Proceedings of the Institute of Acoustics

APPROXIMATING ARBITRARY LENGTH fBm SEQUENCES USING AN ADAPTIVE SPECTRAL SHAPING FILTER

P. DARLINGTON (1) & Y. WANG (2)

(1) Department of Applied Acoustics, University of Salford, Salford, M5 4WT, U.K.
(2) Department of Electrical Engineering, University of Wyoming Laramie, WY., 82071, U.S.A.

0. ABSTRACT

This paper describes a new class of adaptive filter. The filter attempts to match the variance spectrum of its output to a desired spectrum vector, specified by the user. The filtering operation is performed in the time domain, although the system is "programmed" and adapts in the frequency domain. This cross-domain operation is not typical of previously reported systems. The finite order approximation of arbitrary length fractional Brownian motion ("fBm") noise sequences is described as an application of the adaptive system.

1. INTRODUCTION

Although early applications of adaptive signal processing were developed in the time domain, the concept of "frequency domain adaptive filters" [1] has produced a useful set of devices now commercially exploited in equalisation systems etc.. These systems offer one approach to the realisation of adaptive filters having an autoregressive impulse response (or, at least, "AR-like") and this quality motivated much of the initial research into frequency domain implementations.

This paper describes a new system in which a transversal (time domain) adaptive filter's weights are updated according to a "stochastic gradient" [2] search of a frequency domain performance surface. The filter attempts to match the variance spectrum of its output with a "desired" spectrum input, applied to the system by the user as a programming input.

Since the system is only attempting to match the power spectrum of its output with a desired spectrum, the adaptive filter does not exploit or need any correlation between the signals at its "reference" and "desired" inputs. This is markedly different from most adaptive filtering algorithms, which usually rely upon such correlation.

2. THE ADAPTIVE SPECTRAL SHAPING ALGORITHM

Given a finite order transveral filter, characterised by its impulse response vector W, length N, the filter's response can be written as:

$$\underline{y}_k = [X_k]\underline{W} \qquad (1)$$

where $\underline{y}_k$ is the vector of the N most recent outputs, y:

$$\underline{y}_k = [\, y_k,\ y_{k-1},\ \ldots\ldots\ y_{k-(N-1)}\,]^T$$

and $[X_k]$ is a matrix of filter input samples:

$$[X_k] = \begin{bmatrix} x_k & x_{k-1} & x_{k-2} & \ldots\ldots & x_{k-(N-1)} \\ x_{k-1} & x_{k-2} & \ldots\ldots\ldots & & x_{k-N} \\ \ldots & \ldots & \ldots & \ldots & \\ \ldots & & & & \\ \ldots & & & & \\ x_{k-(N-1)} & \ldots & \ldots & \ldots & x_{k-2(N-1)} \end{bmatrix}$$

Equation 1, whilst valid for a fixed weight vector, <u>is approximately true of an adaptive system, provided that the update speed is small</u>.

The frequency content of the output vector $\underline{y}_k$ can be evaluated by applying a discrete Fourier transform, using a DFT matrix [3]:

$$\underline{Y} = [F]\ \underline{y}_k \qquad (2)$$

where $\underline{Y}$ is the vector representing the DFT of y (N elements) and [F] is the DFT matrix. The nth. element of $\underline{Y}$ may be evaluated using:

$$Y_n = F_n^T \underline{y}_k \qquad (3)$$

where F_n^T is the nth. row of the DFT matrix.

A raw estimate of the power content of y lying in spectral bin n is given by:

$$\begin{aligned} S_n &= Y_n\ Y_n^* \\ &= F_n^T \underline{y}_k\ .\ F_n^{\dagger} \underline{y}_k \qquad (4) \end{aligned}$$

where * denotes complex conjgate and † denotes conjugate transpose.

If the output desired output spectrum is specified by a vector $\underline{D}$ (which has N elements) then the raw spectral error in bin n is given by:

$$\begin{aligned} e_n &= d_n - S_n \\ &= d_n - \underline{y_k}^T F_n F_n^{\dagger} \underline{y_k} \end{aligned} \qquad (5)$$

A positive definite measure of spectral error is obtained by squaring equation 5:

$$\begin{aligned} e_n^2 = d_n^2 - 2d_n \underline{y_k}^T F_n F_n^{\dagger} \underline{y_k} + \\ (\underline{y_k}^T F_n F_n^{\dagger} \underline{y_k})^2 \end{aligned} \qquad (6)$$

A standard approach to the design of adaptive filter update algorithms is the "stochastic gradient" technique, in which the controllable parameters of the filter are adjusted according to the gradient of an instantaneous cost function.

In the adaptive spectral shaping application, it is required to minimise the mean square spectral error in each of the N spectral bins. Using the stochastic gradient method, an update algorithm for the filter weights W is defined from the gradient of the instantaneous spectral error, equation 6.

Substituting for $\underline{y_k}$ in equation 6, from equation 1, gives:

$$\begin{aligned} e_n^2 = d_n^2 - 2d_n W_k^T [X_k]^T F_n F_n^{\dagger} [X_k] W_k \\ + (W_k^T [X_k]^T F_n F_n^{\dagger} [X_k] W_k)^2 \end{aligned} \qquad (7)$$

which can be differentiated with respect to the controllable weight vector, W, to yield:

$$d/dW_k \; (e_n^2) = -2\{y_n [X_k]^T F_n^* \} e_n \qquad (8)$$

The equation above describes the gradient of the spectral error in bin n with respect to the time domain weights W. In order to minimise the error in each spectral bin, the total update applied to the weight vector must comprise a contribution from each bin (i.e. from each value of frequency index n). The simplest approach uses an unweighted summation over frequency to define the total update :

$$W_{k+1} = W_k + \alpha \sum_n y_n [X_k]^T F_n^* e_n \qquad (9)$$

where α is the conventional update speed parameter. Stability of the update process is dependant upon the statistics of the x input sequence, the desired spectrum, filter order and choice of alpha. The adaption rates used in simulations of the system were chosen by trial and error - in view of the complexity of the update algorithm, it is suggested that an exact analysis of the stable bounds of the system would be intractable.

Although equation 9 represents the simplest possible update equation for the stochastic gradient spectral shaping filter, the authors have evaluated several modifications, the most important of which are described below.

2.1 Filter Update Rate

Although equation 9 defines a filter update step for each iteration index, the authors have found that it is possible for the system to converge to acceptable solutions if the weights are updated at frequencies lower than 1 update / N samples.

This reduced update rate also allows for a limited amount of smoothing on the power spectral estimates, which appears to assist the adaptive filter's convergence speed (such smoothing of the cost function reduces the update noise inherant in the stochastic gradient approach).

2.2 Data Windowing

The windows applied to the data before transforming in the analysis above are all rectangular. This can introduce spectral leakage from adjacent bins which limits the performance of the spectral shaping system. Laboratory simulations of the adaptive system applied standard Hanning windows to the data vectors before the DFT operations.

The structure used to implement an adaptive filter controlled by an update algorithm such as equation 9 is shown in figure 1.

3. EXAMPLE - APPROXIMATING fBm SEQUENCES

One dimensional "fractional Brownian motion" noise sequences may be characterised by their auto power spectra, which have form:

$$S(f) = A\, f^{-\beta} \qquad (10)$$

where A is a positive real scaling factor and β is the "spectral exponent". Although such noise sequences are widespread in biological, physical and electronic systems, the mechanisms responsible for their generation are not well understood. It is not, for example, possible to derive a finite order generating filter which will operate upon a white noise input to generate 1/f noise [4].

Current techniques for generating these noise sequences, of which the "Inverse FFT" method is the most widely used [5], yield only finite length results. If a longer noise sequence is required then either a larger FFT must be used, or multiple records must be spliced together. The approximation of fBm sequences is an ideal application for the adaptive spectral shaping filter described above, particularly for small values of spectral exponent.

If the desired noise spectrum is applied to the "D" input of the shaping filter and white noise, <u>or any reasonably flat noise sequence</u>, is applied at the "x" input then, after convergence, the filter produces a finite order estimate of the specified noise. The sequence length is only limited by the length of the input noise sequence, which could be derived from a random number generator and so be of effectively infinite length.

An example of the power spectrum of the output of the adaptive spectral shaping filter when approximating 1/f noise is presented as Figure 2. The data presented in Figure 2 was generated by an adaptive filter having 32 controllable weights, giving 15 controllable frequency bins (plus d.c. and half sample frequency). The low frequency bins were power limited, as the infinite power required at d.c. is obviously impractical.

The order of the filter used in the spectral shaping system can be increased to give a larger number of controllable frequency bins, although the order is effectively limited by the speed of the hardware available; the update algorithm, 9, represents a considerable computational load.

4. CONCLUDING REMARKS

It has been demonstrated that it is possible to control the weights of an adaptive transversal filter using a stochastic gradient search of a frequency domain performance surface. Although the authors have considered only the spectral shaping application described in this paper, it it possible that such an approach could be usefully applied in other adaptive modelling and estimation applications; there remains considerable scope for further investigation of the "cross domain" adaptive filter architecture.

REFERENCES

[1] M DENTINO, J McCOOL and B WIDROW, "Adaptive Filtering in the Frequency Domain", Proc. IEEE, 66, p1658 (1978).
[2] M L HONIG and D G MESSERSCHMITT, Adaptive Filters: Structures, Algorithms, and Applications, Kluwer, 1984.
[3] D MANSOUR and A H GRAY, "Unconstrained Frequency-Domain Adaptive Filter", IEEE Trans. Acoust., Speech, Signal Processing, ASSP-30, p726, (1982)
[4] R F VOSS, "1/f (flicker) noise: A brief review", Proc. 32nd. Annual Symposium on Frequency Control, Atlantic City, p40, (1979)
[5] H-O PEITGEN and D SAUPE (editors), The Science of Fractal Images, Springer-Verlag, (1988)

ACKNOWLEDGEMENTS

The authors gratefully acknowledge financial support from the Department of Electrical Engineering, University of Wyoming, and from Lucas CEL Instruments Ltd..

Figure 1. Adaptive Spectral Shaping System

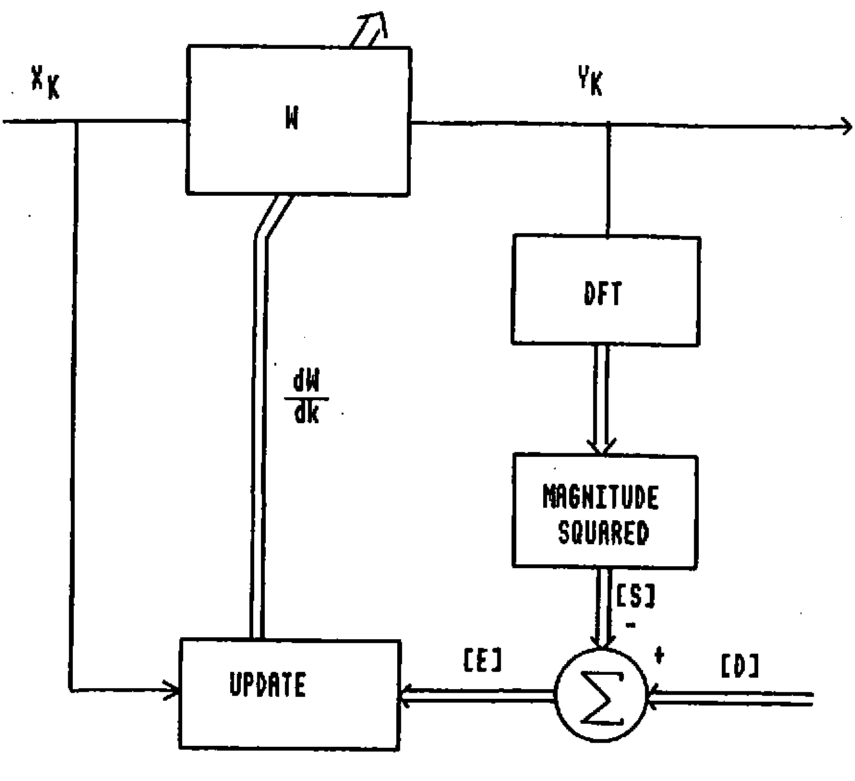

Figure 2. Power Spectrum of 31st. Order Estimate of 1/f Noise
(white noise used as input)