi static" approach to adaptive noise control in complicated systems.

CONCLUDING REMARKS

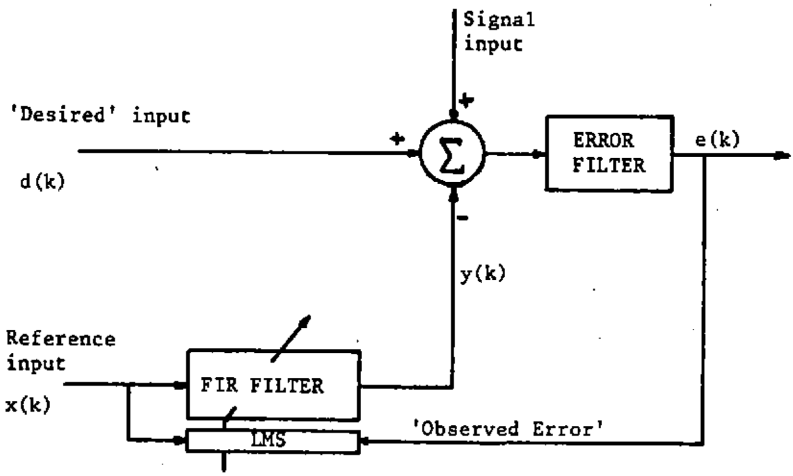
It has been demonstrated by simulation, supported by a theoretical discussion, that the performance of an adaptive noise cancelling system controlling a resonant system will be significantly different from that expected in a similar signal conditioning task. The pure phase delay across the system under control imposes a maximum cancelling frequency in highly damped systems whereas a lightly damped application is stable only to a lower reference frequency.

The multiresonant and mixed phase characteristics of typical acoustic systems may both be discussed in terms of such a simple error filter, explaining one of the difficulties of applying conventional adaptive cancelling strategies to acoustic noise control problems.

REFERENCES

[1] B. Widrow, J.R. Glover, et al, 'Adaptive noise cancelling; principles and applications', Proc. I.E.E.E., Vol. 63, no. 12, 1692-1716, (1975).
[2] D.R. Morgan, 'An analysis of multiple correlation cancellation loops with a filter in the auxiliary path', I.E.E.E. Trans. on A.S.S.P., Vol.ASSP-28, no.4, 454-467, (1980).
[3] J.R. Glover, 'Adaptive noise cancelling applied to sinusoidal interferences', I.E.E.E. Trans. on A.S.S.P., Vol. ASSP-25, no.6, 484-491, (1977).
[4] P.M. Clarkson, 'Adaptive approaches to signal enhancement and deconvolution', Ph.D. Thesis, University of Southampton, (1983).
[5] S.J. Elliott and P. Darlington, 'Adaptive cancellation of periodic, synchronously sampled interference', Accepted for publication in I.E.E.E. Trans. on A.S.S.P.

fig. 1 The adaptive noise canceller with an error filter



ADAPTIVE CONTROL OF PERIODIC DISTURBANCES IN RESONANT SYSTEMS

fig. 2 Response of the LMS noise canceller + 2nd. order error filter to a sinusoidal input.
(f_0 = .5, ζ = .2)

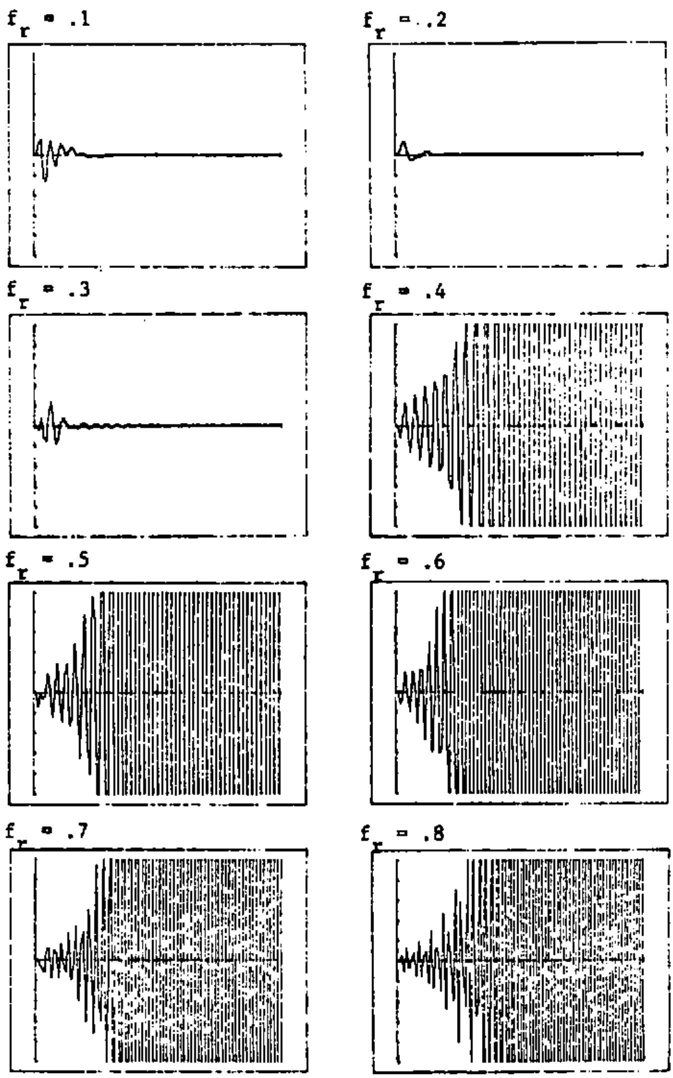


fig. 3
POLE LOCATIONS
(ζ = .2, f_r/f_0=1)

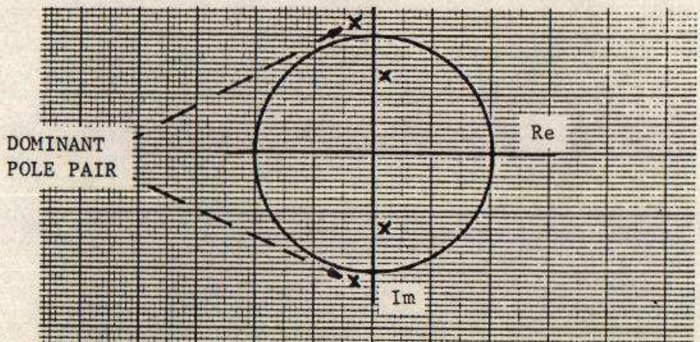


fig. 4
MODULUS OF DOMINANT
POLE PAIR
(ζ = .2)

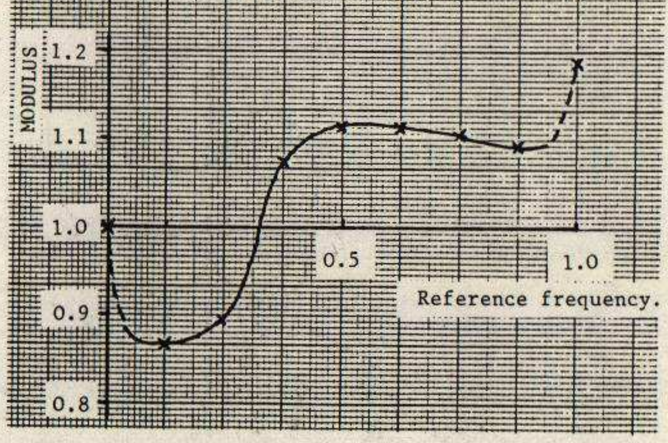


fig. 5 Error filter frequency responses (f_0=.5, ζ=.1, .2, .3, .4, .5)

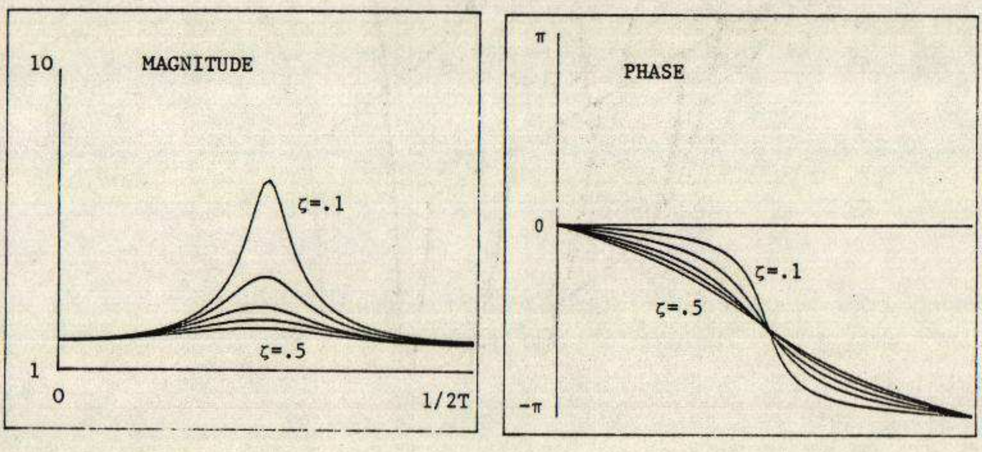


fig. 6 Modulus of dominant pole pair.
(Second order error filter, $f_0 = 1/4T$)